"*p*-Beauty Contest" With Differently Informed Players: An Experimental Study

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Abstract

The beauty contest stems from Keyne's famous book where he uses a baeuty contest game to illustrate how investors make their decisions in financial markets. In this paper we analyse an experimental p-beauty contest. In this game players choose a number from the closed interval. The winner of the game is a players whose chosen number is closest to 2/3 of the mean of all chosen numbers. In this game players have to eliminate dominated strategies, and the game has the unique equilibrium. Players are divided in different rationality levels according to their ability to make iterated elimination of dominated strategies. More precisely, we calculate rationality level of players by using the logic of iterated best responses. The game is played in two rounds with large number of players. In the first round all players are symmetric, whereas in the second round we divide players into three groups: uninformed, semi-informed and informed. Our objective is to study the impact of different information on players' decisions. Uninformed players don't know the previous round mean, semi-informed players know the previous round mean and informed players obtain information about previous round mean and equilibrium outcome. We find the fastest convergence to equilibrium for informed players, followed by semi-informed and uninformed. We also find that informed players significantly increase their depth of reasoning in the second round, while uninformed and semi-informed players do not increase significantly their depth of reasoning in the second round.

Key Words: Behavioural economics; Dominance solvable games; Iterated best responses; Nash equilibrium; Level of rationality

JEL Classification: D03, C70

1. INTRODUCTION

The beauty contest stems from Keyne's (1936) famous book where he describes the following game to illustrate how investors make their decisions in financial markets. Competitors have to choose six prettiest faces from a hundred of photographs. The winner of the game is a player who has chosen a person that is the most beautiful to all competitors. In this game a player has to form beliefs about average choices of others and about what average beliefs expect average beliefs to be. This game resembles on stock market where investors have to deduce which stocks will be most attractive to the majority of investors.

The game which replicates the Keynes's idea is called *p*-beauty contest and it is studied experimentally by several researchers. In this game players choose a number from the closed interval [0,100]. The winner of the game is a player who chooses the number closest to the mean of all chosen numbers multiplied by a fixed parameter *p*. Parameter *p* is a scale parameter that could by less or higher than 1, and this parameter is common knowledge. The winner receives a fixed prize which is independent of the chosen number and the scale parameter *p*. In the case when more than one player chooses the same winning number, the prize is divided equally among the winners. This game is repeated several times, and after each round the mean, *p* times the mean, the winning numbers, and the payoffs are presented to players.

This game has a unique equilibrium in which the equilibrium strategy of each player is to choose 0. In order to find the equilibrium strategy, a player must eliminate dominated strategies infinitely many times. To illustrate the argument, suppose that $p=2/3$. If all players choose 100, then 2/3 times the mean could not be higher than 67. Thus, it is not rational for a player to choose a number higher than 67, because this strategy is dominated by a strategy of choosing a number in the interval [0,67]. A player who eliminates numbers higher than 67 obeys one step of iterated dominance. If a player thinks that others obey one step of iterated dominance and choose 67, he can conclude that 2/3 times the mean could not be higher than 44. Therefore, a player who chooses a number in the interval [0,44] obeys two steps of iterated dominance, and a player who chooses a number in the interval [44,67] obeys one step of iterated dominance, but not two. By induction, a player who obeys three steps of iterated dominance chooses a number in the interval [0,29], and a player who chooses a number in the interval [29,44] obeys two steps of iterated dominance, but not three. Proceeding in this way by infinitely eliminating dominated strategies, we find the unique equilibrium in which the optimal strategy for each player is to choose 0. Games in which iterated elimination of dominated strategies leads us to a unique equilibrium are called *dominance solvable games*. Extensive treatment of behavioural game theory and in particular of dominance solvable games can be found in Camerer (2003).

In this paper we will analyse a *p*-beauty contest game which is played in two rounds with large number of players. We employ two rounds in order to study the impact of different levels of information on the speed of convergence and the dynamics of players' rationality levels. In contrast to other papers who study only the speed of convergence to equilibrium in several rounds with symmetricaly informed players, we study the *impact of information on different paths leading to equilibrium*. We use only two rounds since the comparison of results of these two rounds unambigously show the tendency of differents paths of convergence and rationality levels for the three types of differently informed players that we introduce. In the first round all players are symmetric and we differentiate players in the second round. We make a distinction between uninformed players who are completely ignorant of the previous round results, semi-informed players who receive information about the previous round mean, and informed players who obtain information about the equilibrium outcome in addition to the previous round mean. We find the fastest convergence to equilibrium among the informed players followed by semi-informed. The uninformed players exhibit the slowest convergence. The other important finding of our paper is that the depth of reasoning does not increase significantly from round 1 to round 2 for uninformed and semi-informed players, but increases significantly for informed players.

The rest of the paper is organised as follows. In the second part we review results of previous research. In the next section we give a brief description of the experimental design. The third section deals with the results of experimental research and the last section concludes the paper.

2. PREVIUOS RESEARCH

The first paper dealing with experimental examination of behaviour in beauty contest games was Nagel (1995). In this experiment players have to choose a number from the interval [0,100] and the game is repeated in 4 rounds. In the first round a player has to form beliefs about choices of other players on a different basis than in other rounds. In subsequent rounds a player can update his prior beliefs by using the data about choices of other players in previous rounds. Nagel (1995) uses three different values of the threshold parameter *p*: 1/2, 2/3 and 4/3. Experimental groups consisted of 15-18 players. The winner in each round received a fixed prize.

In the first round none of the players chose the equilibrium strategy 0 in games with p=1/2 and p=2/3 and 6% chose numbers below 10. However, a small fraction of players chose strategies that violate one step of iterated dominance in the first round. In the game with p=1/2, 14% chose 50 or a higher number, and in the p=2/3 game 16% chose 66, 67 or a higher number. In the subsequent rounds the behaviour of players converged towards the equilibrium. In the fourth round more than a half of players chose a number less than 1 in the p=1/2 game, but only 6% chose 0. In the p=2/3 game only one player chose a number less than 1.

Nagel (1995) observes that strategies converge faster to equilibrium for smaller values of *p*. She defines the rate of decrease of means and medians from round 1 to round 4:

$$w_{mean} = \frac{(mean)_{t=1} - (mean)_{t=4}}{(mean)_{t=1}}, \quad w_{med} = \frac{(median)_{t=1} - (median)_{t=4}}{(median)_{t=1}}. \tag{1}$$

The rate of decrease of medians is statistically significantly higher in the p=1/2 game than in the p=2/3 game. On the other hand, the rates of decrease of means are not significantly different in these 2 games.

Nagel (1995) groups players according to their ability to make steps of iterated elimination of dominated strategies. She defines the depth of reasoning *d* as follows:

$$x_i(t) = \mu_{t-1} p^d, \tag{2}$$

where $x_i(t)$ is a choice of subject *i* in round *t*, and μ_{t-1} is the mean from the previous round. Nagel (1995) classifies chosen numbers in intervals $[50 p^{d+1/4}, 50 p^{d-1/4}]$ by using the logic of *iterated best responses* to determine the boundaries of intervals. The concept of iterated best responses assumes that one player thinks that other players use one level lower depth of reasoning. For example, a level 0 player chooses a random number from the interval [0,100] and the expected number chosen by this type of player is 50. Level 1 player chooses a best reply given his belief that other players are level 0 players. His best reply is to choose 50*p*. Level 2 player chooses a best reply given his belief that other players are level 1 players. His best reply is to choose $50p^2$. By induction, a level *k* player chooses a best reply given his belief that other players are level *k*-1 players. For example, for *d*=1 and *p*=2/3 the best response of level 1 player is to choose 33,33 and the above interval is [30,37] which means that subjects that have chosen numbers in this closed interval are level 1 players. Likewise, for *d*=2 this interval is [20,25] and all choices in this interval are from level 2 players and so on. Numbers between these intervals are classified as neighborhood intervals. Nagel (1995) finds that the majority of subjects are level 1 and level 2 players. Even in the further rounds of play players do not increase their depth of reasoning. Only in the fourth round of the *p*=1/2 game 10% of players are classified as level 3 players.

Finally, Nagel (1995) tests the learning direction theory by defining the adjustment factor as the relative deviation of the chosen number from the previous round mean and compares that adjustment factor with the optimal adjustment factor defined as $p \cdot (mean)_t / (mean)_{t-1}$. She finds that majority of players increase their adjustment factor if it was lower than the optimal adjustment factor in the previous round and decrease the adjustment factor if it was higher than the optimal adjustment factor.

Duffy and Nagel (1997) study *p*=1/2 games in which players have to guess one half of the *mean, median or the maximum*. The game is repeated in 10 rounds. Outliers have no impact on the median as they have on the mean and faster convergence to equilibrium is expected in the median game. In the maximum game players focus their attention on outliers and the convergence, if it exists at all, will be the slowest among the three games. In the first round there is no significant difference in choices between mean and median games. On the other hand, there exists a significant difference in choices between mean and maximum games and between median and maximum games. In other words, chosen numbers in the maximum game are higher than choices in the mean or median games in the first round. In the following rounds chosen numbers in median and mean games are lower than in maximum game and there is a faster convergence to equilibrium in the median game than in the mean game, while choices in the maximum game show no apparent trend. Duffy and Nagel (1997) find no evidence that subjects increase their depth of reasoning in first four rounds of the three games. There exists only an increase in depth of reasoning in rounds 7-10 of the median game.

Ho, Camerer and Weigelt (1998) study *finite and infinite threshold games*. In finite threshold games the parameter p is higher than 1 and in the Nash equilibrium players choose the highest number. For example if players have to choose numbers from the interval [100,200] and p=1,3, players who choose numbers from the interval [100,130] violate one step of iterated dominance. Likewise, players who choose numbers from the interval [130,169] obey one step of iterated dominance but not two. Finally, players who choose a number from the interval [169,200] obey two steps of iterated dominance. Therefore, in this game finite steps of iterated elimination of dominated strategies are needed to ensure convergence to equilibrium. On the other hand, in games with p<1, infinite steps of iterated elimination of dominated strategies are needed to ensure convergence to equilibrium. They find that numbers chosen in the first round are far from equilibrium and that chosen numbers converge to equilibrium in further rounds. However, chosen numbers in the first round are closer to equilibrium in finite threshold games than in infinite threshold games.

Ho, Camerer and Weigelt (1998) study the effect of group size on convergence to equilibrium. Large groups consisting of 7 players choose higher numbers in earlier rounds and exhibit faster convergence to equilibrium in both type of games than groups consisting of 3 players. This result is counter intuitive, because it would be natural to expect faster convergence in small groups where players have higher impact on the mean.

Ho, Camerer and Weigelt (1998) compare choices of subjects with different levels of experience. Subjects with experience in an infinite threshold game participated in a finite threshold game, and subjects with experience in a finite threshold game participated in an infinite threshold game. The chosen numbers of experienced players were not significantly different from choices of unexperienced players in the first round, but choices of experienced players converge faster to equilibrium.

Ho, Camerer and Weigelt (1998) construct probability density functions that one player estimates for choices of other players and use these probability density functions to construct log likelihood function. They get a more precise estimate of the depth of reasoning of players than Nagel (1995). They find more level 0 players and fewer level 2 and level 3 players.

Güth, Kocher and Sutter (2002) study *interior and boundary equilibria with homogenous and heterogeneous players*. In a boundary equilibrium the equilibrium strategy is to choose the lowest or the highest number. If a constant is added to the mean and this sum is multiplied by *p*, we obtain an interior equilibrium. In an interior equilibrium the equilibrium strategy is to choose a number from the open interval (0,100). When all players have the same parameter *p*, players are homogenous. In the case of heterogeneous players, half of the players has one value of *p* and the other half has different value of *p*. An additional feature of this game is the continuous payoff function for each player defined as:

$$\Pi_i(s_i) = W - c\,|\,s_i - p[\mu + k]\,|, \tag{3}$$

where *k* is a constant which is added to the mean, *W* is the amount of money that a player receives at the beginning of the game and *c* is a cost of a unit deviation from the target number. With these modifications they wanted to make the game more similar to stock markets where payoffs are continuous, interior equilibria are more frequent than boundary equilibria and investors are asymmetric rather than symmetric

Güth, Kocher and Sutter (2002) find that in all 10 rounds of play choices in the game with homogenous players and interior equilibrium are closer to the target number than choices in a game with boundary equilibrium. Choices of heterogeneous players are closer to equilibrium only in first 5 rounds. Moreover, the equilibrium choices of homogenous players are two times more frequent in games with interior than in games with boundary equilibrium. In both cases the equilibrium choices are incomparably more frequent in these two games with continuous payoffs than in Nagel’s (1995) game where only the winner receives the prize. Finally, Güth, Kocher and Sutter (2002) find that heterogeneous players need more time to choose a number than homogenous players, but choices of homogenous players are closer to equilibrium.

Kocher and Sutter (2005) study beauty contest games with *p*=2/3 in which players are *individuals or groups of three person*. In the game with individuals only individuals take part, whereas in group games only groups participate. The game is repeated in four rounds. In the first round the mean and median in the group game are smaller than in the individual game, but the difference between choices in the two games is not statistically significant. However, in rounds 2-4 groups choose significantly lower numbers than individuals, which indicates that groups learn faster.

Kocher and Sutter (2005) find that there exists an increase in depth of reasoning in group games from round 1 to round 2 and from round 2 to round 3, but there is no further increase in depth of reasoning from round 3 to round 4. On the other hand, individuals show no increase in depth of reasoning from round 1 to 3, but they only exhibit an increase in depth of reasoning from round 3 to round 4. One possible explanation for this result is that groups expect other groups to reason deeper than individuals expect other individuals to reason. To clarify this phenomenon, Kocher and Sutter (2005) run a second experiment in which individuals compete against groups. In the first round of this experiment groups chose higher numbers than individuals, but in rounds 2-4 groups chose lower numbers than individuals. In round 1 individuals win more often than groups, but in round 2 groups win more often than individuals. In rounds 3 and 4 groups win more often than individuals, but the difference is not statistically significant. Averaged over all rounds, groups win more often and have higher payoffs than individuals and this difference is statistically significant.

Sutter (2005) compares *groups of different size* in a beauty contest game with p=2/3 and four rounds of play. In fact, he compares performances of groups consisting of 4 players with groups of 2 players and an individual. In the first three rounds, numbers chosen by teams of 4 players were lower than numbers chosen by 2 player teams and individuals. In round 4, numbers chosen by individuals were higher than numbers chosen by either team size, with no significant difference between choices of 2 and 4 player teams. In this experiment, larger groups clearly outperform smaller groups and individuals, with groups of 4 player winning in 58 cases, groups of 2 players winning in 32 cases and individuals winning in 31 case.

Burnham et al. (2009) study the impact of cognitive ability of subjects on the choice of first-order dominated strategies in a beauty contest game. They find that in the pool of subjects with the highest cognitive abilities only 7 out of 142 violate dominance and in the pool of subjects with the lowest cognitive abilities 58 out of 150 violate dominance. This study finds that there exists a negative correlation between cognitive abilities and the distance of the chosen number from the Nash equilibrium. More precisely, one standard deviation decrease in the cognitive ability leads to 7-10 points decrease in the chosen number. One possible interpretation of this result is that people with higher cognitive abilities have more precise beliefs about choices of other players. The other explanation is that people with lower cognitive abilities have computational difficulties to calculate the equilibrium strategy.

Bosch-Domènech et al (2002) study beauty contest game in two daily business *newspapers*: the Financial Times in UK and Expansión in Spain. This experiment differs from laboratory experiments in some important aspects. First, larger number of heterogeneous players can participate and they have more time to make their choices. Second, players have an option to run a parallel game among their friends and use that experience in the original game. The experiment shows that players who run a parallel experiment were closer to the winning number. Finally, in the newspaper version of the experiment there is a possibility for players to form coalitions which is excluded in laboratory experiments.

In this experiment players take part in a p=2/3 game. If they use iterated best responses, the frequency distribution of choices should have spikes at 33.33, 22.22, 14.81, 9.88 and so on. The results of the experiment show that this is exactly the case. Frequency distribution has spikes at 33.33, 22.22 and 0. This indicates that there is a concentration of level 1 and level 2 players.

Another feature of this experiment is that players were supposed to write their comments about choices they made. 64% of players said that they used iterated best responses, but 15% of them are level 0 players. Furthermore, 81% of players who have calculated the Nash equilibrium strategy have chosen a larger number than 0. This behaviour could be explained by the fact that rational players expect others to behave irrationally. Players in the newspaper version of the game had more time to make a choice, but this option had no considerable effect on the number of equilibrium choices compared to laboratory experiments. The only effect was an increase in the number of comments describing the equilibrium.

3. THE EXPERIMENTAL DESIGN

In our experiment we conducted a p=2/3 beauty contest game in which players had to choose an ordinary number from the closed interval [1,100]. Subjects were second year undergraduate students from the Faculty of Economics in Belgrade with some background in economics and no knowledge of game theory. The game was played in two rounds. In the first round all players were symmetric and this round was conducted in 10 separate sessions in which a total of 737 individuals participated. Payoffs were discontinuous where only the winner received the fixed prize, and other players received no reward.

In the second round we introduce differently informed players. Uninformed players had no feedback information concerning the results of the previous round. The second group we call semi-informed players who had information about the previous round mean and the target number. This is the group that was most commonly used in previous research. The third group consisted of informed players whose information set contained the same information as the information set of semi-informed players and they were also informed about the Nash equilibrium of this game. We also gave some basic information about non-cooperative game theory to this group. Second round was conducted in three sessions having one type of players in each competing just among themselves. We run three sessions in the second round in the following order: uninformed, semi-informed and informed, thus preventing information externalities. In contrast to usual practice, we conducted second round several days later. The reason was to give some time to uninformed players to see whether this will significantly change their beliefs.

Typical session was conducted as follows. Players received a written instruction where they were told about the design of the game (instructions available at the request from the authors). On top of this, instructor read the instruction, clarified the game, gave one example and answered the questions, if there were any. Subjects were seated in a way that prevented any possible communication.

4. EXPERIMENTAL RESULTS

In this part we will present results of our experimental research. First, we give results about first round choices and first round rationality levels when all players are symmetric. After that we focus our attention on second round choices of differently informed players, the speed of convergence of their choices and examine the dynamics of their rationality levels.

First round choices and rationality levels

In the first round subjects had no information concerning the behaviour of other subjects and they had to base their guess solely on their beliefs, and it is natural to expect that guesses of subjects are further from equilibrium in first round than in second round.

The experimental results show that 28 out of 737 subjects (3,8%) chose dominated strategies, i.e. they violate first-order iterated dominance. 27 from the sample of 737 subjects (3,66%) chose a number not higher than 10. There is only one individual who chose equilibrium strategy of 1. Figure 1 shows the frequency distribution of first round choices. The frequency distribution is asymmetric and the K-S test rejects the null hypothesis of normal distribution.

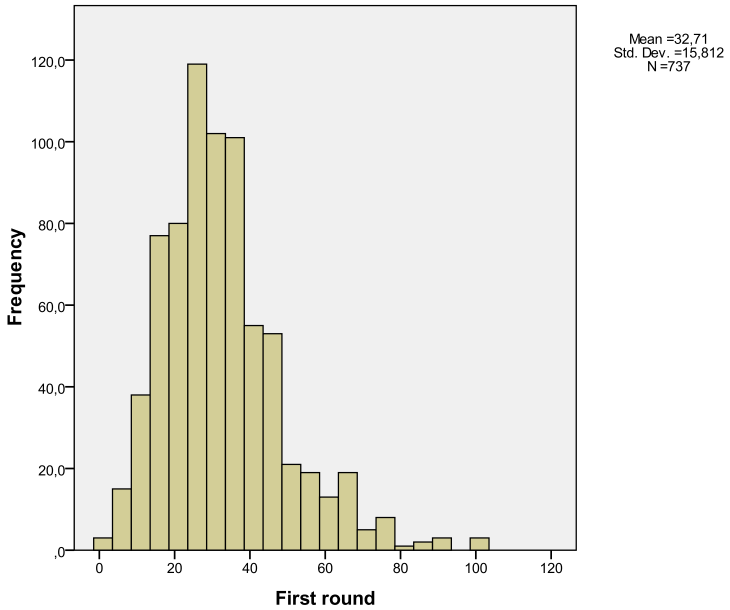

Figure 1. *Histogram of frequency distribution of first round choices*

The mean of the first round was 32,71, median was 30 and the target number was 22. There were 20 winners of the first round who guessed the target number.

We calculated rationality level of players by using Nagel's (1995) methodology of iterated best responses where subjects are divided into intervals in the following fashion. Subject has a level *d* of depth of reasoning if his choice falls in the interval: $[50{,}5\, p^{d+1/4}, 50{,}5\, p^{d-1/4}]$, for *d*=0,1,2,… We will clarify the intuition behind this classification. We take 50,5 as an initial reference point, since a player who chooses randomly from the interval will on average choose expected value of 50,5. This player has a level 0 of rationality. We construct interval $[50{,}5\,(2/3)^{0+1/4}, 50{,}5\,(2/3)^{0-1/4}]$ to capture all choices of players with level 0 of rationality. Player with level 1 of rationality thinks that other players are level 0 players and he will choose the best reply given his belief, i.e. he will choose $50{,}5\,(2/3)^1 = 33{,}33$. We construct interval

$[50{,}5\,(2/3)^{1+1/4}, 50{,}5\,(2/3)^{1-1/4}]$ to capture all level 1 players. Level 2 players think that others are level 1 players who choose 33,33 and level 2 players best reply given this belief by choosing $50{,}5\,(2/3)^2 = 22{,}22$. All players who fall in the interval $[50{,}5\,(2/3)^{2+1/4}, 50{,}5\,(2/3)^{2-1/4}]$ are level 2 players and so on. Intervals between these boundaries are called interim intervals.

Unlike Nagel's (1995) classification of intervals, we introduced two changes. First, we introduced irrational players who have chosen first-order dominated strategies. Second, we used the above formula for all levels in contrast to Nagel (1995) who uses the upper bound of 50 for level 0 players. The classification of levels of rationality according to the previous procedure is shown in Table 1.

Interval classification for first round			
Interval name	*Interval*	*Frequency*	*Percentage*
Irrational	67-100	28	3,8%
Interim 0	57-66	33	4,5%
Level 0	46-56	55	7,5%
Interim 1	38-45	100	13,6%
Level 1	30-37	171	23,2%
Interim 2	26-29	90	12,2%
Level 2	20-25	116	15,7%
Interim 3	18-19	24	3,3%
Level 3	14-17	64	8,7%
Interim 4	12-13	24	3,3%
Level 4	9-11	14	1,9%
Level 4+	0-8	18	2,4%

Table 1. *Rationality levels in first round*

The distribution of first round levels of rationality in our experiment is shown in Figure 2.

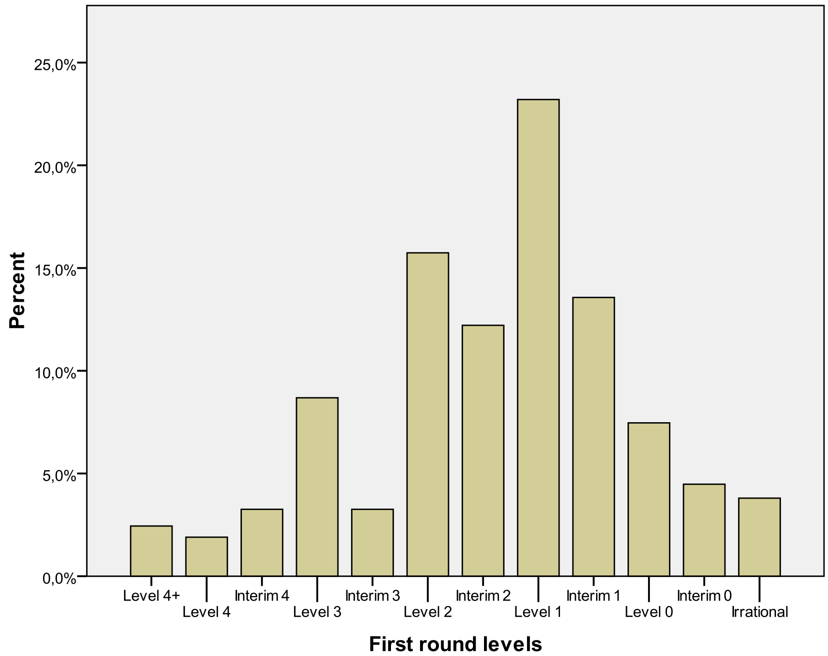

Figure 2. *Histogram of distribution of first round rationality levels*

We see that the majority of subjects are level 1 players (23,2%), followed by level 2 (15,7%). In other words, 38,9% of players are level 1 or level 2. This result is in line with other experimental research where the largest part of players belongs to these two levels and our large sample didn't change the qualitative features of the results. Thus, this result is robust with respect to number of players. The number of players of higher levels of rationality decrease as expected with the level of rationality but at a slower pace comparing to other experimental results. Moreover, due to the large sample size we identify a small number of level 4 players (1,9%).

Second round choices

In the second round we compared behaviour of different types of players. As we mentioned before, we run three sessions with uninformed, semi-informed and informed players. Table 2 represents descriptive statistics for round 1 and round 2.

Descriptive statistics of round 1 and round 2 choices

Table 2. *Descriptive statistics*

Group	Number of participants	Mean	Median
First round	737	32,714	30
Uninformed	187	27,54	25
Semi-informed	121	21	19
Informed	197	12,518	12

From the table we can conclude that the mean as well as the median in the second round in all three sessions were lower than in first round indicating that subjects' choices converge to equilibrium. It is obvious that choices of semi-informed players were lower than choices of uninformed and that choices of informed players were lower than choices of semi-informed players.

Frequency distribution of choices of uninformed players is more asymmetric to the right than the first round choices which indicate convergence to equilibrium (Figure 3).

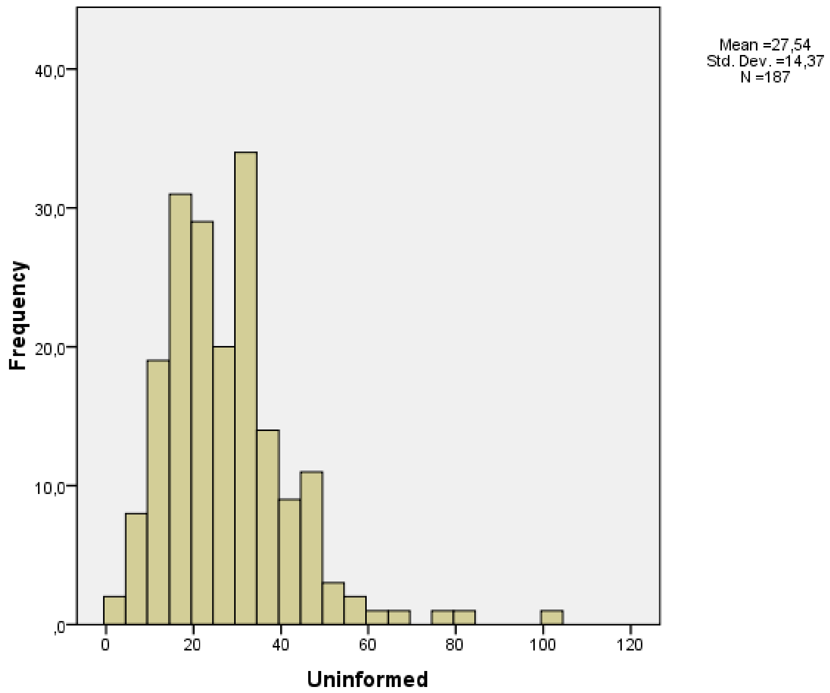

Figure 3. *Histogram of frequency distribution of uninformed players*

On the other hand, frequency distribution of semi-informed players shows some differences in players behaviour. It seems that one part of players decreased their choices to a greater extent than the other part of players. However, both groups of players converge to equilibrium (Figure 4).

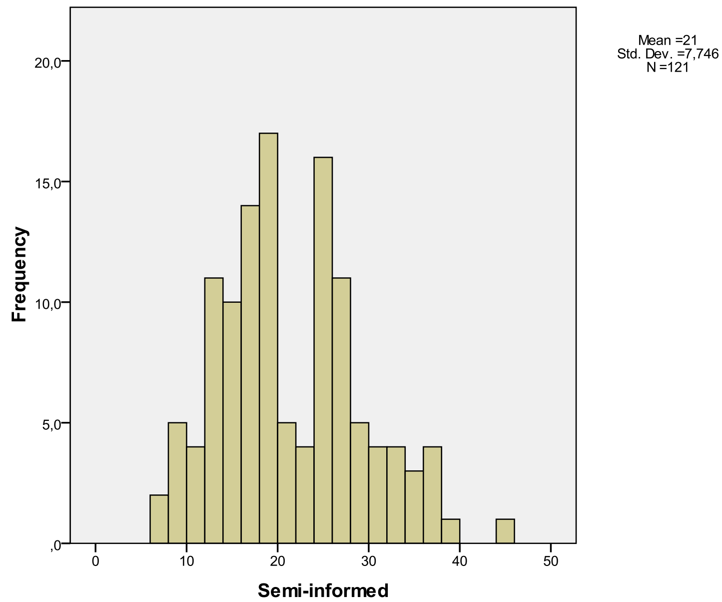

Figure 4. *Histogram of frequency distribution of semi-informed players*

The frequency distribution of informed players is the most asymmetric to the right indicating the fastest convergence of this group.

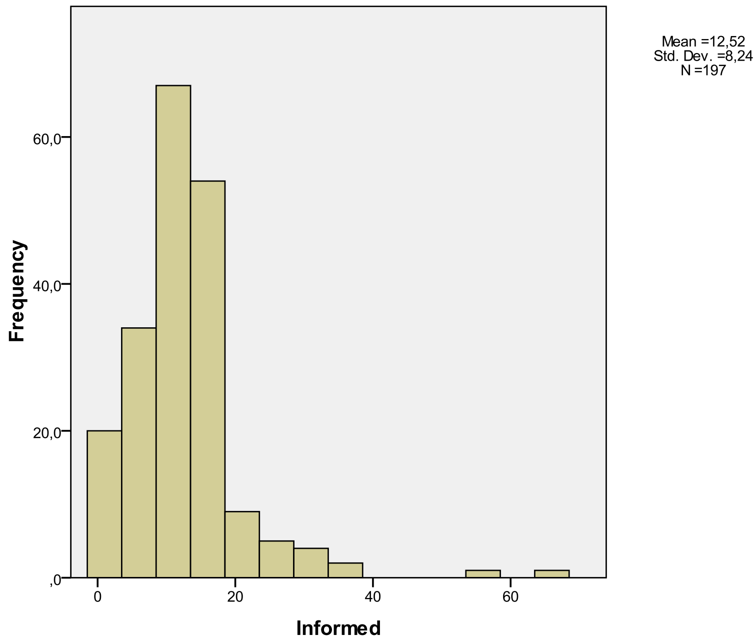

Figure 5. *Histogram of frequency distribution of informed players*

We conducted Mann-Whitney U test with 1% significance level to examine whether the choices are significantly different for the three types of players in the second round. The results are shown in Table 3.

	First round	Uninformed	Semi-informed	Informed
First round		355026,5	338406	403482,5
Uninformed			32078,5	49103,5
Semi-informed				26737
Informed				
a) We use one sided Mann-Whitney U test				
b) All values are significant for α=0,01.				

Table 3. *Mann Whitney U test*

The table shows that there is a significant difference between second round choices of three types of players at 1% significance level. Very interesting result is the difference between first round choices and choices of uninformed players in the second round, where we can see that players' beliefs evolved through the time. Furthermore, we see that choices of semi-informed players are significantly lower than choices of uninformed players, indicating that information released after the end of the first round increased the speed of convergence. Finally, we see that the mean of choices of informed players is lower than the mean of choices of semi-informed players and that the decrease in choices is larger than the decrease in choices between uninformed and semi-informed players. This result indicates that *the information* *about the equilibrium outcome increased the speed of convergence to a greater extent than the information about the previous period choices*.

The rationality levels in second round of semi-informed and informed players

We compared the rationality levels in the second round for the three types of players. In the second round we use again the iterated best response argument, assuming that level 0 players choose the previous round mean μ_1. Level 1 players choose best reply given their belief that other players are level 0 players, i.e. they choose $\mu_1(2/3)^1$. Level 2 players best reply given their belief that others are level 1 players and they choose $\mu_1(2/3)^2$ and so on. We construct the intervals around iterated best responses in the same fashion as in the first round. In general, in round *k* subject has a level *d* of depth of reasoning if his choice falls in the interval: $[\mu_{k-1} p^{d+1/4}, \mu_{k-1} p^{d-1/4}]$, for *d*=0,1,2,…, where μ_{k-1} is the previous round mean.

Interval classification for semi-informed			
Interval name	*Interval*	*Frequency*	*Percentage*
Irrational	37-100	3	2,5%
Level 0	30-36	14	11,6%
Interim 0	25-29	22	18,2%
Level 1	20-24	19	15,7%
Interim 1	17-19	28	23,1%
Level 2	13-16	19	15,7%
Interim 2	12	5	4,1%
Level 3	9-11	7	5,8%
Level 3+	0-8	4	3,3%

Table 4. *Interval classification for semi-informed players*

Comparing these results with first round rationality levels, we determine a slight increase in relative frequency of level 0 players, a slight decrease in relative frequency of level 2 and level 3 players and a moderate decrease in relative frequency of level 1 players. We also identify an increase in relative frequency of players whose choices belong to interim intervals. However, the majority of subjects are still clasiffied as level 1 and level 2 players with a small part of level 3 players. This is in accordance with previous research where a significant increase in the depth of reasoning from round 1 to round 2 is not identified for this type of players.

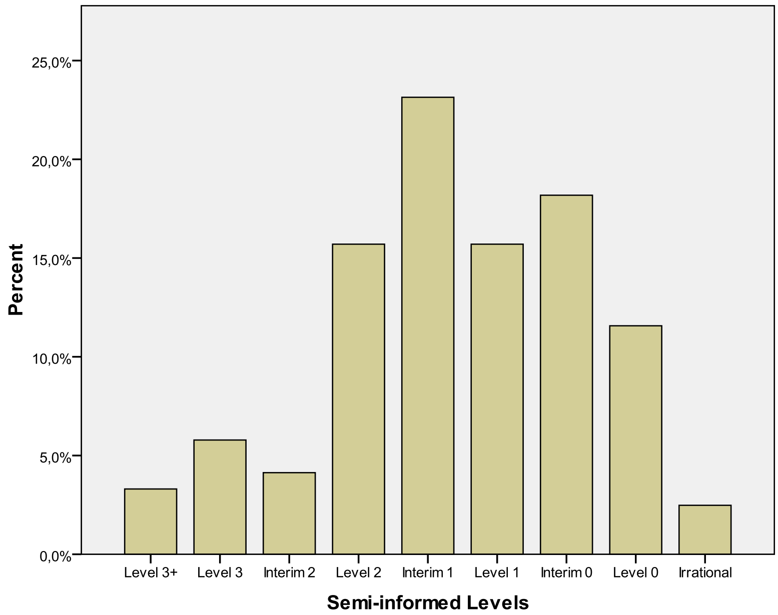

Figure 6. *Histogram of distribution for semi-informed rationality levels*

We now examine the rationality levels of informed players. Intervals for these players are the same as for semi-informed players. The interval classification for informed players is given in Table 5.

Interval classification for informed			
Interval name	*Interval*	*Frequency*	*Percentage*
Irrational	37-100	4	2%
Level 0	30-36	3	1,5%
Interim 0	25-29	5	2,5%
Level 1	20-24	9	4,6%
Interim 1	17-19	17	8,6%
Level 2	13-16	59	29,9%
Interim 2	12	6	3%
Level 3	9-11	40	20,3%
Interim 3	8	12	6,1%
Level 4	6-7	16	8,1%
Level 5	4-5	6	3%
Level 5+	0-3	20	10,2%

Table 5. *Interval classification for informed players*

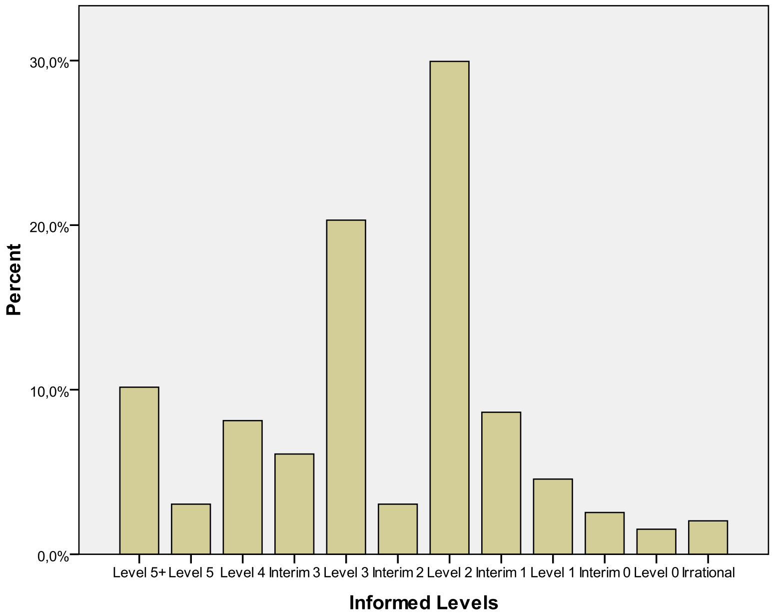

Figure 7. *Histogram of distribution of informed rationality levels*

The most interesting result concerns the levels of reasoning of informed players. Comparing to first round, there is an apparent increase in frequency of level 2 and level 3 players. More than a half of players are now classified as level 2 or level 3 players. Moreover, there is a significant number of level 4, level 5 and even higher levels. This result is different from results of previous research and it shows us a tremendous impact of the knowledge of the game and the equilibrium outcome on players' rationality.

The rationality levels in the second round of uninformed players

Uninformed players didn't receive any information concerning the previous round results, which means that they form their beliefs in the second round in a different way than other players. Therefore, the interval classification for rationality levels rests the same as in the first round. Level 0 players still choose randomly from the interval [1,100] and higher level players best reply to the choice of players of one lower level of rationality. The classification of rationality levels is shown in Table 6.

Interval classification for uninformed players

Interval name	*Interval*	*Frequency*	*Percentage*
Irrational	67-100	4	2,1%
Interim 0	57-66	2	1,1%
Level 0	46-56	11	5,9%
Interim 1	38-45	18	9,6%

Level 1	30-37	43	23,0%
Interim 2	26-29	14	7,5%
Level 2	20-25	35	18,7%
Interim 3	18-19	15	8,0%
Level 3	14-17	21	11,2%
Interim 4	12-13	6	3,2%
Level 4	9-11	11	5,9%
Level 4+	0-8	7	3,7%

Table 6. *Interval classification for uninformed players*

It is interesting to note that there is almost linearly decreasing number of players with the rationality levels. Comparing to first round results, there are less irrational players (2,1% vs 3,8%). There are also less level 0 players in the second round than in the first round (5,9% vs 7,5%). The relative freqency of level 1 is almost the same as in the first round (23% vs 23,2%) and there is a slight increase in the relative frequency of level 2 players (18,7% vs 15,7%). In general, there is a tendency of an increase in the relative frequency of all rationality levels higher than 1. This absolute difference is greater the higher the rationality level. For instance, in the first round there were 1,9% level 4 players, whereas there are 5,9% level 4 players in the second round. However, the majority of players are still classified as level 1 and level 2 players. The distribution of rationality levels is shown in the following graph.

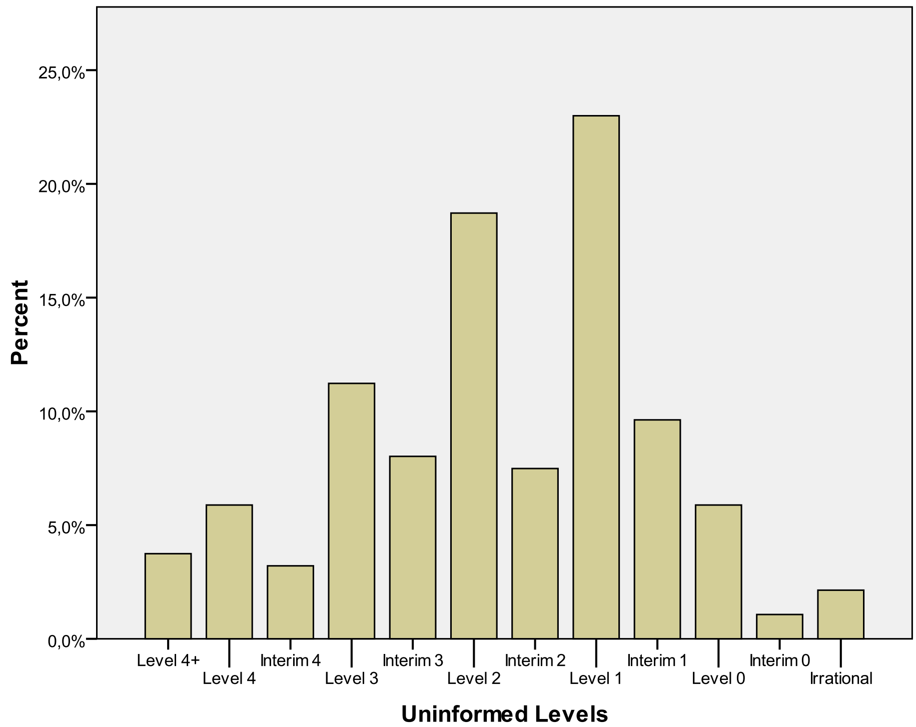

Figure 8. *Histogram of distribution of uninformed rationality levels using first round intervals*

We can conclude that uninformed players learned the structure of the game and converged to equilibrium. The speed of convergence is slower than within other groups of players. Furthermore, we identify a slight increase of rationality levels for this group of players but the majority of players belong to first two rationality levels. Thus, we can conclude that there is no significant increase in the levels of rationality for uninformed players.

5. CONCLUDING REMARKS

In this paper we have analysed a *p*-beauty contest game. This game is simple, but a powerful tool for studying how players behave in reality when they have to find optimal strategies. The beauty contest game captures some features of decision making in financial markets. Therefore, the knowledge of actual behaviour of players in beauty contest games can help us to understand how investors actually behave in financial markets. This game shows us that there always exists a small number of irrational players. Moreover, there is an important number of rational players who are not able to calculate the equilibrium strategy and they make only few steps of iterated elimination of dominated strategies. Finally, there are rational players who know the equilibrium strategy, but they play some other strategy believing that other players will not choose the optimal strategy. Thus, *p*-beauty contest games show us that in reality players are not rational to the extent that the game theory predicts. However, in real life, and in particular in stock markets, only players whose behaviour is the closest to optimal are successful and continue to operate. This argument justifies the assumption of game-theoretic models that players behave rationally. Our research shows us that this rational behaviour is obtained after several repetition of the game and that in the limit players' behaviour converge to optimal behaviour predicted by game theory. We have found that the speed of this convergence depends on information that players posses.

The contribution of our paper is a clarification of the process of decision making in games of imperfect information. We have analysed beauty contest game with differently informed players. The objective of our research was to determine whether different information that players receive after the first round have an influence on players' choices in the second round. We have found that uninformed and semi-informed players show a tendency of convergence to equilibrium and that informed players converge the fastest. Furthermore, we have found that uninformed and semi-informed players do not increase their level of rationality whereas informed players significantly increase their rationality level from first to second round.

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