DOI: 10.20472/EFC.2018.010.043

ALEKSANDRA ŁUCZAK
Poznań University of Life Sciences, Poland

MAŁGORZATA JUST
Poznań University of Life Sciences, Poland

AGNIESZKA KOZERA
Poznań University of Life Sciences, Poland

APPLICATION OF THE POSITIONAL POT-TOPSIS METHOD TO THE ASSESSMENT OF FINANCIAL SELF-SUFFICIENCY OF LOCAL ADMINISTRATIVE UNITS

Abstract:

In this paper we propose new approach to the construction of synthetic measure, where the objects are described by the characteristics with strong asymmetry and outliers. The aim of the paper is to present the application potential of the tools of the Extreme Values Theory (EVT) i.e. Peaks over Threshold Model (POT) in constructing a synthetic measure based of positional Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), that utilize the spatial median of Weber. POT model has been used for identification of outliers and to determination of the positive and negative ideal solutions of financial self-sufficiency of local administrative units (LAUs). This approach is used in the assessment of financial self-sufficiency of LAUs in Poland in 2016.

Keywords:

positional POT-TOPSIS, Extreme Values Theory, Peaks over Threshold Model, synthetic measure, financial self-sufficiency, local administrative units

JEL Classification: C10, C49, H70

1 Introduction

The procedure for constructing a synthetic measure (or synthetic indicator) is a multi-stage process in which the principal researcher must take many decisions at every stage which relate to accepting the method of selecting simple characteristics for research, normalization procedures, or methods of aggregating the values of simple characteristics. These tasks are not simple, especially when there are outliers in real data sets, which may result from the specifics of the complex phenomenon covered by the study. These observations, as noted by Łuczak and Wysocki (2013) and also Kozera and Wysocki (2016) may have a significant impact on the results of the analysis, which is why they require special attention. In such cases, the assumption that the maximum and minimum values of the characteristics in reference methods for linear ordering of objects (e.g. TOPSIS) are module objects leads to excessive remoteness from typical values of the considered characteristics and consequently narrows the range of variation of the constructed synthetic measure and makes it difficult to correctly identify the types of the examined objects on its basis (Łuczak, Wysocki, 2013). It is important to seek optimal methods for identifying outliers in view of this problem, in order to eliminate their impact on the process of determining model values i.e. the positive ideal solution and the negative ideal solution in linear ordering methods.

In this paper we propose new approach to the construction of synthetic measure, where the objects are described by the characteristics with outliers. The aim of the paper is to present the application potential of the tools of the *Extreme Values Theory* (EVT) i.e. *Peaks over Threshold Model* (POT) (McNeil, 1999) in constructing a synthetic measure based on TOPSIS (*Technique for Order of Preference by Similarity to Ideal Solution*) method (Hwang, Yoon, 1981). TOPSIS method is useful in creating the ranking of objects describing by many characteristics. The method allows to find the shortest distance from the positive ideal solution and the farthest from the negative ideal solution for solving multiple-criteria decision making problems. The synthetic measure is constructed on the base of the distances. The higher values of synthetic measure denote of the better position of objects in ranking. In this paper, we propose application of a positional TOPSIS method that utilizes the spatial median of Weber (cf. Rousseeuw, Leroy, 1987; Wysocki, 2010). POT model has been used for identification of outliers and to determination of the positive and negative ideal solutions. This approach was used in the assessment of financial self-sufficiency of local administrative units in Poland in 2016.

2 Methods

TOPSIS is the most known technique for solving *Multi-Criteria Decision Making* (MCDM) problems. It has numerous advantages, especially it classical version is easy to use. Classical TOPSIS and its various versions are popular approaches and have been widely used in many issues (Behzadian et al., 2012; Velasquez, Hester, 2013; Mardani et al., 2015; Afsordegan et al., 2016; Nădăban et al., 2016) i.e.: logistics, design, engineering, manufacturing systems, business, management, marketing strategy, health, safety, environment, human resources management, energy, chemical engineering, planning and many others. TOPSIS was extended base on e.g.: triangular fuzzy numbers (Chen, 2000), interval data (Jahanshahloo et al., 2006), interval-valued fuzzy sets (Chen, Tsao, 2008), interval type-2 fuzzy sets (Chen, Lee, 2010), interval-valued intuitionistic fuzzy sets (Li, 2010) and multi-granularity linguistic assessment information (Liu et al., 2013). The number of studies relating to the theoretical and empirical approaches of TOPSIS is impressive and can be counted in tens of thousands of studies.

The procedure to construct a synthetic measure, based on positional POT-TOPSIS method, includes six basic stages:

1. selecting characteristics on the complex phenomenon (i.e. the financial self-sufficiency of the LAUs),
2. determining the nature of characteristics in relation to the main criterion,
3. normalizing the values of characteristics,
4. calculating the distance of each object (LAU) from positive and negative ideal solutions,
5. calculating values of the synthetic measure,
6. linearly ordering and identifying the types.

The first and second stages include the selection of characteristics describing objects, as well as determination of the direction of their preferences in relation to the main criterion. The selection of characteristics is based on substantive and statistical criteria. The set of characteristics describing the financial self-sufficiency of LAUs usually includes strongly asymmetric items or outliers. The problem may be solved by to approach with the application of:

I. *Peaks over Threshold Model* to establish threshold (limit) of outliers,
II. the positional formulation of TOPSIS method, that utilize the spatial median of Weber to limit the impact of the strong asymmetry of characteristics.

First, the *Peaks over Threshold Model* was used to establish limits of outliers. In this approach, the tail of the distribution of the characteristic is modeled using the *Generalized Pareto Distribution* (GPD), and its beginning is determined by establishing a threshold value (ul_k). In POT (McNeil, 1999), the starting point for the considerations is the conditional distribution of excess over ul_k of random variable X_k (k^{th} characteristic), defined with the formula above:

$$F_{ul_k}(y_k) = P(X_k - ul_k \le y_k | X_k > ul_k) = \frac{F(y_k + ul_k) - F(ul_k)}{1 - F(ul_k)} \tag{1}$$

where $y_k = x_k - ul_k > 0$, F is the unknown distribution function of random variable X_k. According to the Pickands–Balkema–de Haan theorem, for a sufficiently large ul_k, the distribution function F_{ul_k} is well approximated by the generalized Pareto distribution:

$$G_{\xi,\beta}(x_k - ul_k) = \begin{cases} 1-(1+\xi(x_k - ul_k)/\beta)^{-1/\xi}, & \xi \neq 0 \\ 1-\exp(-(x_k - ul_k)/\beta), & \xi = 0 \end{cases}, \tag{2}$$

where:
$\beta > 0$, $x_k - ul_k \geq 0$ for $\xi \geq 0$ and $0 \leq x_k - ul_k \leq -\beta/\xi$ for $\xi < 0$.
The distribution has two parameters:
ξ – shape parameter responsible for the thickness of the tail,
β – scale parameter.
Positive values of the shape parameter indicate the occurrence of fat tails, which is associated with an increased probability of extreme characteristic values whereas the negative values of the shape parameter indicate that the distribution has thinner tails than the normal distribution. The choice of the threshold value ul_k has an impact on the values of GPD parameter estimators. An excessive threshold value ul_k causes few observations to exceed the ul_k threshold, which translates into a large variance, while an excessively low value will result in a highly biased estimator. If K is the number of observations, and K_{ul_k} is the number of observation in excess of ul_k, the estimator of the distribution function F is expressed with the following formula:

$$\hat{F}(x_k) = 1 - \frac{K_{ul_k}}{K}\left(1 + \hat{\xi}\,\frac{(x_k - ul_k)}{\hat{\beta}}\right)^{-1/\hat{\xi}}. \tag{3}$$

Selecting the ul_k threshold should depend on the specifics and number of characteristics considered[1]. In this study, the method for determining the ul_k threshold will consist in analyzing the stability of estimated GPD distribution parameters obtained for different threshold values[2]. If the threshold is properly selected, the estimates of the shape parameter for higher threshold values should be similar, whereas the scale parameter estimates should change linearly. The lower limit (ll_k) of the characteristic is determined by performing calculations for the values of the characteristic multiplied by -1.
The selected characteristics have a stimulating or destimulating effect on the phenomenon (stage 2). Characteristics that have stimulating effect, contribute to increasing the level of the phenomenon, while characteristics that have destimulating effect, decreasing the level of the phenomenon. Characteristics considered to have a destimulating effect may be converted into stimulating characteristics with the use of a negative coefficient transformation:

---

[1] For example, McNeali and Frey (2000) assumed that the tail consists of 10% of extreme observations, while Hull (2011) stated that the upper tail of the distribution should contain about 5% of extreme observations.
[2] The methods for selecting the threshold are presented by Coles (2001) and in other publications.

$$x_{ik} = a - b \cdot x_{ik}^{D}, \tag{4}$$

where:

x_{ik}^{D} – value of the k^{th} characteristic, a destimulant $(k \in I_D)$ in the i^{th} object (i = 1, …, N),
x_{ik} – value of the k^{th} characteristic (k = 1, …, K) transformed into a stimulant,
a, b – arbitrary constants, e.g. a = 0 and b = 1.

In the third stage, values of characteristics are normalizing. For this purpose, it was applied the modified median standardization of Weber[3] based on threshold values of characteristics ul_k and ll_k (k=1, 2, …,K) (cf. Lira et al., 2002):

$$z_{ik} = \begin{cases} \dfrac{ll_k - m\tilde{e}d_k}{1.4826 \cdot m\tilde{a}d_k} & \text{for } x_{ik} \le ll_k \\ \dfrac{x_{ik} - m\tilde{e}d_k}{1.4826 \cdot m\tilde{a}d_k} & \text{for } ll_k < x_{ik} < ul_k \\ \dfrac{ul_k - m\tilde{e}d_k}{1.4826 \cdot m\tilde{a}d_k} & \text{for } x_{ik} \ge ul_k \end{cases}, \tag{5}$$

where:

x_{ik} – value of the k^{th} characteristic in the i^{th} object,
$m\tilde{e}d_k$ – L_1-median[4] vector component corresponding to the k^{th} characteristic,
$m\tilde{a}d_k = med_i \left| x_{ik} - m\tilde{e}d_k \right|$ – median absolute deviation of k^{th} characteristic values from the median component of the k^{th} characteristic,
1.4826 – a constant scaling factor corresponding to normally distributed data, $\sigma \approx E\left(1.4826 \cdot m\tilde{a}d_k\left(X_1, X_2, ..., X_K\right)\right)$, σ – standard deviation.

The median standardization of Weber[5] is calculated for winsorized data. Winsorization (or winsorizing) is a process of replacing a specified number of extreme values of characteristics with a constant (smaller or bigger) value. We propose to adopt threshold values of characteristics ul_k and ll_k (k = 1, 2, …, K) in winsorization of data.

In fourth stage the coordinates of positive ideal solution (PIS) and negative ideal solution (NIS) were calculated as (see e.g. Wysocki 2010):

PIS $$A^{+} = \left(\max_i\left(z_{i1}\right), \max_i\left(z_{i2}\right), ..., \max_i\left(z_{iK}\right)\right) = \left(z_1^{+}, z_2^{+}, ..., z_K^{+}\right), \tag{6}$$

NIS $$A^{-} = \left(\min_i\left(z_{i1}\right), \min_i\left(z_{i2}\right), ..., \min_i\left(z_{iK}\right)\right) = \left(z_1^{-}, z_2^{-}, ..., z_K^{-}\right). \tag{7}$$

[3] The distribution of thus standardized characteristic values is considered close to the normal distribution of zero expectation and unitary standard deviation (Młodak, 2006).
[4] The L_1-median is also called the Weber median, geometric median, median center, spatial median or the Weber point (Lira et al., 2002).
[5] The Weber median standardization should be used when the empirical distribution of the studied characteristics is strongly asymmetric (Lira et al., 2002).

The positive ideal solution[6] (PIS) are the best values of characteristics. PIS includes the maximum (ideal) values of each characteristic. Whereas, the negative ideal solution[7] (NIS) are the worst values of characteristics. NIS contains the minimum (nadir) values of each characteristic, which are stimulant or are transformed into stimulant.

Next, distances for each object from the positive ideal solution (A^+) and the negative ideal solution (A^-) were calculated based on the median absolute deviation (stage 4) (Wysocki, 2010):

$$d_i^+ = med_k\left(\left|z_{ik} - z_k^+\right|\right), \quad d_i^- = med_k\left(\left|z_{ik} - z_k^-\right|\right), \tag{8}$$

where:

med_k – marginal median for the k^{th} characteristic.

The construction of a synthetic measure (stage 5) used the TOPSIS method (Hwang, Yoon 1981):

$$S_i = \frac{d_i^-}{d_i^- + d_i^+}, \ (i = 1,\dots, N), \qquad 0 \le S_i \le 1. \tag{9}$$

The smaller the distance of the PIS, and thus greater from the NIS, the closer to 1 is the value of the synthetic measure. Established values of the synthetic measure are used in rank ordering of objects and are the base for identification of their typological classes (stage 6). Identification of classes for the entire range of variation of a synthetic measure may be performed using statistical methods based on the mean and standard deviation from values of the synthetic measure, or in an arbitrary manner, assuming e.g. numerical ranges of values for synthetic measure.

3 Results of research

Study concerning the level of financial self-sufficiency of municipalities in Poland (N= 2412) are based on statistical data from 2016 coming from the Central Statistical Office of Poland (*Local Data Bank*). In the first stage 5 indicators were preliminarily selected based on a substantive and statistical analysis:

x_1 – own income per capita (in PLN),

x_2 – share of own income in total income (%),

x_3 – transfer income (including specific grants and the general subsidy) per capita (in PLN),

x_4 – share of ratio of tax income (tax bill of agricultural, forestry, real estate, from funds transport of civil law, income from taxation, income from mining fee) to current income (budgetary revenue other than income property) (%),

[6] The positive ideal solution is also called the ideal solution, the pattern or the ideal object.

[7] The negative ideal solution is also called the anti-pattern or the anti-ideal object.

x_5 – self-financing rate (share of operating surplus and capital income in capital expenditure).

Table 1: Descriptive statistics of the characteristics of municipalities in Poland in 2016

Specification	Characteristics				
	x_1	x_2	x_3	x_4	x_5
	initial data				
Mean	1575.14	38.17	2442.17	15.04	191.10
Median	1382.34	35.98	2441.40	14.01	141.59
Max	45340.71	95.04	4521.20	59.74	18507.88
Min	508.17	13.08	1163.22	2.24	-134.22
St. Dev.	123.45	13.20	550.34	6.86	487.88
Skewness	20.35	0.58	0.18	1.34	29.95
Ex. kurtosis	667.11	-0.15	-0.37	3.30	1026.09
	trimmed characteristics*				
Mean	1436.03	37.72	2424.53	14.44	151.83
Median	1364.13	35.98	2434.72	13.91	139.55
Max	2686.06	63.32	3395.97	27.88	322.52
Min	724.68	19.14	1484.30	5.90	68.75
St. Dev.	449.68	11.21	473.81	4.97	56.52
Skewness	0.66	0.39	0.00	0.43	0.93
Ex. kurtosis	-0.20	-0.87	-0.94	-0.49	0.28
	winsorized characteristics**				
Mean	1489.87	38.01	2435.12	14.77	160.44
Median	1382.34	35.98	2441.40	14.01	141.59
Max	2689.97	63.39	3397.31	27.88	322,56
Min	724.34	19.12	1481.88	5.90	68.38
St. Dev.	539.26	12.49	522.89	5.86	71.34
Skewness	0.73	0.40	0.02	0.55	0.94
Ex. kurtosis	-0.27	-0.85	-0.91	-0.40	-0.02

* Characteristics cleaned from extreme values, ** Extreme values replaced with lower and upper limits.
Source: Own calculations based on data from the Central Statistical Office (Local Data Bank).

Table 1 presents descriptive statistics of the characteristics – indicators of municipalities in Poland in 2016. All characteristics demonstrated positive skewness, with extremely high skewness in the case of x_1 (the level of own income per capita) and x_5 (self-financing rate). The distributions of three characteristics (own income per capita, share of ratio of tax income and self-financing rate) were demonstrated positive kurtosis, which means a high probability of extreme values.

The parameters of the generalized Pareto distribution for the left and right tail of distributions of the considered characteristics were estimated in the next stage of the study (Table 2). The results confirmed the occurrence of fat right tails of the distribution of x_1 and x_5 and suggested the existence of a fat left tail in the distribution of x_5. Other distributions of characteristics, especially x_1 and x_3, had thin left tails. After removing the extreme characteristics, i.e. trimming the tails, the distribution of characteristics for trimmed data demonstrated small or moderate skewness, and their kurtosis was close to that of normal distribution. The same was true for winsorized data.

In order to limit the influence of extreme values, the limits of extreme values of indicators were established based on POT model (Table 2). Then, it was assumed that four characteristics have a stimulating effect (x_1, x_2, x_4, x_5) and only one characteristic (x_3) has a destimulating effect on the level of financial self-sufficiency of municipalities. The characteristic with a destimulating effect was converted into an opposite characteristic.

Table 2: Estimates of the parameters of the GPD for characteristics of municipalities in Poland in 2016

Specification	Left tail			Right tail		
	ll	$\hat{\xi}$	$\hat{\beta}$	ul	$\hat{\xi}$	$\hat{\beta}$
x_1	724.34	-0.48	112.10	2689.97	0.43	762.68
x_2	19.12	-0.31	2.48	63.39	-0.05	6.35
x_3	1481.88	-0.44	151.01	3397.31	-0.10	271.08
x_4	5.90	-0.33	1.38	27.88	-0.06	6.61
x_5	68.38	0.25	16.48	322.56	0.68	133.82

Source: Own calculations based on data from the Central Statistical Office (Local Data Bank).

In next stage, the values of indicators were standardized using the modified Weber's median standardization (the calculations were performed with *robustX* in *R*) (Stahel, Maechler, 2012). The standardized values of characteristics allowed us to calculate the distance of each municipality (LAU) considered from the positive and negative ideal solutions with the use of the median absolute deviation. Subsequently, the values of the synthetic measure of financial self-sufficiency of municipalities were calculated using the positional POT-TOPSIS method, becoming the basis for identifying six types of municipal financial self-sufficiency levels in Poland (Table 3). Proposed approach (approach I) was compared with classical TOPSIS (approach II) and positional TOPSIS (approach III).

Table 3: Typological classification of municipalities in Poland in terms of the level of financial self-sufficiency in 2016

Class typology (level of financial self-sufficiency)	S_i	Approaches I		II		III	
		Positional POT-TOPSIS		Classical TOPSIS		Positional TOPSIS	
		N_c	%	N_c	%	N_c	%
I (very high)	⟨0.80; 1.00⟩	135	5.60	0	0.00	0	0.00
II (high)	⟨0.60; 0.80)	179	7.42	0	0.00	1	0.04
III (medium-high)	⟨0.50; 0.60)	242	10.03	2	0.08	6	0.25
IV (medium-low)	⟨0.40; 0.50)	319	13.23	1	0.04	16	0.66
V (low)	⟨0.20; 0.40)	889	36.86	2	0.08	167	6.92
VI (very low)	⟨0.00; 0.20)	648	26.87	2407	99.79	2222	92.12

N_c – the number of object in *c*-th class.
Source: Own calculations based on data from the Central Statistical Office (Local Data Bank).

The rankings show big differences in the arrangement of objects and values of synthetic measures (Tables 3-5). This is confirmed by the low values of Spearman's and Kendall's rank correlation coefficients calculated for approach I and approach II. The rankings in approach I and III show a high consistence of linear ordering results (0.99 and 0.91), contrary to the rankings in approaches II and III (0.19 and 0.13).

The positional POT-TOPSIS[8] method is robust to outliers and to defined values of the positive ideal solution and the negative ideal solution determined by the POT method.

[8] Similar effects of object ranking are obtained using POT-TOPSIS with trimmed data and winsorized data with classical standardization; in this case, the ranges of variation in the values of synthetic measures are slightly narrower, and the distributions are more platykurtic as opposed to approaches I-III that have leptokurtic distributions. In particular, the synthetic measure values in approaches II and III are more concentrated than in approach I.

Compared to other synthetic measures, it resulted in the largest range of variation of the synthetic measure (from 0.013 to 1.000), which allowed to more accurately determine the ranks and types of municipalities (including six levels of financial self-sufficiency of municipalities, from “very high” to “very low”) (Table 3).

Table 4: Values of rank correlation coefficients of the synthetic measures

Approaches*	Spearman's rank correlation coefficient			Kendall’s rank correlation coefficient		
	I	II	III	I	II	III
I	1.00	0.21	0.99	1.00	0.13	0.91
II	0.21	1.00	0.19	0.13	1.00	0.13
III	0.99	0.19	1.00	0.91	0.13	1.00

* I – Positional POT-TOPSIS, II – classical TOPSIS, III – positional TOPSIS.
Source: Own calculations based on data from the Central Statistical Office (Local Data Bank).

In turn, approaches II and III provided a narrower range of variation of the synthetic measures. Synthetic measures calculated using the classical and positional TOPSIS method do not reflect satisfactorily the inter-class differences in the scope of financial self-sufficiency of municipalities. Considering the ranges of variation (in the case of classical TOPSIS and positional TOPSIS, the synthetic measure values fall within the following intervals: ⟨0.011; 0.515⟩ and ⟨0.002; 0.732⟩, respectively). Most municipalities, i.e. almost 100% in approach II and over 92% in approach III, could qualify for a class representing a very low level of financial self-sufficiency. The synthetic measure values obtained in these approaches (especially the values close to zero) do not allow for a meaningful identification of types of financial self-sufficiency of municipalities. It should also be noted that the values of the synthetic measure in approaches II and III were characterized by extreme positive skewness and positive kurtosis, which may also indicate an incorrect distribution of objects in the classes (Table 5).

Table 5. Descriptive statistics of the synthetic measures of financial self-sufficiency of municipalities in Poland according to approaches

Specification	Approaches		
	I	II	III
	Positional POT- TOPSIS	Classical TOPSIS	Positional TOPSIS
Max	1.000	0.515	0.732
Min	0.013	0.011	0.002
Range	0.987	0.504	0.730
Median	0.318	0.082	0.091
Mean	0.360	0.085	0.105
Skewness	0.976	3.309	2.087
Ex. kurtosis	0.698	40.106	7.754

Source: Own calculations based on data from the Central Statistical Office (Local Data Bank).

4 Conclusion

The following statements and conclusions can be formulated based on the calculations and analyzes carried out. The proposed approach to linear ordering of objects based on the positional POT-TOPSIS method (using the Weber spatial median) can be used to determine the synthetic measure in cases where outliers or strong asymmetry appear in the set of characteristics describing the examined objects. Even a single outlier (very large or very small) for a given object may significantly affect the attribution of an excessively high (or low) rank in the final classification of objects, which is particularly apparent when using the classical TOPSIS method. The rankings slightly improve by using the positional TOPSIS method, which is recommended for use when the empirical distribution of the characteristics under consideration is strongly asymmetric. This is because the essence of the classical TOPSIS method lies in the calculation and aggregation of the squared deviations of each multi-characteristic object from the positive ideal solution and the negative ideal solution. In turn, the application of the positional approach (i.e. using the absolute deviation from median) enables locating the center of the set of absolute differences between each multi-characteristic object and the the positive and negative ideal solutions, which makes it possible to limit the impact of outliers on the construction of the synthetic measure.

The positional POT-TOPSIS method reduces (eliminates) the impact of extreme values of characteristics (which are a common occurrence in the assessment of financial self-sufficiency of LAUs) on the value of the synthetic measure. The typology of municipalities created on its basis reflects well the inter-class differences well in financial self-sufficiency of municipalities, and is composed of six classes of municipalities, spanning from a very low to a very high level of financial self-sufficiency.

Due to the most synthetic values being close to zero, neither the classical nor the positional TOPSIS method did give grounds for the correct typology of municipalities.

5 Recommendations

It is recommended to use the POT method to determine the threshold of extreme values along with other substantive criteria and statistical methods in order to avoid mechanical and excessive winsorization, which can lead to incorrect placement of objects in extreme classes (with high and very high levels or with low and very low levels).

References

AFSORDEGAN, A., SÁNCHEZ, M., AGELL, N., ZAHEDI, S., CREMADESL, L. V. (2016). Decision making under uncertainty using a qualitative TOPSIS method for selecting sustainable energy alternatives. *International Journal of Environmental Science and Technology*, Vol. 13, No. 6, pp.1419–1432.

Bank Danych Lokalnych (Local Data Bank), Central Statistical Office, Poland (www.stat.gov.pl).

BEHZADIAN, M., KHANMOHAMMADI OTAGHSARA, S., YAZDANI, M., IGNATIUS, J. (2012). A state-of the-art survey of TOPSIS applications. *Expert Systems with Applications*, Vol. 39, No. 17 pp. 13051–13069.

CHEN, C.-T. (2000). Extension of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy Sets and Systems,* Vol. 114, No. 1, pp. 1–9.

CHEN, S.-M., LEE, L.-W. (2010). Fuzzy multiple attributes group decision-making based on the interval type-2 TOPSIS method. *Expert Systems with Applications*, Vol. 37, No.4, pp. 2790–2798.

CHEN, T.-Y., TSAO, C.-Y. (2008). The interval-valued fuzzy TOPSIS method and experimental analysis. *Fuzzy Sets and Systems*, Vol. 159, No. 11, pp. 1410–1428.

COLES, S. (2001). *An Introduction to Statistical Modeling of Extreme Values*. London: Springer.

HULL, J.C. (2011). *Zarządzanie ryzykiem instytucji finansowych* (*Risk management and financial institutions*). Warszawa: Wydawnictwo Naukowe PWN.

HWANG, C.L., YOON, K. (1981). *Multiple attribute decision-making: Methods and applications*. Berlin: Springer.

JAHANSHAHLOO, G. R., LOTFI, F. H., IZADIKHAH, M. (2006). An algorithmic method to extend TOPSIS for decision-making problems with interval data. *Applied Mathematics and Computation*, Vol. 175, No. 2, pp. 1375–1384.

KOZERA, A., WYSOCKI, F. (2016). Problem ustalania współrzędnych obiektów modelowych w metodach porządkowania liniowego obiektów (*The problem of determining the coordinates of model objects in object linear ordering methods*). *Prace Naukowe Uniwersytetu Ekonomicznego we Wrocławiu, No. 427, Taksonomia 27: Klasyfikacja i analiza danych – teoria i zastosowanie*, pp. 131–142.

LI, D.-F. (2010). TOPSIS-based nonlinear-programming methodology for multiattribute decision making with interval-valued intuitionistic fuzzy sets. *IEEE Transactions on Fuzzy Systems*, Vol. 18, No. 2, pp. 299–311.

LIRA, J., WYSOCKI, F., WAGNER, W. (2002). Mediana w zagadnieniach porządkowania obiektów wielocechowych (*Median in the ordering problems of multi-feature objects*) [In:] *Statystyka regionalna w służbie samorządu terytorialnego i biznesu*. (eds.) J. Paradysz. Poznań: Akademia Ekonomiczna, pp. 87–99.

LIU, S., CHAN, F. T., & RAN, W. (2013). Multi-attribute group decision-making with multi-granularity linguistic assessment information: An improved approach based on deviation and TOPSIS. *Applied Mathematical Modelling,* Vol. 37, No. 24, pp. 10129–10140.

ŁUCZAK, A., WYSOCKI, F. (2013). Zastosowanie mediany przestrzennej Webera i metody TOPSIS w ujęciu pozycyjnym do konstrukcji syntetycznego miernika poziomu życia (*The application of spatial median of Weber and the method TOPSIS in positional formulation for the construction of synthetic measure of standard of living*). *Prace Naukowe Uniwersytetu Ekonomicznego we Wrocławiu, No. 278, Taksonomia 20: Klasyfikacja i analiza danych – teoria i zastosowanie*, pp. 63–73.

MARDANI, A., JUSOH, A., ZAVADSKAS, E.K. (2015). Fuzzy multiple criteria decision-making techniques and applications – Two decades review from 1994 to 2014. *Expert Systems with Applications*, Vol 42, No. 8, pp. 4126–4148.

McNEIL, A.J. (1999). *Extreme Value Theory for Risk Management*. Zurich: Mimeo ETZH Zentrum.

McNEIL, A.J., FREY, R. (2000). Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of Empirical Finance*, Vol. 7, No. 3–4, pp. 271–300.

MŁODAK, A. (2006). *Analiza taksonomiczna w statystyce regionalnej* (*Taxonomic analysis in regional statistics*). Warszawa: Difin.

NĂDĂBAN, S., DZITAC, S., IDZITAC, I. (2016). Fuzzy TOPSIS: A General View. *Procedia Computer Science*, Vol. 91, pp. 823–831.

ROUSSEEUW, P.J., LEROY, A.M. (1987). *Robust regression and outlier detection.* New York: John Wilney and Sons.

STAHEL, W., MAECHLER, M. (2012). Package 'robustX'. eXperimental eXtraneous eXtraordinary ... Functionality for Robust Statistics. Version 1.1-3. (cran.r-project.org/web/packages/robustX).

VELASQUEZ, M., HESTER, P.T. (2013). An Analysis of Multi-Criteria Decision Making Methods. *International Journal of Operations Research*, Vol. 10, No. 2, pp. 56–66.

WYSOCKI, F. (2010). *Metody taksonomiczne w rozpoznawaniu typów ekonomicznych rolnictwa i obszarów wiej*skich (*Taxonomic methods in recognizing economic types of agriculture and rural areas*). Poznań: Wydawnictwo Uniwersytetu Przyrodniczego w Poznaniu.