DOI: 10.20472/TEC.2016.003.022

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TEACHING ABOUT THE REINVESTMENT RATE ASSUMPTION AND CONFLICTS IN CAPITAL BUDGETING

Abstract:

Much has been written in finance textbooks and scholarly publications about the capital budgeting methods derived from Discount Cash Flow (DCF) models: Net Present Value (NPV), Internal Rate of Return (IRR), Modified Internal Rate of Return (MIRR) and Profitability Index (PI). However, there is a divergent understanding about the role of the reinvestment rate assumption in some of these models. In this paper I will discuss a number of issues regarding the assumption of the reinvestment rate and clarify its correct usage. This discussion will naturally lead to the Modified Internal Rate of Return as the only method where a reinvestment rate assumption makes sense. In addition, for the first time in the literature I will propose a method of determining if the NPV and MIRR will produce conflicting decisions in advance of the MIRR computation. The literature has a clear method to determine if the there is a conflict between NPV and IRR but no method for the NPV-MIRR conflict is described. Interestingly the Profitability Index will have a role in determining if the NPV and MIRR are in conflict. The paper is useful to all those who teach the topic of capital budgeting in corporate finance courses of any level.

Keywords:

Capital Budgeting, reinvestment rate, conflict in ranking,

JEL Classification: G30, G31

Introduction

One of the most critical issues in creating corporate value is selection of wealth creating projects. The methods employed vary from sound financial methods to some that are not that sound but are popular. As such methods based on Discount Cash Flow (DCF) models: Net Present Value (NPV), Internal Rate or Return (IRR), Modified Internal Rate of Return (MIRR), Profitability Index (PI) are considered sound financial methods while Payback, Discounted Payback, Return on Investment and similar others are in the other category. The first group represents methods backed by financial theory while the latter is based on accounting concepts (see for example Ehrhardt and Brigham (2011))

While all the DCF methods are able to identify good projects when they are stand alone, when ranking of projects is necessary methods give conflicting results. The most well-known ranking conflict is between NPV and IRR. Less is known or even discussed in textbooks about the conflict between NPV and MIRR. In fact, despite the fact that MIRR is one of the most interesting methods of analyzing investments going back to the 18th century and the work of Duvillard (see Biondi (2003)) less attention is paid to this method in textbooks.

In this paper I will discuss the idea of reinvestment rate as well as the conflicts between the principal methods of capital budgeting. Addressing the issue of conflict a new I will present a new method of determining if there is a conflict between MIRR and NPV.

The paper continues with section 2 in which I will present a brief discussion of a number of randomly selected textbooks, section 3 in which I will address the conflict issues and a conclusion section.

Review of Textbooks

An extensive review of textbooks is not necessary in this case. I pulled at random five textbooks at different editions. Three of them are well known textbooks. Two are at their latest edition: Brigham and Daves (BD, 11th edition, 2013), Brealy, Myers and Allen (BMA, 11th edition, 2014) while the third, Ross, Westerfield and Jaffe (RWJ, 6th edition, 2002) is at an earlier edition. I supplemented with a less known text, Clauss (C, 2010) and with the gold standard text for Capital Budgeting, Bierman and Smidt (BS, now at the 9th edition, 2007).

All the texts discuss the NPV and IRR ranking conflicts. However, not all the texts agree on the reinvestment rate assumption. BD and C clearly state that both NPV and IRR rest on the assumption of reinvestment rate, IRR at the IRR rate and NPV at the opportunity cost of capital (or Weighted Average Cost of Capital, WACC). BMA and RWJ do not comment on the issue while BS make a very important point: neither IRR not NPV methods rest on the assumption of reinvestment rate. It is interesting to note

that very early in the modern history of Capital Budgeting discussions Dudley (1972) made this very important point so the fact that text books continue to maintain that the reinvestment rate is at the base of the NPV and IRR methods is disturbing.

In the words of BS:

“It is sometimes stated that the internal rate of return methods assumes reinvestment at the internal rate of return rate. At best this claim is inexact. The internal rate of return of an investment can be computed without any assumption about the utilization of the funds generated by the investment. For example, an investment generating cash flow that are consumed will have the same internal rate of return as an investment whose cash flows are invested, if the cash flow of the two investment are equal”

On the issue of MIRR the selected text reveal an even more interesting result. BE and C discuss the MIRR at length and provide methods of computations. BMA, addresses the issue in a footnote and provide an unusual computation that overestimates the MIRR. Finally, BS provide a discussion about the reinvestment rate and provide a correct computation of MIRR but do not name the procedure as such. None of the texts discuss explicitly the conflict in ranking between MIRR and NPV.

Faced with such diversity of opinions it is useful to put some order in the topics and provide instructors a clear view on the correct teaching of capital budgeting techniques.

Analyzing the Issues

The Net Present Value (NPV) formula involves discounting all the cash flows of the project CF by an appropriate rate of return called Weighted Average Cost of Capital (WACC). The formula is well known:

$$NPV = \sum_{i=0}^{N} \frac{CF_i}{(1+WACC)^i} \qquad (1)$$

The fact that Internal Rate of Return (IRR) is obtained as a solution to the following equation:

$$\sum_{i=0}^{N} \frac{CF_i}{(1+IRR)^i} = 0 \qquad (2)$$

It is well known that NPV and IRR could give conflicting rankings (see for example [2]). This means that projects that are better according to NPV might or not be better according to IRR. It is easy to identify if such an occurrence will exist. All one has to do is to find a solution to the following equation:

$$\sum_{i=0}^{N} \frac{CF_i(A) - CF_i(B)}{(1+IncRR)^i} = 0 \quad (3)$$

The existence of one or more Incremental Rate of Return (IncRR) means that the conflicting ranking would be present and selection must proceed with caution.

To avoid such conflict another method called Modified Internal Rate of Return (MIRR) was developed.

$$(1+MIRR)^N = \frac{\sum_{i=0}^{N} CF_i^+ (1+WACC)^{N-i}}{\sum_{i=0}^{N} \frac{CF_i^-}{(1+WACC)^i}} \quad (4)$$

The common understanding is that MIRR and NPV rank projects in the same way. However, this is not true.

This papers develops a methodology to predict when MIRR and NPV would give conflicting rankings that is when: NPV (A)> NPV (B) and MIRR(A)< MIRR(B).

I will introduce the following notations: PV^+ the sum of all positive cash flows present values (cash flows discounted at WACC) and PV^- the sum of all negative cash flows. With these notations it is easy to rewrite (4) as:

$$(1+MIRR)^N = \frac{PV^+ (1+WACC)^N}{PV^-} \quad (5)$$

Since both project A and B are discounted at the same WACC a simple inspection of (5) leads to the following inequality when MIRR (A) < MIRR (B):

$$\frac{PV^+(A)}{PV^-(A)} < \frac{PV^+(B)}{PV^-(B)} \quad (6)$$

Subtracting 1 from both sides, leads to the following condition

$$\frac{PV^+(A) - PV^-(A)}{PV^-(A)} < \frac{PV^+(B) - PV^-(B)}{PV^-(B)} \quad (7)$$

Which can be easily written as:

$$\frac{NPV(A)}{PV^-(A)} < \frac{NPV(B)}{PV^-(B)} \quad (8)$$

Finally one can write:

$$\frac{NPV(A)}{NPV(B)} < \frac{PV^-(A)}{PV^-(B)} \quad (9)$$

In cases where NPV (A) is greater than NPV (B) to ensure that MIRR(B) is not greater than MIRR (A) one has to inspect (9). As long as the ratio of PV^{-}(A) and PV^{-}(B) is not greater than the ratio of the two NPVs there will be no conflicting ranking. If the projects do not have any negative cash flow except for the initial investment to have a consistent ranking between NPV and MIRR all it takes is to have the ratio of NPVs greater than the ratio of initial investments.

Therefore, the paper developed a simple rule to understand when the MIRR and NPV could be in conflict.

The following example is clarifying this approach.

Year	Project A Cash Flows	Project B Cash Flows
0	-$1,900	-$1,200
1	$ 700	$ 500
2	$ 900	$ 600
3	$1,300	$ 800

At a WACC of 15% standard computations give NPV (A) =243.99, MIRR(A)=19.72% and NPV(B)=214.47, MIRR(B)=21.47%. While both projects will be accepted as stand alone projects since NPVs are positive and MIRRs are greater than WACC there is a clear conflict in the selection of the two projects. NPV favors project A while IRR favors project B.

A quick substitution in (9) gives the ratio of NPVs as 1.137 while the ratio of the initial investments is 1.58. Since (9) is satisfied one can be sure of the conflict.

If now we consider the same projects where the initial investment in project A is decreased by 100 to 1800, we get NPV(A)=343.99 and MIRR(A)=21.9. Clearly there is no more conflict between the NPV and MIRR selection. Inspecting (9) one can see that the ratio of NPVs is now 1.6 while the ratio of initial investments is 1.5. Since (9) is not satisfied we could have been certain that there would have been no conflict in the ranking between NPV and MIRR without even performing the computation for MIRR.

Conclusion

In conclusion, after clarifying that there is no reinvestment rate assumption for IRR and NPV but one is needed for MIRR, this paper found a very simple test to predict if MIRR and NPV would be in conflict. The test is even simpler to apply than the test for the NPV, IRR conflict as it requires only elementary inspection of the two projects.

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