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DUPLEX-MULTI-AGENT DECISION-MAKING OF MARINE DISASTER INSURANCE BASED ON THE GREY GAME MODEL

Abstract:

This paper introduces reinsurance institutions as a fundamental decision agent and rebuilds the grey game matrix to find the optimal pure strategy of the insurance participants. The conclusions verified that when an insurance company chooses to offer marine disaster insurance, the subsidy ratio of the government plays an active role in the equilibrium strategy. When the public chooses insure and the compensation subsidy ratio is up to the threshold value, the commercial insurance company could sufficiently supply compensation for marine disasters，Similarly, the reinsurance companies choose to join the marine insurance market restricted to the compensation subsidy rate of the government. This paper presents the application of the grey game model and illustrates its usefulness as a tool to solve duplex-multi-agent decision-making in marine disaster insurance.

Keywords:

marine disaster insurance, grey game model, duplex-multi-agent, decision-making

1 Introduction

As a main marine country, China is faced with a serious threat of marine disasters. The pecuniary loss caused by storm surge and sea ice was estimated at 13.6 billion Yuan in 2014. Marine disasters have become important factors for the social and economic development of China's coastal areas. In 2014, Premier Li KeQiang presented a plan called “China-ASEAN plan for disaster reduction cooperation” during the East Asian Leaders Conference that emphasized the allocation of overall resources, prevention of land and environmental disasters. Flood, earthquake and other disaster insurance had been implemented in countries such as the United States and Britain since the 1950s. In 2011, Japan and South Korea, two of the ASEAN "10+3" members, established a regional disaster insurance fund. In July 2014, Shenzhen catastrophe insurance officially launched, and in November 2014, Ningbo catastrophe insurance system officially began.

However, as a result of the single relief mode, whereby the government is chief, and the backwardness of development in the insurance market, China faced the “double bottleneck" phenomenon, which includes finite financial resources for disaster relief and insufficient commercial means. The Swiss Reinsurance Company shows that approximately 30% of global economic losses due to natural disasters can be insured, while in developed countries, such as those of Europe and the United States, this value can reach 60% to 70%. However, in China, it is less than 5%. The willingness to participate is limited mainly because the commercial insurance companies' requirements for disaster insurance rates are always high and the insurance premiums are not affordable for all. How can the persistence of long-term cooperation among the insured, the insurance company, the government, and others be ensured? How can their roles be positioned while the main responsibilities of the principals are not equal? These two questions have become the key issues for restricting the development of marine disaster insurance system in China.

2 Global scenario of disaster insurance

2.1 Different insurance mechanisms

Market and non-market insurance mechanism are the main types for policyholders coping with disaster risks. Specifically speaking, these mechanisms could have four classification. The first type is pure-market insurance mechanism. It is operated by commercial insurance company and relies on a high-matured insurance market. Such as the flood insurance in England, for which insurance company takes charge of the insurance premium and investment and the government will not intervene the daily operation. The second type is the cooperative mechanism between commercial insurance and the government. The government will pay for all the funds for the insurance compensation, and also allocates the premium and the expenditure of the insurance fund. The commercial insurance company is actually the "employee" of the government and it takes no risks, so its profit depends on the "commission" issued by the government. Some insurance companies that have been approved by the government can apply for independent management in the insurance mechanism, but they need to pay a certain amount of fees to the government every year, such as the United States flood disaster insurance and the Japanese earthquake disaster insurance. In the United States flood insurance cooperation model, the government's dominant feature is very obvious and established the National Flood Insurance Program. Japan's earthquake disaster insurance introduced a complete reinsurance system, which was in charge by the government and insurance company together. The third one is pure-government mechanism. For this mechanism, disaster insurance is run entirely by the government. However, a commercial insurance company may apply for a license to operate the relevant insurance business within the permission by the Government. Canada's agricultural disaster insurance system is the representative one. The government not only operates insurance business, but also provides subsidy and reinsurance for the scheme. Although the commercial insurance company and the government have various share in the market and non-market mechanism, they are all belong to formal mechanism. And the fourth one is informal insurance mechanism consisted of fishermen, neighbors and investors. Each member who voluntarily joins a cooperative needs to pay a certain premium, then the benefit for the affected people is allocated from the premium paid by all the members. The government will give policy support through the system instead of involving in the process directly. France's agricultural disaster insurance cooperation model and the German flood disaster insurance cooperation model are typical types of the disaster insurance mechanism. France's agricultural insurance has a strict management level from the spontaneous, insurance companies to the top the central insurance company. And its association has reached more than ten thousand until 2015.Germany's flood disaster insurance originated much earlier. But around 1990, the government began to intervene in the insurance system to balance the growing premium and declining demand.

2.2 The states of disaster insurance in China

In 1992, the People's Insurance Company of China launched a disaster insurance product for houses in rural areas of Jiangxi Province. The loss of house caused by the natural disasters is in the range of compensation. The premium amount was set to 62 Yuan with the maximum amount was 2,000 Yuan. The government does not bear any liability, but to assist the insurance company to collect premiums. After 1997, due to the continued decline in bank interest rates, the interest income generated by premiums was decreasing. In 1998, a huge flood disaster took place in the Yangtze River Basin and Jiangxi Province suffered serious damage. The insurance company suffered huge compensation, which had been exceeding the limit of the company. Forced by operating pressure, PICC decided to withdraw from the market and the disaster insurance cooperation pilot failed.

China's insurance market was undeveloped for a long period of time; the unique product of disaster insurance was not introduced in modern society until agricultural insurance appeared in 2007 and catastrophe insurance appeared in 2016. Agricultural insurance is a typical and popular risk transfer channel for natural disasters that has been used in a variety of forms and has performed remarkably over the past few decades(Goodwin and Smith, 1995; Young et al., 2001; Enjolras et al., 2012).Government involvement is vital in terms of legal arrangements and fiscal support for the initiation and promotion of this insurance(Yang et al., 2015; Liu et al., 2010; Wang et al., 2012; Zhang and Stenger, 2014).Similarly, China's agricultural insurance is famous for its subsidies and flexibility, which means premiums are subsidized by the government, and the rating varies in accordance with the demand and risk level. This mechanism contains crops, forest, livestock, and fishery mutual insurance, nearly 95% of which belongs to policy-oriented agricultural insurance, and subsidies from the central government reach almost 40% in northeast and mid west areas and 35% in east areas in China. Fishery mutual insurance has a direct relationship with fishery, but it mainly focuses on fishing boats and fishermen's life insurance, and the aquaculture premium accounts for less than one in several million insurance premiums. As the fiscal subsides vary with society's level of economic development, the real coverage of the insurance is fairly low, less than 10% (NBSC, 2014). Moreover, despite the huge systemic risk of disasters in rural areas, insurance companies are not keen on operating these.

Earthquake insurance became the breakthrough for catastrophe insurance in China in 2016. It provides security for residential products, and its basic coverage is 50,000yuan, with the highest amount paid being less than 1,000,000 Yuan. Catastrophe insurance can offer benefits to poor and remote populations in a relatively fair manner (Hellmuth et al., 2009). Insurance in the marine sector has just been organized and does not have a long process like crop insurance in China. Typhoon index insurance came into effect in Guangdong in August 2016. The insurance company will compensate the government directly, and the government will be in charge of the recovery as both the policyholder and the insured party.

3 literature review

As early as the beginning of twentieth century, Britain, the United States, and other developed countries began to establish a system for flood insurance, hurricane insurance and other disaster insurance. In the long-term evolution and reform, disaster insurance mainly includes a pure government mode (such as flood insurance in the United States), a pure commercial mode (such as earthquake insurance in Japan), and a government-commercial insurance company cooperation mode (such as New

Zealand’s earthquake insurance). Mutenga S.(2007) confirmed that commercial insurance was explored along with its effectiveness and possible caveats in disaster risk transferring. Arthur Charpentier, Benoit Le Maux(2014) pointed out the natural catastrophe insurance industry is characterized by economies of scale. The government should consequently encourage the emergence of a monopoly and discipline the industry through regulated premiums. It is also shown that government intervention of last resort is not needed when the risks are highly, as an individual is consequently more likely to accept limited liability when faced with severe natural disasters under this situation.

However, there is no absolute limit among these three categories, and a government funded plan in combination with mainstream policy support is a still different type of form.Born,P. et al.(2006) found that insurers adapt to these catastrophic risks by raising insurance rates, leading to lower loss ratios after the catastrophic event. And these behaviours leaded to the exit of firms from the disaster insurance market. Firms with low levels of homeowners' premiums are most adversely affected by the catastrophes. John R Coomber(2006) analyzed the consequences and questions for the commercial insurance industry for disaster risk and argued that the insurance industry is confronted with ever bigger losses due to large disasters as well as with a series of new risks whose risk profile is extremely difficult to quantify. Meanwhile, Elisabetta Ioss(2012) studied the costs and benefits of public-private partnership and showed that relying on private finance enhances the benefits of bundling only if lenders have enough expertise to assess project risks especially when disaster insurance market being full filled with moral hazard in China. This analysis also highlighted the costs and benefits that bundling planning and implementation as under PPP models-can bring in terms of project design and operational costs under various scenarios, possibly allowing for asymmetric information, moral hazard and renegotiation.

Many studies on hurricane insurance, catastrophe funds, and other aspects of disaster insurance demonstrate that market failure has widely existed in the disaster insurance market, with significant public welfare characteristics, and the introduction of government participation is an important way to resolve the contradiction between supply and demand for disaster insurance （Smith K，2002；Froot K A，2008；Fang Li，2009；Yuexiu Ouyang，2010；Huiyan Ding，2010；Haixia Gao，2012). Craig E. L. and Mohammad R. J. (2010) studied the sensibility of individual residents' participation in proportion to government subsidies in a “Flood Disaster Loss Compensation” plan. Partricia H. B. and Barbara K. B.(2012) analysed the main factors that affect the decision-making of a commercial institution in the disaster loss compensation mechanism. Joanne L. B., Anna V. and Lisa B. （2013）, using the catastrophe model, established a Flood Loss Compensation System that includes public local organizations, government agencies and private insurance agencies.Shi, P., Shuai, J. et al.(2010) proposed that large-scale disaster risk transfer in China should be supported by governments at all levels, operated by insurance companies, and the responsibilities should be shared by all stakeholders, but they did not answer how to realize this cooperation mechanism.

Recent experience with disasters and terrorist attacks in the US indicates that state and local government rely on the federal sector for support after disaster occur.TJ Goodspeed(2012) proposed a second-best transfer scheme, which highlights the trade off between providing appropriate incentives for protection at the local level and insuring actual losses after a disaster occurs.The results indicated that when regional government undertake disaster prevention measures and act non-cooperatively, federal disaster insurance will result in underinvestment in pre-disaster protective investment. But the effectiveness of such a regime requires credible ex-ante commitment by the federal government. This commitment may be difficult to sustain. When the central government cannot commit to the second-best transfers and regions anticipated ex-post equalizing transfers, they will further under-invest to influence anticipated future transfers, taking advantage of a type of soft budget constraint. Bruggeman, V. et al.(2010) noticed that various forms of public-private partnerships have hence developed to solve the market failure. They confronted these practical examples with five main conditions that would have to be fulfilled to make government intervention efficient or at least as little disruptive as possible. They suggested the government could act as reinsurer of last resort or primary insurer in cases where it was held that the market did not offer any catastrophe insurance at all. Marine disaster such as storm surge has a typical characteristic of frequency and huge-losses in China, under this circumstance, the government would suffer a much heavier fiscal burden.

After recognizing the high fiscal deficits and bankruptcy risks faced by the pure government mode and the pure commercial mode（Mills et al.，2001; US Government Accountability Office，2007; Matthewset et al.，1999; Charpentier，2007; Florida Tax Watch Research Institute，2011）,it has been agreed that disaster insurance is complex and requires the participation of the government, commercial organizations and individuals. Accordingly, the public private partnership model (PPP) with multi-collaboration, risk sharing and cooperation has become a popular model for disaster insurance（Busch、Givens，2013；Chen et al.，2013；Lassa，2013；Jing、Besharov，2014；Zairol A. et al.，2014；Shiqing Xie，2009；Changli Zhang，2014). Hart(2003) pointed out under traditional procurement(pure commercial insurance mechanism or non-market insurance mechanism), the builder cannot internalize the impact of his effort neither on benefit nor on costs and, as a result, implements too little of both investments. Under a PPP, the builder partly internalizes the impact of his productive investment whereas he still exerts too much of the unproductive one. Bettett and Iossa(2006) studied the desirability of bundling and giving ownership to the investor in disaster insurance market. The holdup problem was less severe under a PPP, compared with traditional procurement, when there is a positive externality between building and managing assets. Martimort and Pouyet(2008) argued that PPP model and reduce operating costs and provide incentives for lower costs also boots incentives for a better design than non-market disaster insurance mechanism. And Peijun Shi （2014）discussed and structured China's integrated catastrophe system based on the PPP model.

Recently, the game model has become an effective tool to resolve the failure of disaster insurance market, especially in terms of moral hazard control, to devise a strategy for multi equilibrium participation. Einav et al. （2010）provided the endogenous characteristics of an insurance contract in the game model; Geoffroy de Clippel （2005）explored the Shapley Value using incomplete information and examples based on Shapley’s （1953，1967）game model; and Francoise Forges and Roberto Serrano （2011）considered alliance confirmation the most efficient solution in the game of disaster insurance with incomplete information. Additionally, Sadegh M. and Kerachian R. （2011）performed a search for optimal allocation of water resources using a cooperative game model. Furthermore, Arthur Charpentie and Benoît Le Maux （2014）built a game model with nonzero ruin probability, calculated the John Nash equilibrium solution of pure strategy and analysed the optimal government intervention behaviour in different risk areas. Furthermore, the integrated model is used to analyse the equilibrium issue of multiple subjects. While thinking about the market failure problem, Jiazhi Xie （2013）comprehensively considered the transaction costs and operational risks of agricultural catastrophe insurance and used the Shapley method to analyse the main coalition game model of agricultural catastrophe insurance. Zhanqing Sun （2015）constructed a four party alliance under the mechanism and then established a single period static game model and a repeated game model, as well as obtained the conditions for ensuring the operation of the alliance.

In September 2015, Asia Pacific Economic Cooperation(APEC) Finance Ministers acknowledged that the region is now experiencing more than 60% of the world's disasters, with damages reaching $1.2 trillion USD in the past decade(2001-2010).Unfortunately, insurance against low-probability, high-consequence events presents a challenge for individuals at risk, insurers and regulators for good reason. Decision makers have limited experience with these events and even experts are likely to conclude that there is considerable uncertainty as to the probability of these events occurring and their resulting consequences. As a result, insurance decisions often differ from those recommended by normative models of choice.

Most homeowners in marine disaster prone areas do not voluntarily purchase the insurance until after they suffer damage from a disaster. If they then do not experience losses in the next few years, they are likely to cancel their policy. Similarly, demand for disaster insurance in Jiangxi Province China increased significantly after the flood in 1998.In most Asian countries, state-owned insurance companies are involved in disaster insurance. The main reason behind the absence of private players is the dominance of small and marginal products in the production portfolio who operate at limited scale and are therefore unattractive for the private players. Stated insurance regulators sometimes have restricted insurers from setting premium that reflect risk in order to satisfy the policyholders' affordability. However, marine disaster insurance market has high chance of moral hazards and lack of adequate data, that is, limited premium level is prone to a poor operating of commercial insurance companies.

In pure market mode, premium income is the only source of insurance company operating funds. Due to the devastating effects of the marine disasters, it can result in huge insurance payments once a disaster occurs. Insurance companies tend to raise their rates in order to ensure adequate claims and profits, while the cost increases will reduce the willingness of insurers to participate and ultimately reduce the size of the market. In the pure government model, though it can effectively guarantee the public welfare of disaster insurance, the pressure of payment is entirely borne by the financial funds, which in turn leads to the excessive occupation of financial funds, which is also obviously unfair to the residents in non-coastal areas. Almost all of the losses had to be absorbed by the victims and government. Furthermore, as China’s current disaster risk management strategy primarily consists of ex-post relief programs, the huge loss also is a major burden to fiscal government. Numerous studies have highlighted the inefficiency of this system and the necessity of involving commercial insurance in marine disaster risk management.

It was ascertained that the effectiveness and scientific nature of multiple participants in disaster insurance. The development of the marine economy and the serious repercussions of marine disaster support a feasible argument for a marine disaster insurance system, which has an obvious feature of moral hazard and poor ability to pay. Due to the existence of cognitive differences, asymmetric information, systematic structural and stochastic volatility, and other factors, it is obvious that the benefit matrix calculation made by players in advance is biased. Under this situation, the symmetrical PPP model widely used in previous studies failed to obtain a feasible and clear cooperation strategy. To solve this problem, it is necessary to build a new asymmetric PPP model for marine disaster insurance. The innovation of this paper lies in: When using the traditional game model to solve the multi-agent participation in disaster insurance equilibrium strategy, the participants will increase the qualifications or pre-set the strategic choice of the game side, which simplifies the uncertainties in the decision-making of participants. This paper introduces the gray game model, which can take full account of the uncertainty in game decision-making and make up for the shortcomings of the traditional game theory after simplifying the research and finding the low practical value.

4 Marine Disaster Insurance Based on the Grey Game Model

4.1 Asymmetric PPP Cooperation Framework for Marine Disaster Insurance

The most ideal mode of cooperation should be as many participants as possible to participate in the cooperation, each of the partners have a certain degree of economic power to bear part of the risk losses. This paper proposes to establish an asymmetric PPP marine disaster insurance cooperation model of "the original insurance + reinsurance", "market fragmentation + individual retention" by the government-led transition to commercial insurance companies. On one hand, the government provides subsidies and reinsurance support to policyholders and insurance companies. On the other hand, it uses its coercive power to carry out popularization and publicity of prevention awareness among the general public and actively address the issue of moral hazard in the disaster insurance market. With the growth of the market mature government assumes responsibility, the proportion of anti-subsidy will gradually decline. The insurance company is responsible for the operation of the specific policy design, marketing, management and claims, self-financing. The retention of reinsurance and policyholder can help insurance company better spread risk. In a market economy, each participant hopes to get the maximum return with the minimum payment. Therefore, when signing the policy, the insured may hide himself risk or inform the insurer of the fact of being false or non-existent, resulting in adverse selection. Therefore, the insured should also become part of the disaster insurance cooperation mechanism and bear the disaster loss below the retention.

4.2 The process of decision making

A. The grey game model

The grey game model is based on the uncertainty of both sides of the game, and it can fully consider the uncertainty in the game, which has better flexibility than the traditional game model using an interval grey matrix. The use of grey game theory to solve actual economic problems is a new development in the relevant fields in recent years, which is based on the growth of grey game theory. Fang Zhigeng and Liu Sifeng (2003) studied the problem of the grey matrix game based on non-precise mathematics. They analysed the relation and difference between the grey matrix game and the classical matrix game based on pure strategy and set up a strict standard and a normal grey matrix game. According to the solution of the grey game matrix, which cannot be directly determined by the interval grey number, Mi Chuanmin and Fang Zhigeng (2005) proposed a superior strategy law under the grey number potential based on the decision rules of interval grey numbers and defined a pure strategy solution. Luo and Wu Shunxiang (2005) discussed two person finite zero sum game problems using a hybrid strategy according to the thought and method based on Grey System Theory and presented the concept of two people with a mixed strategy and the grey game theory. Fang Zhigeng, Liu Sifeng, and Ruan Aiqing (2006) proposed the concept of advantage/equilibrium and disadvantage of the interval grey number and defined the pure strategy solution of the grey number potential. In addition, they presented the necessary and sufficient conditions for the existence of the pure strategy solution (or grey saddle point). Fang Zhigeng, Liu Sifeng, and Chen Hong (2007) defined the concept of an extended square matrix of grey full rank matrix between player 1 and player 2 and gave player 1 and player 2 the optimal grey game strategy and the optimal grey game worth solving method. Additionally, Xu Huafeng, Li Lingling, and Fang Zhigeng (2012) provided the general form of the grey matrix game under the situation of the payment matrix and grey constraint conditions and proved that the game equilibrium of the grey double matrix can be obtained by solving a nonlinear programming problem. Furthermore, Zhou Xingwang, Fu Zhong, and Pang Shichun (2015) put forward a new model based on the information loss of the grey game network and applied the grey system in the economic field in their dissertations.

The matrix of the grey game is as follows: $G'(\otimes)=\{S'_1,S'_2;A'(\otimes)\}$, where $S'_i=\{\alpha'_1,\alpha'_2,\cdots,\alpha'_m\}$, S'_i is defined as the players' strategy, and $A'(\otimes)$ is defined as they players' grey profit and loss value matrix based onearly judgement. The two person grey game benefit matrix is shown in Figure 1.

		Player2			
		ST 1	ST 2	…	ST n
Player 1	ST 1	$[a_{11,}b_{11}]$, $[c_{11},d_{11}]$,	$[a_{12,}b_{12}]$, $[c_{12},d_{12}]$,	…,…	$[a_{1n,}b_{1n}]$ $[c_{1n},d_{1n}]$
	ST 2	$[a_{21,}b_{21}]$ $[c_{21},d_{21}]$	$[a_{22,}b_{22}]$ $[c_{22},d_{22}]$	…,…	$[a_{2n,}b_{2n}]$ $[c_{2n},d_{2n}]$
	…	…,…	…,…	…,…	…,…
	ST m	$[a_{m1,}b_{m1}]$ $[c_{m1},d_{m1}]$	$[a_{m2,}b_{m2}]$ $[c_{m2},d_{m2}]$	…,…	$[a_{mn,}b_{mn}]$ $[c_{mn},d_{mn}]$

Figure 1Grey game benefit matrix

where a_{ij},b_{ij}, c_{ij},d_{ij}（i=1，2，…，m；j=1,2，…，n）are the values of the interval grey number of player 1 and player 2, respectively. Taking into account people's rational behaviour, players not being excessively risky, and realizing that the opponent is trying to gain as much income as possible, it should be feasible to choose the potential maximum values from the minimum situations to determine the grey game equilibrium strategy.

B. Basic assumptions

In the process of establishing the grey game model, the following main factors for participants are included in the process:

(1) Players. A player is an individual or organization that is independent of the outcome of the game. The two players of the game are: 1-- the public and insurance companies; and 2-- reinsurance institutions and insurance companies. The player makes two assumptions: ①the "economic man" hypothesis. Everyone wants to achieve his or her own utility maximization goal under certain conditions; and ②although there may be differences of opinion and conflict among the members of the group, the asymmetry of power and responsibility mainly occur in market powers and non-market-oriented powers, assuming that their internal contradictions have been eliminated.

(2) Strategy set. Strategy sets are collections of all possible strategies for a player. Each player can choose a variety of methods when making decisions, and at least two different strategies should be set for each strategy. The strategy for reinsurance companies is whether to operate catastrophe reinsurance; similarly, the strategy for insurance companies is whether to accept reinsurance. The strategy for the publicist whether to pay for the insurance, while the strategy for an insurance company is whether to join the marine disaster insurance market.

(3) Players’ payoffs. When all the players have adopted the final strategy, they will obtain the corresponding benefit. We use payoff function to represent the benefit of the player, which is also the function of all the players.

(4) The order of play. The original insurance transaction and the reinsurance transaction are the main aspects of marine disaster insurance. Hence, there minder of the paper will present the analysis of these two parts.

C. Solution for the equilibrium strategy

Model Ⅰ: Game between an insurance company and a reinsurance institution. The basic set of variables is as follows:

A: Reinsurance company's compensation for the insurance company, $A=\gamma L$;

B: Insurance company's expenses for reinsurance (here only considering the most traditional form of reinsurance);

L: The insurance company's compensation when disaster occurs;

E: Operating costs of the reinsurance company;

ε : The government's subsidy ratio to the reinsurance company;

λ :The government's subsidy for the insurance company buying reinsurance;

γ: The compensation rate from the reinsurance company to the insurance company;

For reinsurance companies are participating in the marine disaster insurance market and the insurance company buying reinsurance, $[B-A-E, B-A-E+\varepsilon L]$ and $[A-B-L, A-B-L+\lambda B]$ represent their respective payoffs. If the insurance company does not choose reinsurance, their payoffs are -E,-L. The grey game matrix of the two is shown in Table 1:

Table 1.

THE GREY GAME MATRIX BETWEEN ANINSURANCE COMPANY AND THE REINSURANCE ORGANIZATION

		Reinsurance Company	
		Offer	Does not Offer
Insurance Company	Insures	$[A-B-L, A-B-L+\lambda B]$, $[B-A-E, B-A-E+\varepsilon L]$	-L,0
	Does Not Insure	-L,E	-L,0

When the government does not provide subsidies for reinsurance companies and insurance companies, that is $\lambda=0, \varepsilon=0$, the grey game model of the insurance company and the reinsurance company becomes a traditional game model, and the benefit value of both sides is changed into certain white numbers from the interval grey number. Using the marking method for comparing the size of the benefits, the result shows that when the reinsurance companies engage in marine disaster reinsurance, insurance companies will choose reinsurance for risk sharing. Regardless of whether an insurance company chooses to share the risk through reinsurance, reinsurance companies will not offer it. Therefore, the equilibrium strategy of the game is that the reinsurance agency does not operate marine disaster reinsurance, and the insurance company does not cover marine disaster insurance. The PPP participation mode fails. How can the optimal equilibrium be achieved under the PPP model? We introduce the third main body: the government.

Case2: The government participates in marine disaster insurance, and the ratio of subsidies are ε and λ, then there is

Th.1: If the reinsurance company operates marine disaster insurance, the game results are that insurance companies choose to share the risk through reinsurance, which is independent ofthe insurance subsidy ratio λ.

Proof: Let $\pi_5(\otimes)\in[a_5,b_5]=[A-B-L,A-B-L+\lambda B]$, $\pi_6(\otimes)\in[a_6,b_6]=[-L,-L]$

The dominance degree of grey number $\pi_5(\otimes)$ with respect to the number $\pi_6(\otimes)$ is

$$M_{s5}=\frac{(b_5-b_6)}{(b_5-a_5)}=\frac{A-B+\lambda B}{\lambda B}$$

The cost of reinsurance is far less than the insurance company's compensation, therefore $A-B+\lambda B>0\Rightarrow M_{s5}>0$,

The inferior degree of grey number $\pi_5(\otimes)$ with respect to the number $\pi_6(\otimes)$ is $\pi_5(\otimes)>\pi_6(\otimes)$ $\Rightarrow$ insurance companies choose reinsurance.

According to the "economic man" hypothesis, everyone wants to achieve their own utility maximization goal, and when $A-B+\lambda B>A-B$, the reinsurance subsidy ratio supplied by the government will play a positive role to encourage insurance company to choose reinsurance. When the government gives no subsidy for reinsurance, insurance companies will choose reinsurance to share the risk. Similarly, the insurance company will be more inclined to buy reinsurance if there is a government subsidy, giving rise to the growth of reinsurance.

Th.2: If $\varepsilon>\frac{A-B+E}{A}$, when the insurance company chooses to insure, the result of the game is that the reinsurance company chooses to purchase marine disaster reinsurance.

Proof: Let $\pi_7(\otimes)\in[a_7,b_7]=[B-A-E,B-A-E+\varepsilon A]$, $\pi_8(\otimes)\in[a_7,a_8]=[0,0]$.

The dominance of grey number $\pi_7(\otimes)$ with respect to the grey number $\pi_8(\otimes)$ is $M_{s6} = \frac{B-A-E+\varepsilon A}{\varepsilon A}$,

As $M_{S6} > 0 \Rightarrow \varepsilon > \frac{A-B+E}{A}$, the inferior degree of grey number $\pi_7(\otimes)$ with respect to the grey number $\pi_8(\otimes)$ is $M_{16} = 0$, $\Rightarrow$ $M_{s6} + M_{16} > 0$, $b_7 - b_8 = B - A - E + \varepsilon A > 0 \Rightarrow b_7 > b_8$, $b_8 - a_7 = 0 - (B - A - E) = A - B + E > 0$, $\Rightarrow$ $b_8 > a_7$, $\Rightarrow b_7 > b_8 > a_7$,

$\Rightarrow$ $\pi_7(\otimes) > \pi_8(\otimes)$, $\Rightarrow$ reinsurance companies choose to operate marine disaster reinsurance.

We define $\frac{A-B+E}{A}$ as the real compensation cost of reinsurance. When $\varepsilon < \frac{A-B+E}{A}$, $\pi_7(\otimes) < \pi_8(\otimes)$, reinsurance companies do not choose to operate marine disaster reinsurance. In addition, with the increase of ε, $\pi_7(\otimes) < \pi_8(\otimes)$ shows a downward trend; when $\varepsilon > \frac{A-B+E}{A}$, $\pi_7(\otimes) > \pi_8(\otimes)$, reinsurance companies begin to choose to operate marine disaster reinsurance, and with the increase of λ, $\pi_7(\otimes) > \pi_8(\otimes)$ shows an upward trend, and the enthusiasm of reinsurance companies gradually shows an upward trend.

Game model two: the process of the grey game between the public and insurance companies. The basic set of variables is as follows:

L: The compensation of the insurance company to the public when marine disaster risk occurs;

P: The public’s premium;

R: Actual total lossfrom marine disaster;

C: Operating costs of insurance companies;

α: Government subsidy ratio to insurance companies;

β:Government subsidy ratio to the public;

γ: Compensation ratio to reinsure $[P-L-C, P-L-C+\alpha L+\gamma L]$ insurance companies;

When the insurance company chooses to operate catastrophe insurance, the public might obtain compensation for loss if they choose to insure. Their benefit is $[L-P-R, L-P-R+\beta P]$, and the insurance company's benefit is $[P-L-C, P-L-C+\alpha L+\gamma L]$.If the public does not choose to insure, the benefit for them is $[-R,-C]$. If the insurance company does not choose to operate this type of insurance, the benefit would be $[-R,0]$. The grey game matrix for this scenario is shown in Table 2:

Table 2. The grey game matrix between the public and the insurance company

		Insurance Company	
		Offer	Does not Offer
Public	Insures	$[L-P-R, L-P-R+\beta P]$, $[P-L-C, P-L-C+\alpha L+\gamma L]$	-R,0
	Does Not Insure	-R,C	-R,0

Case 1: The government does not provide any subsidies for marine disaster insurance, and reinsurance companies do not participate in the insurance market, that is, α=0,β=0,γ=0. The public and the insurance company's grey game model becomes a traditional game model, so the benefits of the public and insurance companies are transferred from the grey number into a certain white number. By comparing the two sides' benefits, we know that whether the public chooses to be insured depends on the insurance company's decision to offer marine disaster insurance or not. However, regardless of how the public chooses, the insurance company would operate this insurance. Therefore, the equilibrium strategy is: do not insure, do not operate, and PPP mode failure. Therefore, how can we achieve the optimal equilibrium under the PPP scheme? We will add a third main subject: the government.

Case 2: the government provides subsidies to insurance companies and the public and the subsidy ratios are α and β, thus:

Th. 1: If the insurance companies operate marine disaster insurance, then the result of the game is that the public chooses to insure, which is almost decided by β.

Proof: Let $\pi_1(\otimes)\in[a_1,b_1]=[L-P-R, L-P-R+\beta P]$, $\pi_2(\otimes)\in[a_2,b_2]=[-R,-R]$

The dominance of the grey number $\pi_1(\otimes)$ with respect to the grey number $\pi_2(\otimes)$ is $M_{s1}=\frac{(b_1-b_2)}{(b_1-a_1)}=\frac{L-P+\beta P}{\beta P}$, One specification for disaster insurance is that the premium is always less than the compensation, so $L-P+\beta P>0 \Rightarrow M_{s1}>0$.

The inferior degree of grey number $\pi_1(\otimes)$ with respect to the grey number $\pi_2(\otimes)$ is $M_{11}=0 \Rightarrow$ $\pi_1(\otimes)>\pi_2(\otimes) \Rightarrow$ the public chooses to insure. Under the "economic man" hypothesis, as $L-P+\beta P>L-P \Rightarrow$ the government subsidy for insurance positively affects the public's decision to insure. When β=0, the public will choose to insure according to their own interest. In addition, as β increases, the enthusiasm of the public shows an upward trend.

Th.2: If $\alpha+\gamma>\frac{L-P+C}{L}$, when the public chooses to insure, the result of the game is that insurance companies choose to operate marine disaster insurance.

Proof: Let $\pi_3(\otimes)\in[a_3,b_3]=[P-L-C,P-L-C+\alpha L+\gamma L]$, $\pi_4(\otimes)\in[a_3,a_4]=[0,0]$.

The dominance of grey number $\pi_3(\otimes)$ with respect to the grey number $\pi_4(\otimes)$ is $M_{s3}=\frac{P-L-C+\alpha L+\gamma L}{\alpha L+\gamma L}$.

$\because \pi_4(\otimes)\ \alpha+\gamma>\frac{L-P-C}{L} \Rightarrow M_{S3}>0$ the inferior degree of grey number $\pi_3(\otimes)$ with respect to the grey number is $M_{13}=0$, $\Rightarrow M_{s3}+M_{13}>0$, $\Rightarrow b_3>b_4$, $b_4-a_3=0-(P-L-C)=L-P+C>0$, $\Rightarrow b_3>a_4$, $\Rightarrow b_3>b_4>a_3$, $\Rightarrow \pi_3(\otimes)>\pi_4(\otimes)$, $\Rightarrow$ insurance companies choose to operate marine disaster insurance.

We define $\frac{L-P+C}{L}$ as the real compensation cost of insurance. When $\alpha+\gamma<\frac{L-P+C}{L}$, $\pi_3(\otimes)<\pi_4(\otimes)$ and insurance companies do not choose to operate marine disaster insurance. As α increase, $\pi_3(\otimes)<\pi_4(\otimes)$ shows a downward trend. Until $\alpha+\gamma>\frac{L-P+C}{L}$, there will be $\pi_3(\otimes)>\pi_4(\otimes)$, the insurance company will join this market, and $\pi_3(\otimes)>\pi_4(\otimes)$ will show an upward trend as α increases. Furthermore, the enthusiasm of the insurance companies gradually increases, as well.

5 Stability ANALYSES of equilibrium strategies

Simulate the game model established above with the Monte Carlo Simulation, and use uniform distribution and normal distribution sampling calculation with each parameter, compare the results of different distributions, then the equilibrium condition of gray game analysis is obtained, in order to calculate the risk probabilistic of participating in the cooperation of different parameters, and provides reference for participation in decision-making.

5.1 Monte Carlo simulation based on uniform distribution

At the first, the simulation operation which based on the uniform distribution is carried out. In the simulation of the game, six parameters need to be estimated in advance, parameters of simulation are as follows: B_b and B_b are defined as the minimum possible parameter and the maximum possible parameter of premiums that insurance companies must pay for Marine disaster reinsurance, respectively; A_a and A_b are defined as the minimum and the maximum possible parameters of reinsurance company's expenditure for compensation liability, respectively; E_a and $E_b$are defined as the minimum and the maximum possible parameters of the cost when a reinsurance company operate marine disaster insurance, respectively. Suppose that$W = B - A - E$, the prediction of the above six parameters is shown in Table 4-1.

The probability density function of the uniform distribution (continuous) is as follows:

$$f(x) = \begin{cases} \dfrac{1}{b-a}, a \le x \le b \\ 0, \qquad others \end{cases}$$

The cumulative distribution function is as follows:

$$F(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x < b \\ 1, & x \ge b \end{cases}$$

In the above expression, X is a random selected sample, and $ab$are the minimum possible parameters and maximum possible parameters, respectively.

Table 4-1: The Parameter Estimated Value of B_a ,B_b, A_a, A_b, E_a, E_b

Parameter Estimated Value	The Minimum Estimated Value (unit: 1000,000 Yuan)	The Maximum Estimated Value (unit: 1000,000 Yuan)
B	0	122
A	0	231
E	0	12

Specify the number of simulations, set $N = 2000$, for the six available values of the table 4-1, the simulation process based on uniform distribution is adopted. Through the simulation operation, we get the frequency statistics of the benefit value obtained by each participant in the cooperation process under the equilibrium strategy.

According to the Th2 of the previous chapter, where $\varepsilon > \frac{A - B + E}{A}$, insurance companies decide to insure marine disaster reinsurance, the result of the game is that reinsurance companies choose to operate marine disaster reinsurance. This means $W = B - A - E > 0$.Statistics the probability of $W > 0$, the probability of the reinsurance company participates in cooperation can be calculated if the insurance company chooses to purchase marine disaster reinsurance.

Call the $Unifcdf$ command function in $Matlab$, calculate the cumulative probability of $W \le 0$, and obtain a result:

$$P_{(W \le 0)} = 0.6693,$$

$$\because P_{(W > 0)} = 1 - P_{(W \le 0)},$$

$\therefore$ The probability of $W = B - A - E > 0$ can be calculated:

$$P_{(W > 0)} = 1 - 0.6693 = 0.3307$$

In other words, when the insurance company chooses to purchase marine disaster reinsurance, the loss rate of reinsurance companies participating in cooperative is 33%, and its cumulative probability and risk distribution is shown in figure 4-2:

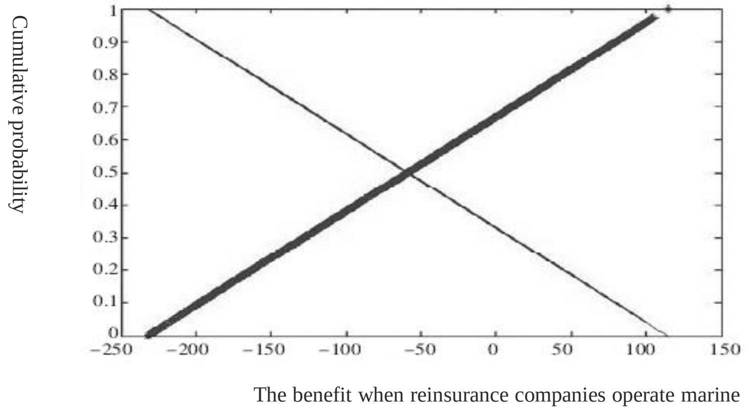

Figure 4-2: The Cumulative Probability and Risk When Reinsurance Companies Operate Marine Disaster Reinsurance

As the figure 4-2 shown, if reinsurance companies choose to participate in the cooperation, the thicker line represents the cumulative probability, and the thinner line is the risk curve. According to figure 4-2,as long as we get the benefit value of reinsurance companies participating in cooperation, the probability of corresponding risks can be calculated; On the contrary, if we get the risk probability of reinsurance companies participate in the cooperation, the benefit value also can be calculated.

5.2 Monte Carlo simulation based on normal distribution.

The second step is to execute a simulation based on a normal distribution. In terms of parameters, we still use the six parameters estimated by table 4-1, the mean is obtained by averaging the maximum and minimum values, which is to say: $\mu_B = \frac{B_a + B_b}{2} = 61$,

$\mu_A = \frac{A_a + A_b}{2} = 115.5$, $\mu_E = \frac{E_a + E_b}{2} = 6$, assume the standard deviation $\sigma=5$,namely $\sigma_A=\sigma_B=\sigma_E=5$.

The probability density function of the normal distribution is as follows:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{(x-\mu)^2}{2\sigma^2}}, x \in (-\infty, +\infty)$$

The cumulative distribution function is as follows:

$$F(x) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{x} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

In the expression above, x is a random selected sample based on normal distribution, μ, σ are defined as population mean and population standard deviation,respectively.

Similar to the previous studies, specify the number of simulations, set $N = 2000$, sampling simulation with normal distribution theory which based on the estimated value of the parameter in above, and draw the corresponding histogram. Figure 4-3 ~ 4-6, in the equilibrium strategy, respectively, are the frequency statistical graph of the premium of the insurance company insure marine disaster, the frequency statistical graph of the compensation of reinsurance company should assume, the frequency statistical graph of the operation cost of reinsurance company, the frequency statistical graph of the benefit value (W) of reinsurance companies while they participate in cooperation.

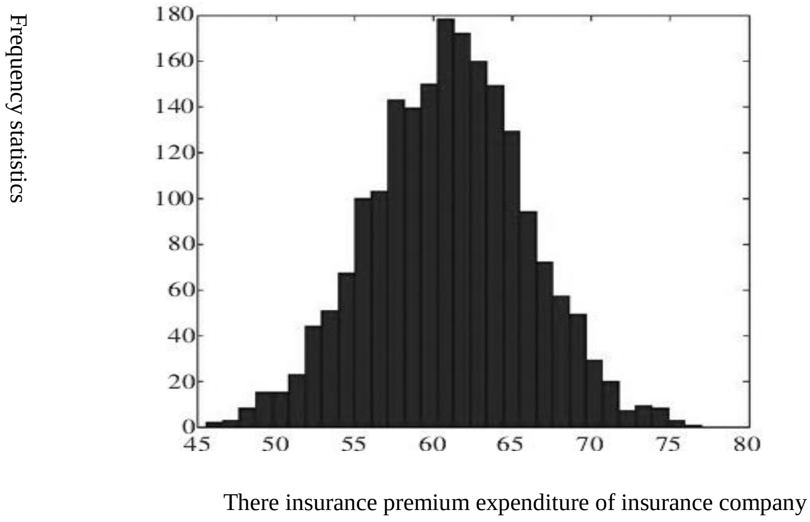

Figure 4-3: The Frequency Statistical Graph of the Reinsurance Premium Expenditure of Insurance Company

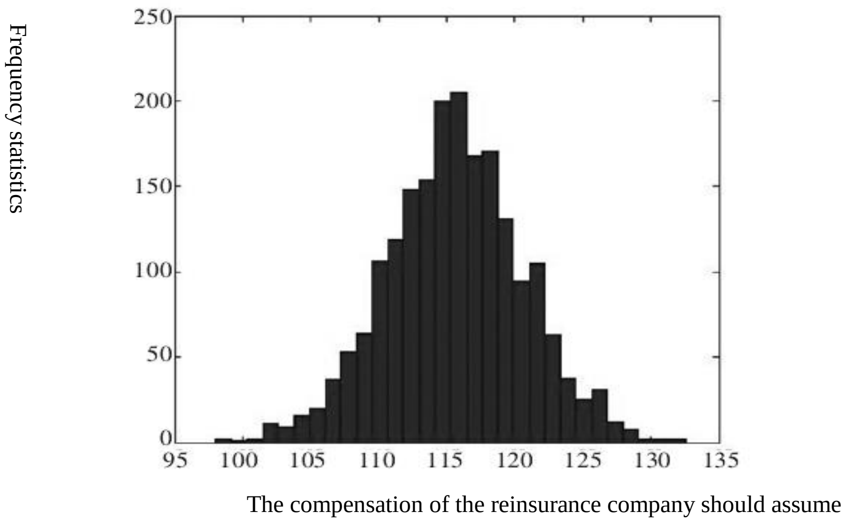

Figure 4-4: The Frequency Statistical Graph of the Compensation of the Reinsurance Company Should Assume

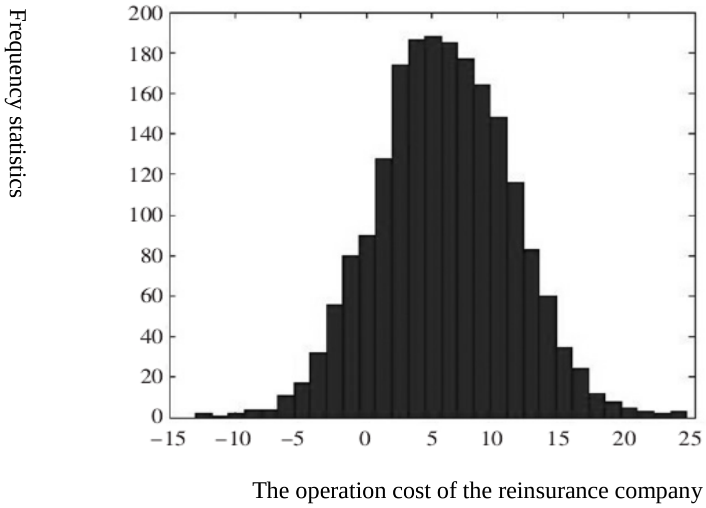

Figure 4-5: The Frequency Statistical Graph of the Operation Cost of the Reinsurance Company

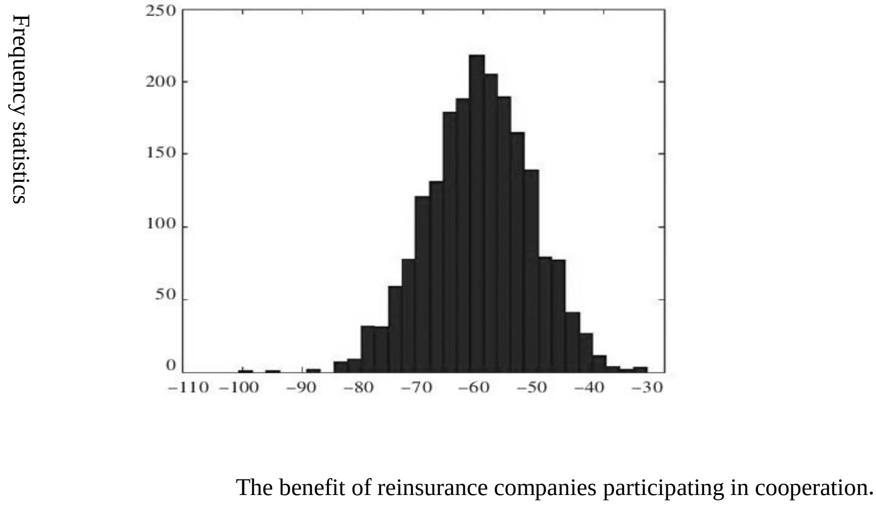

Figure 4-6: The Frequency Statistical Graph of the Benefit Value of Reinsurance Companies Participating In Cooperation.

Analysis figure 4-6, if the reinsurance company chooses to participate in the cooperation, as shown in the figure, all of the benefit values are negative, that is $W = B - A - E < 0$, it does not match the previous assumptions. Call the *normpdf* command function in *Matlab*, calculate the probability of taking various benefit values. As shown in Figure 4-7:

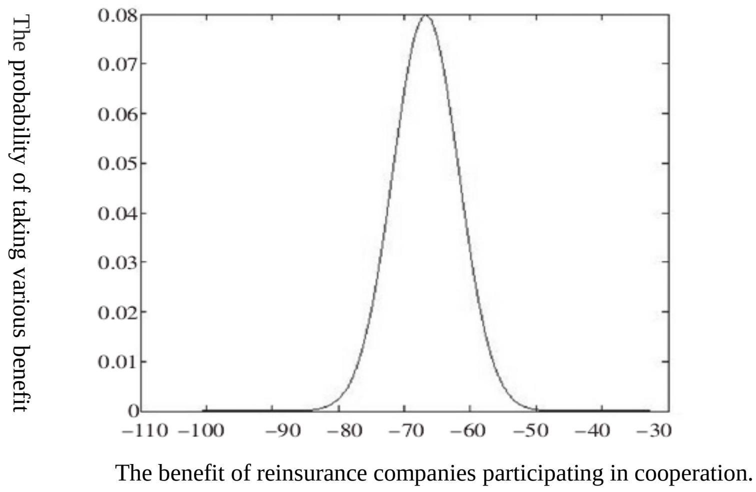

Figure 4-7: The Benefit Probability Graph of Reinsurance Companies Participating In Cooperation

From figure 4-7 we can see that the probability of taking all of benefit values are extremely small, it’s prove that, in terms of probability, reinsurance companies will not participate in cooperation.

6 Conclusions

As time goes on, the economic losses caused by marine disasters are continue to increase, disaster risk prevention becomes the focus of attention. Insurance is an important part of marine disaster risk prevention mechanism, and plays the role of "social stabilizer". At present, marine disaster insurance mechanism has not yet established formally in China. The frequent occurrence of marine disasters poses new challenges and tests for risk diversification, thus creating more insurance demand. The cooperative model of marine disaster insurance in developed countries is constantly developing and improving, however, when China faces marine disaster risk, the national finance assumes most of the compensation responsibility, which resulting the government as "the final insurer" overwhelmed,multiparty cooperation mechanism plays an extremely limited role in marine disaster risk management. Therefore, use the experience of developed countries for reference, to set up a marine disaster insurance cooperation mode that suit for China's national conditions. It is imminent to establish and perfect marine disaster insurance system.

On the basis of predecessors' research, this paper makes a comparative analysis on the basic situation, operation effect and the role orientation of the existing four kinds of disaster insurance cooperation modes. Considering the operational environment of marine disaster insurance, it is necessary to establish a reasonable marine disaster insurance cooperation mode in China. Introducing various participant, and designing a theory framework for marine disaster insurance with two-dimensions and four-parts cooperation, that is “based on commercial mode, the original insurance and reinsurance two-dimensional, policy holder risk retention, government participation”. The cooperation framework is dominated by commercial insurance companies and reinsurance companies, operating marine disaster insurance and reinsurance business independently. Government and the policy holder are participants, the government is focused on policy making and providing subsidies, the policy holder insured and takes a part of self-retention risk.

In order to solve the problem existing in the cooperation model of marine disaster insurance, including: policy holders’ over-decentralization; insurance companies facing excessive cost and high risk; the contradiction between the adverse selection, moral hazard and the law of large numbers in the insured process, some measures must be taken to ensure that these problems are resolved and further guarantee the stability of the cooperation mode. In this paper, we introduce the grey game model, and establish a two-dimension grey game based on the two-dimension cooperation framework, to achieve the realization condition of equilibrium strategy of marine disaster insurance with two-dimension four-part cooperation mode.Empirical results show that only if the insurance company operates marine disaster insurance, the public will definitely choose to insure it. The higher the amount of government subsidy, the stronger the enthusiasm of participants involved in cooperation; when the public demands insurance, only when the subsidy ratio provided by the government exceeds the real compensation cost of insurance and reinsurance respectively, the commercial insurance and reinsurance companies will join in the market.

Taking the insurance company and the reinsurance company as examples, employ the Monte Carlo simulation based on uniform distribution and normal distribution respectively to simulate the gray game model, then we can get the frequency statistical graph of the premium of the insurance company insure Marine disaster, the compensation of reinsurance company should assume, the operation cost of reinsurance company, and the benefit value of reinsurance companies while they participate in cooperation. Calculate the risk probabilistic of participating in the cooperation, in order to provide reference for the cooperation mode of marine disaster insurance, and we can conclude that the final cooperation strategy will be different when the distribution of parameters is different. Therefore, against specific marine disaster insurance, we need to analyze the distribution of parameters in order to provide more accurate decision-making reference for each subject of marine disaster insurance. After the traditional stability analysis of the game results, it is concluded that the more effective the government can guarantee the smooth implementation of subsidies, the stronger the stability of the game between the various main elements. Therefore, the smoothness of the government subsidy is also affect the stability of the cooperation model.

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