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ESTIMATION OF THE DISTRIBUTION OF REMANING LIFE TIME OF THE PEOPLE IN TURKEY

Abstract:

The distribution of lifetime is one of the most important factors for the determination of the population volume. Knowing the population volume is of great importance for the future planning. For the future planning depending on the population volume, it is necessary to know the distribution of lifetime. Accordingly in this study, the distribution of lifetime of people in Turkey was examined. Obtained from the Turkish statistical Institute (TUIK) lifetime statistics of the years between 2004 and 2012 were used in this study. Data contain lifetime between 0-105 years. Lifetime distribution was found for each year. Furthermore the probability distribution was found that a person who was known to have lived at least c years might live more at least x years. Lifetime was modelled by three parameters Dagum distribution.

Keywords:

Lifetime Distribution, Dagum Distribution, Parameter, Estimation, Average Life

JEL Classification: C15, C13, C55

1. Lifetime Distribution for General, Male and Female Between 2004-2012 Years

Lifetime was modeled by three parameters Dagum Distribution for general, male and female between 2004-2012 years. For the three parameters Dagum distribution with probability density function (pdf) and cumulative distribution function (cdf), respectively are given below,

$$f(x)=\frac{\alpha k\left(\frac{x}{\beta}\right)^{\alpha k-1}}{\beta\left[1+\left(\frac{x}{\beta}\right)^{-\alpha}\right]^{k+1}}, \qquad x>0$$

$$F(x)=\left[1+\left(\frac{x}{\beta}\right)^{-\alpha}\right]^{-k}$$

where, $\alpha, k>0$ are shape parameters, $\beta>0$ is scale parameter.

2. Modelling of The Lifetime For 2004 Year

The Distrubution of general, male and female lifetime modeling three parameter Dagum distribution for 2004 year.

Table 2.1. General, Male and Female Distribution Values for 2004 Year

Distribution		Parameters	Value	*p* value for K-S Test	Mean	Std. Dev.
General	Dagum	k	0.111	0.516	64.08	18.467
		α	24.51			
		β	86.74			
Male	Dagum	k	0.113	0.560	61.92	18.024
		α	23.80			
		β	84.02			
Female	Dagum	k	0.104	0.436	66.74	18.493
		α	27.45			
		β	89.46			

As seen in Table 2.1. the average life expectancy of females are more than males for 2004. For the year 2004 on the distribution of general, male and female lifetime is below the probability density function (pdf) and cumulative distribution function (cdf) respectively (Şanlı T., 2013)

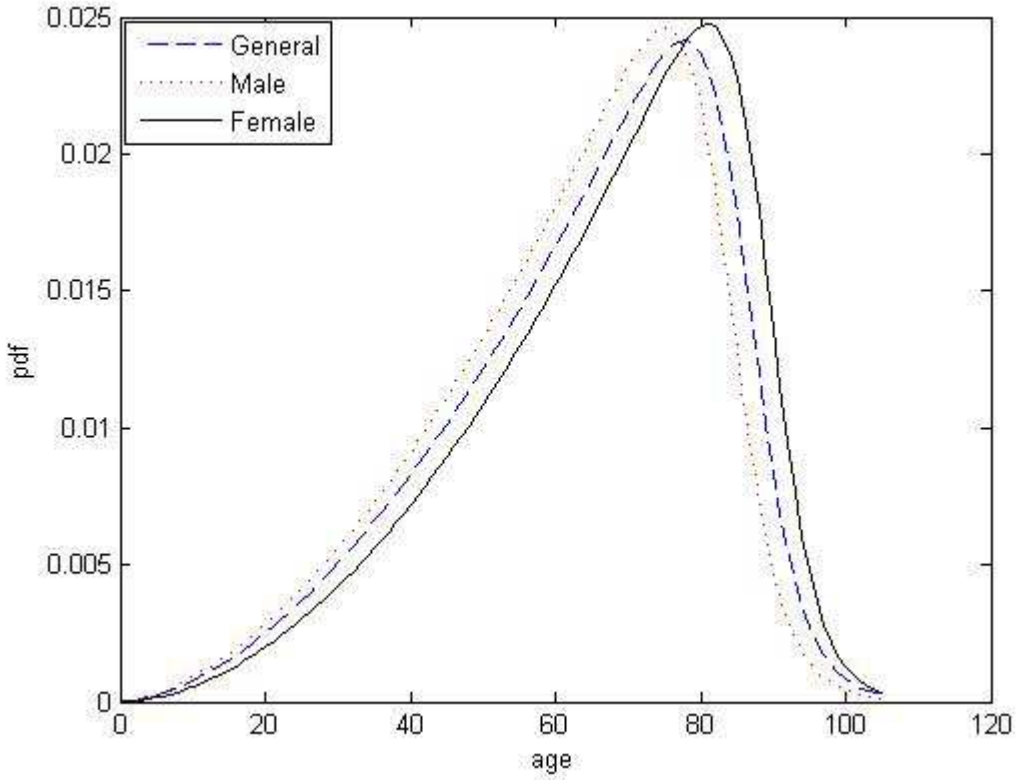

Figure 2.1. Probability Density Function of General, Male and Female's Lifetime for 2004 year

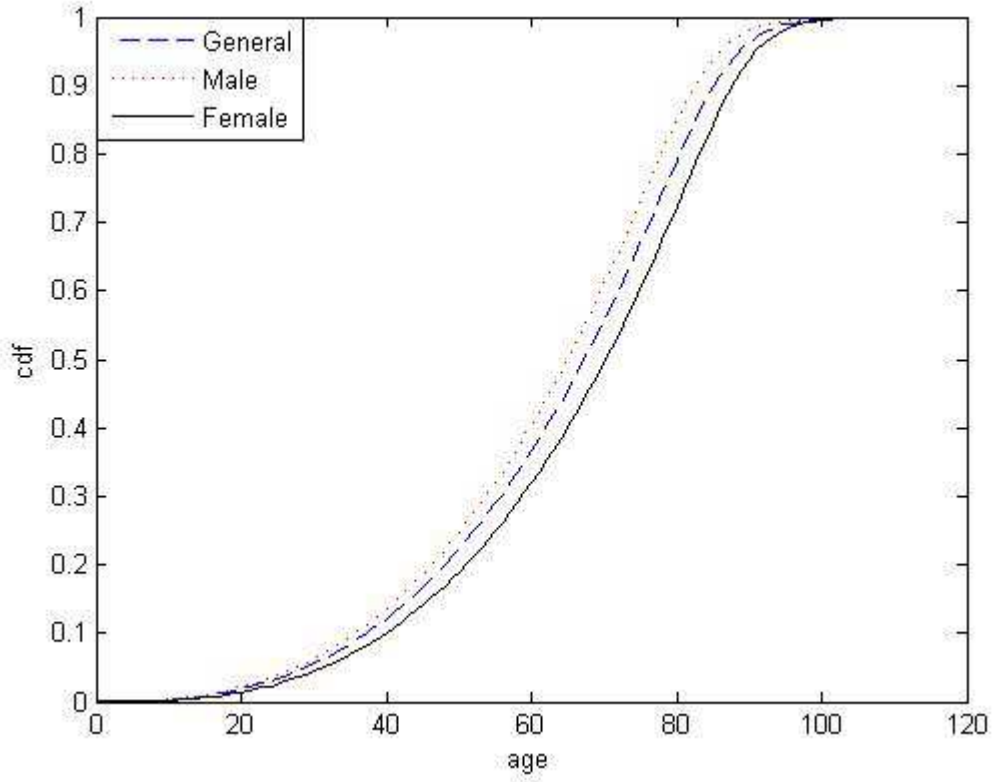

Figure 2.2. Cumulative Distribution Function of General, Male and Female's Lifetime for 2004 year

3. Estimation of the Distribution of Remaining Lifetime

The probability of a person who was known to have lived at least *c* years might live more at least *x* years will be shown as follows,

$$H_c(x) = P[X > c + x / X > c]$$

$$P[X > c + x / X > c] = \frac{P[X > c + x]}{P[X > c]} = \frac{1 - P[X \le c + x]}{1 - P[X \le c]} = \frac{1 - F(c + x)}{1 - F(c)}$$

$$= \frac{1 - \left[1 + \left(\frac{c + x}{\beta}\right)^{-\alpha}\right]^{-k}}{1 - \left[1 + \left(\frac{c}{\beta}\right)^{-\alpha}\right]^{-k}}$$

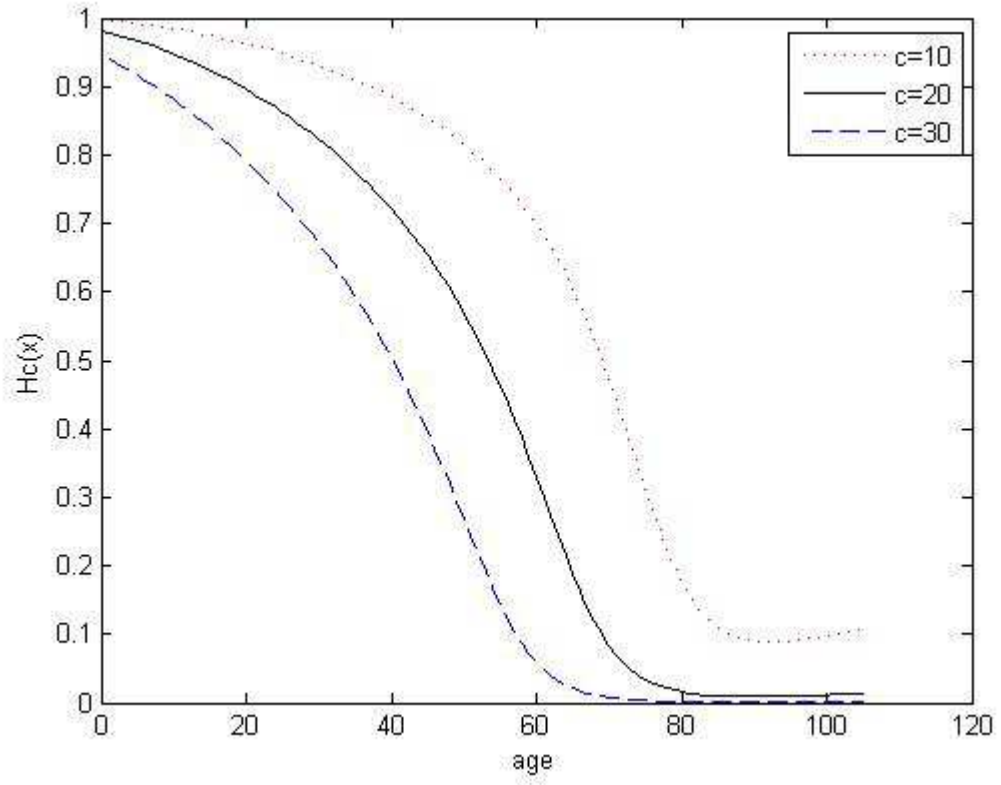

Figure 3.1. $H_c(x)$ Function Figures to General Lifetime for 2004 year

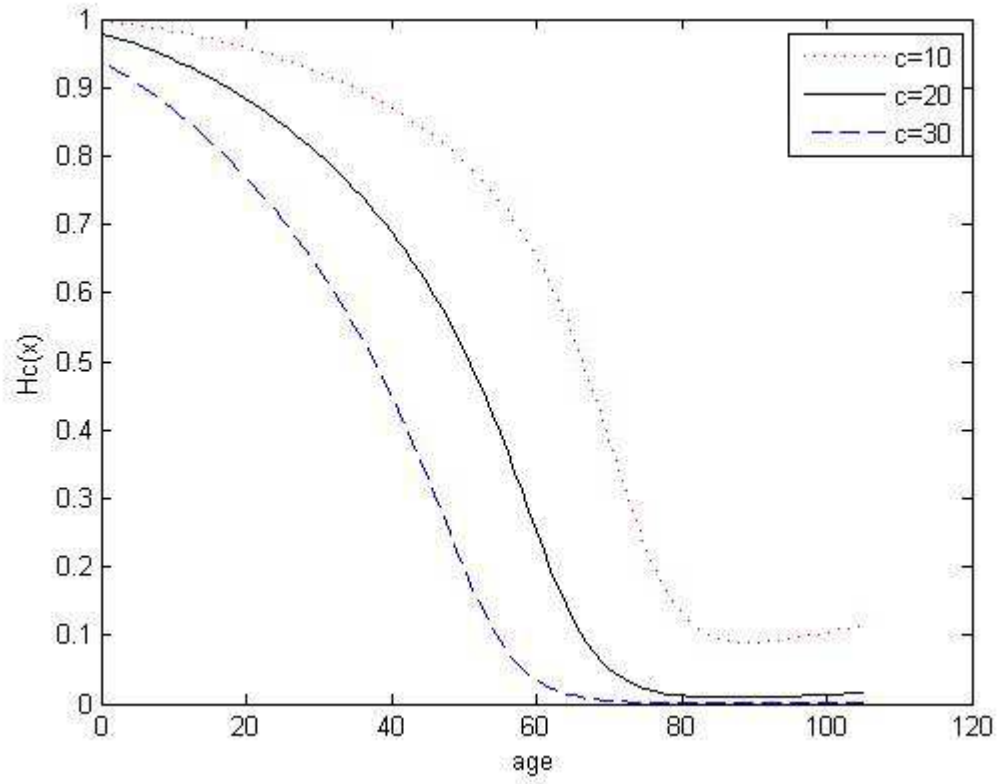

Figure 3.2. $H_c(x)$ Function Figures to Male Lifetime for 2004 year

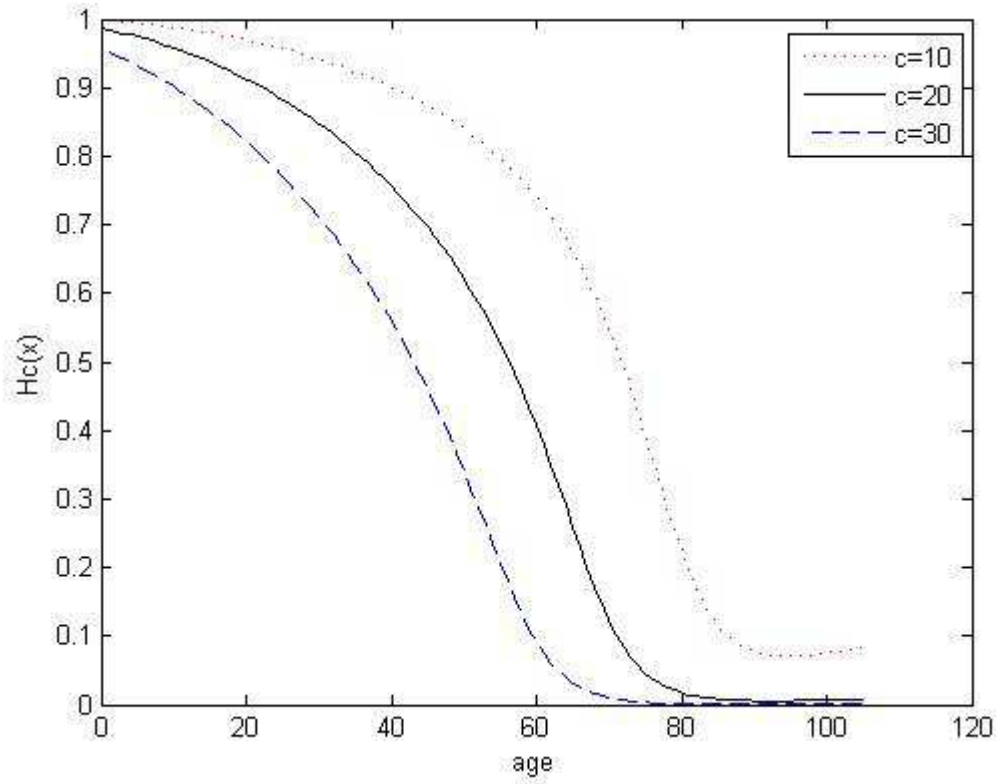

Figure 3.3. $H_c(x)$ Function Figures to Female Lifetime for 2004 year

Table 3.1. $H_c(x)$'s Probabilities to General, Male and Female Lifetime for 2004 Year

Probability of $H_c(x)$	General			Male			Female		
	***c*=10**	***c*=20**	***c*=30**	***c*=10**	***c*=20**	***c*=30**	***c*=10**	***c*=20**	***c*=30**
***x*=20**	0.9621	0.8948	0.7912	0.9575	0.8827	0.7686	0.9693	0.9122	0.8214
***x*=40**	0.8842	0.7209	0.5036	0.8707	0.6894	0.4500	0.9005	0.7563	0.5599
***x*=50**	0.8153	0.5695	0.2691	0.7917	0.5168	0.1993	0.8398	0.6217	0.3415
***x*=60**	0.6986	0.3300	0.0585	0.6528	0.2518	0.0334	0.7403	0.4066	0.0909

As seen in Table 3.1. the average life expectancy of females are more than males for 2004 year.

4. Modelling of The Lifetime for 2012 Year

Table 4.1. General, Male and Female Distribution Values for 2012 Year

Distribution			Parameters Value	*p* value for K-S Test	Mean	Std. Dev.
General	Dagum	k	0.083	0.772	67.24	17.71
		α	35.91			
		β	89.27			
Male	Dagum	k	0.077	0.877	64.84	17.85
		α	36.30			
		β	87.38			
Female	Dagum	k	0.091	0.277	70.20	16.95
		α	35.86			
		β	90.73			

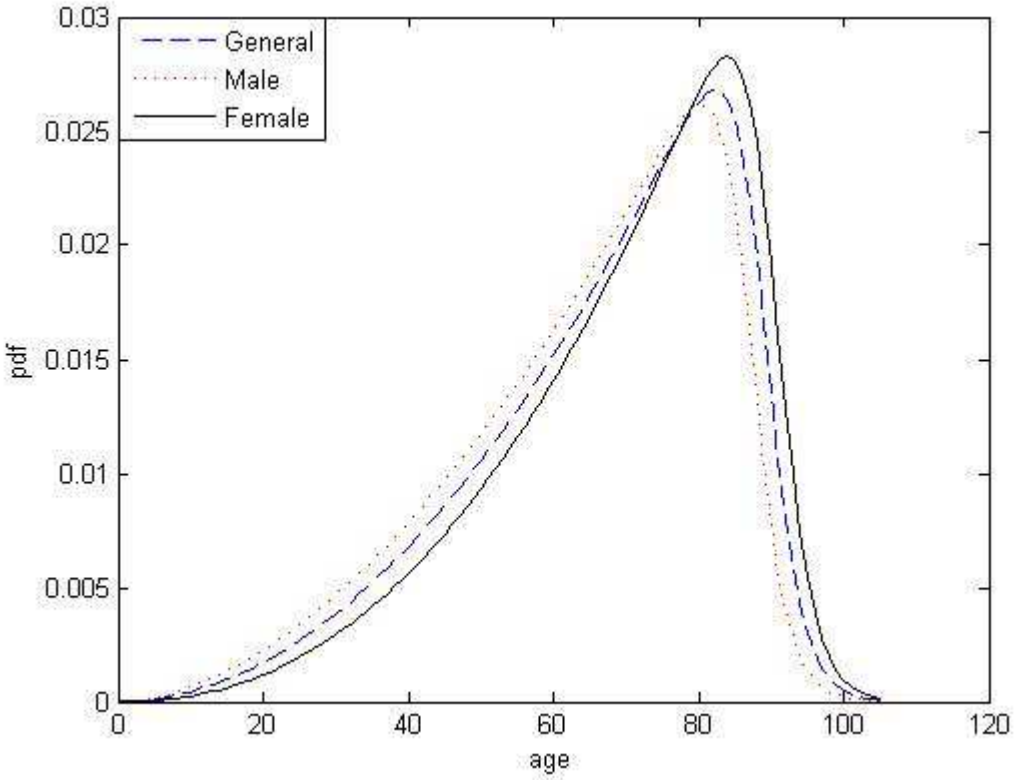

Figure 4.1. Probability Density Function of General, Male and Female's Lifetime for 2012 year (Şanlı T., 2013)

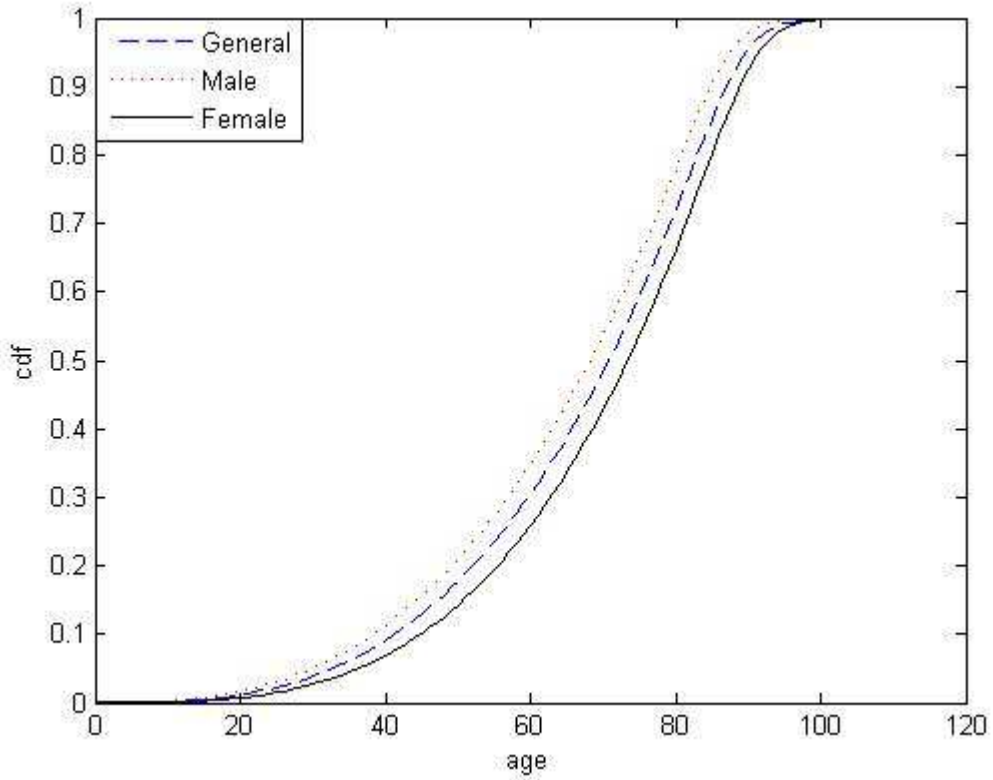

Figure 4.2. Cumulative Distribution Function of General, Male and Female's Lifetime for 2012 year

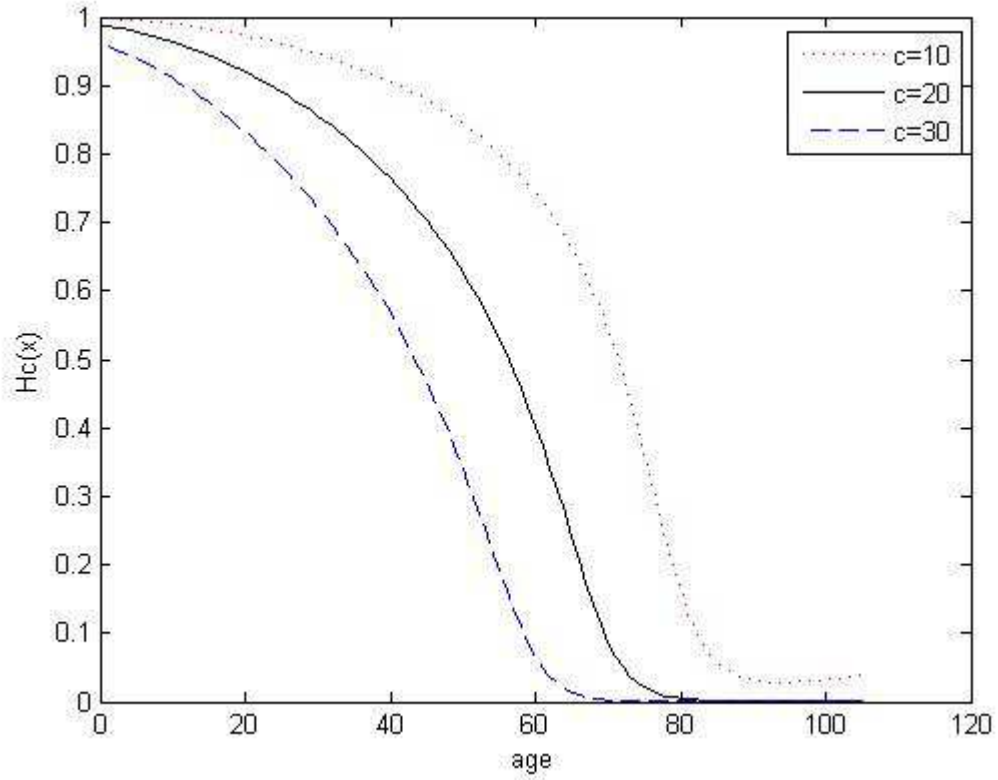

Figure 4.3. $H_c(x)$ Function Figures to General Lifetime for 2012 year

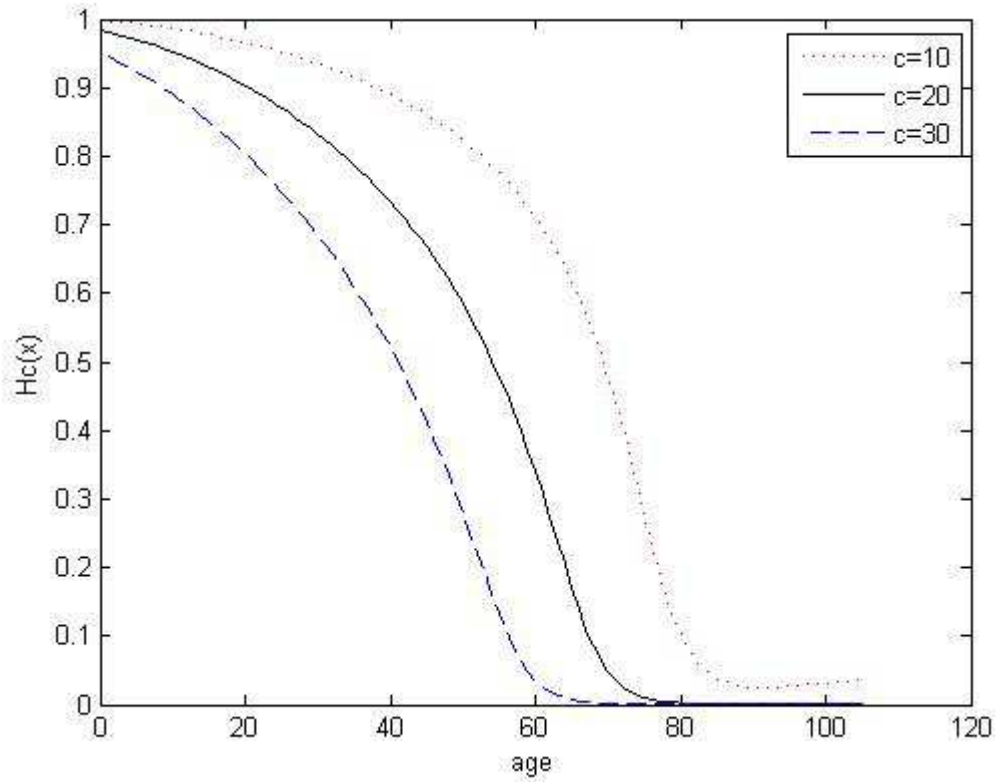

Figure 4.4. $H_c(x)$ Function Figures to Male Lifetime for 2012 year

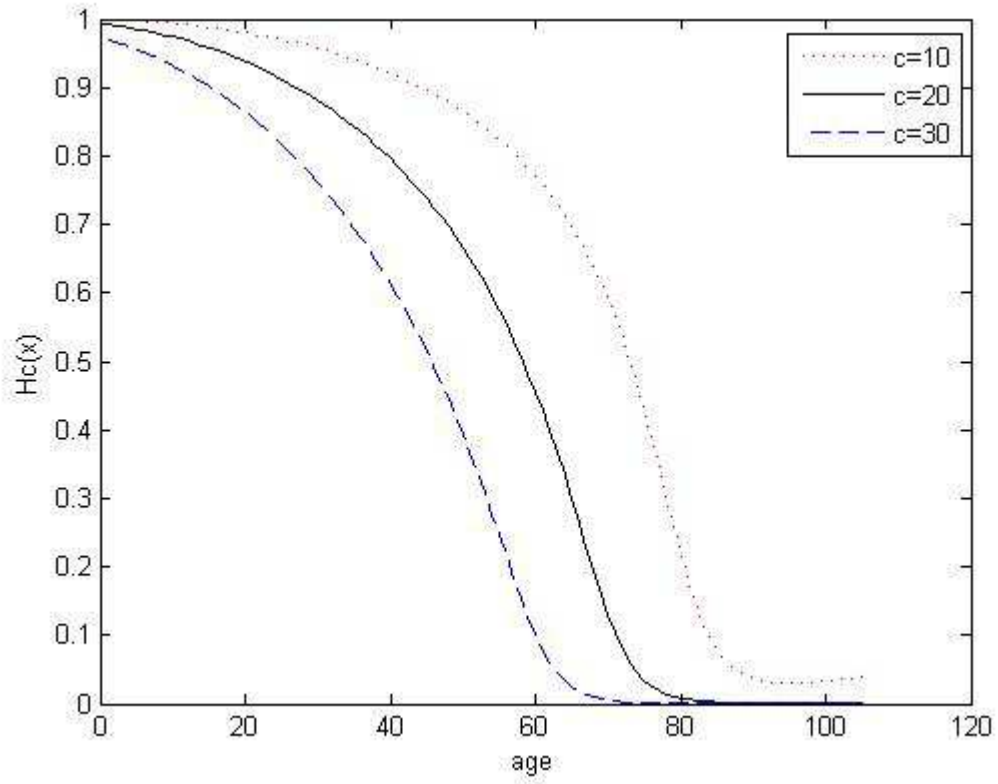

Figure 4.5. $H_c(x)$ Function Figures to Female Lifetime for 2012 year

Table 4.2. $H_c(x)$'s Probabilities to General, Male and Female Lifetime for 2012 Year

Probability of $H_c(x)$	General			Male			Female		
	***c*=10**	***c*=20**	***c*=30**	***c*=10**	***c*=20**	***c*=30**	***c*=10**	***c*=20**	***c*=30**
***x*=20**	0.9725	0.9193	0.8319	0.9653	0.9020	0.8030	0.9800	0.9377	0.8631
***x*=40**	0.9050	0.7638	0.5674	0.8902	0.7328	0.5206	0.9205	0.7956	0.6134
***x*=50**	0.8440	0.6270	0.3404	0.8233	0.5849	0.2797	0.8643	0.6664	0.3938
***x*=60**	0.7429	0.4033	0.0652	0.7104	0.3398	0.0345	0.7711	0.4557	0.1003

As seen in Table 4.2. the average life expectancy of females are more than males for 2012 year.

5. Comparison of the Lifetime Distribution of 2004 and 2012 Years

Table 5.1. Between The Years of 2004-2012 the Average Life Expectancy Table

	2004	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
General	64.08	64.38	65.10	65.72	66.20	64.99	65.90	66.58	67.24
Male	61.92	62.11	62.78	63.25	63.88	62.78	63.56	64.33	64.84
Female	66.74	67.15	67.91	68.74	69.03	67.65	68.70	69.28	70.20

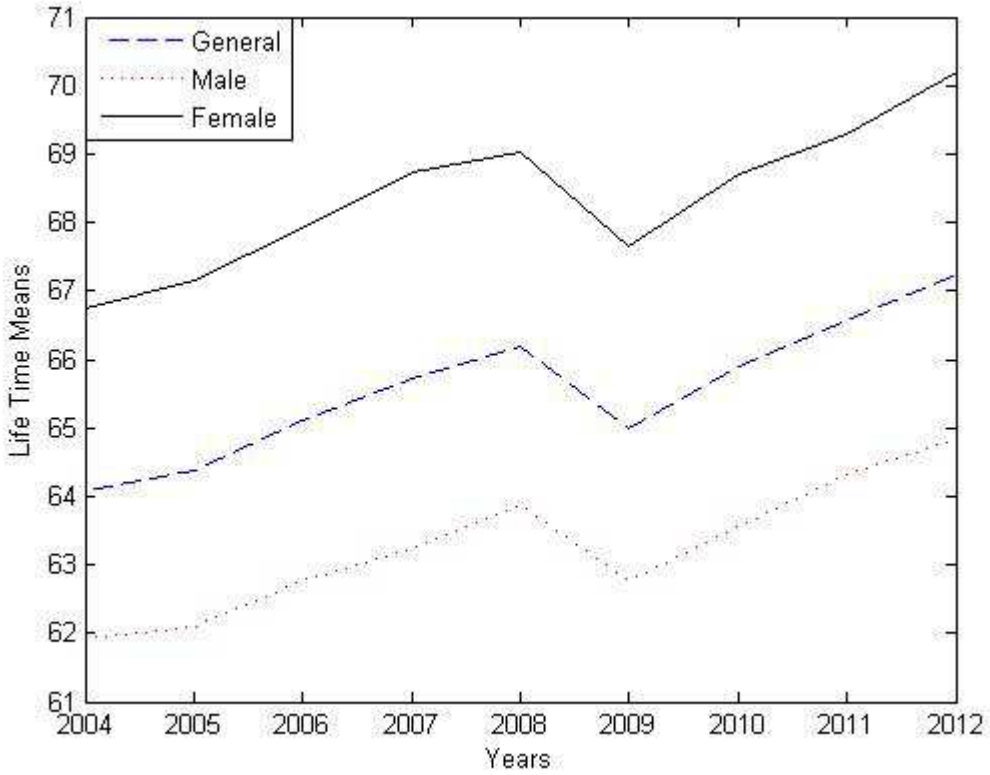

Figure 5.1. Between The Years of 2004-2012 the Average Life Expectancy Figures

Figure 5.1. show that the increase in average life expectancy (Şanlı T., 2013)

Comparison of the probability density function (pdf) of general, male and female lifetime for the years.

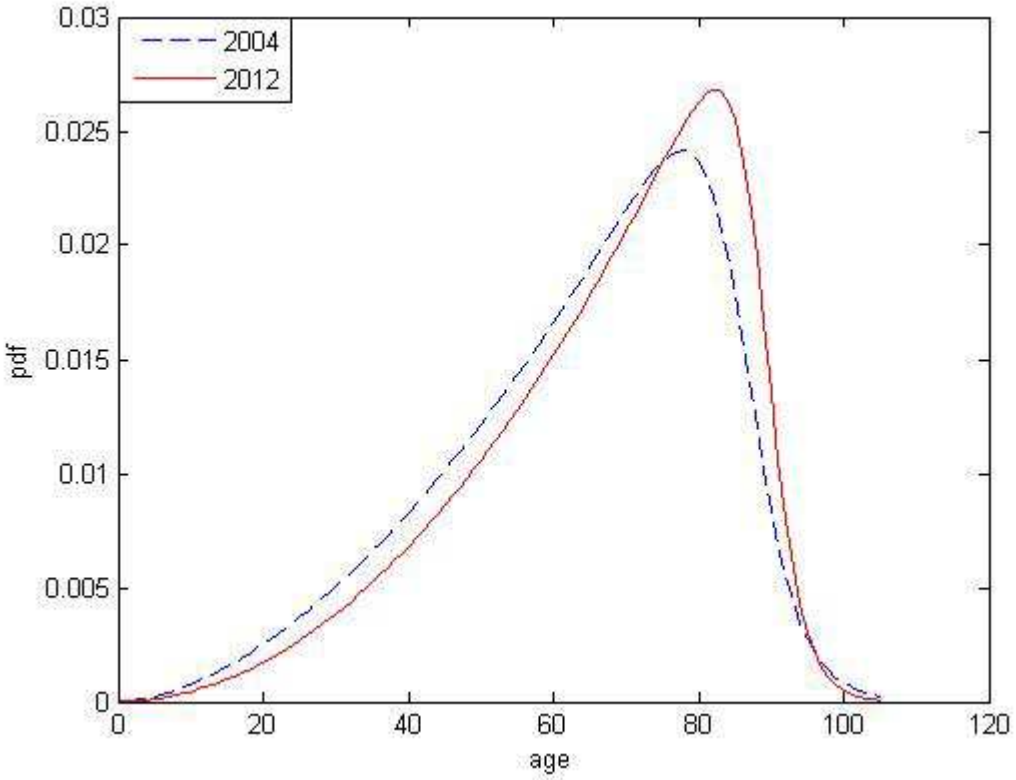

Figure 5.2.Comparison of the pdf of general lifetime for the years.

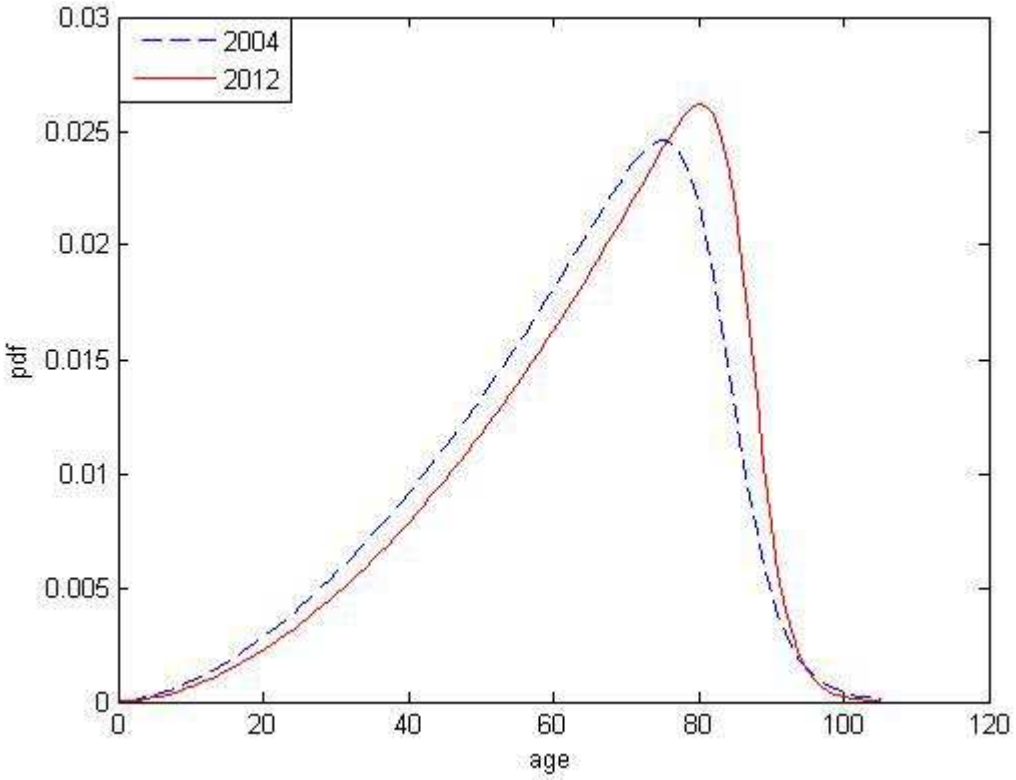

Figure 5.3.Comparison of the pdf of male lifetime for the years.

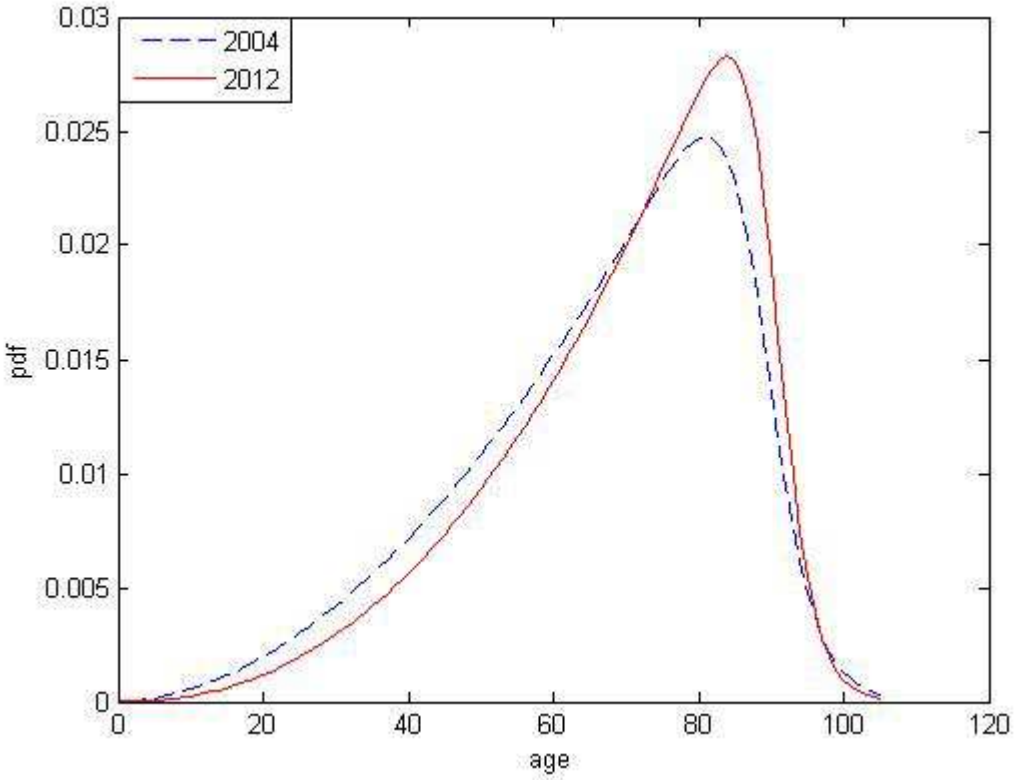

Figure 5.4.Comparison of the pdf of female lifetime for the years.

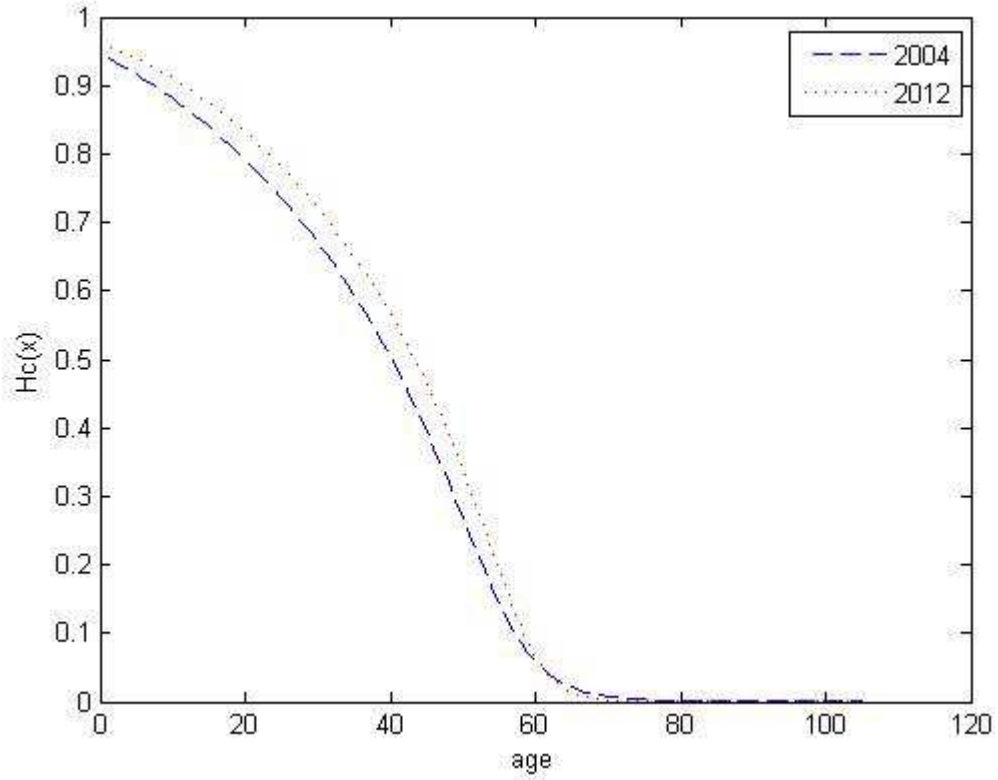

Figure 5.5. $H_c(x)$'s Probabilities to General Lifetime for 2004 and 2012 years(*c=30*)

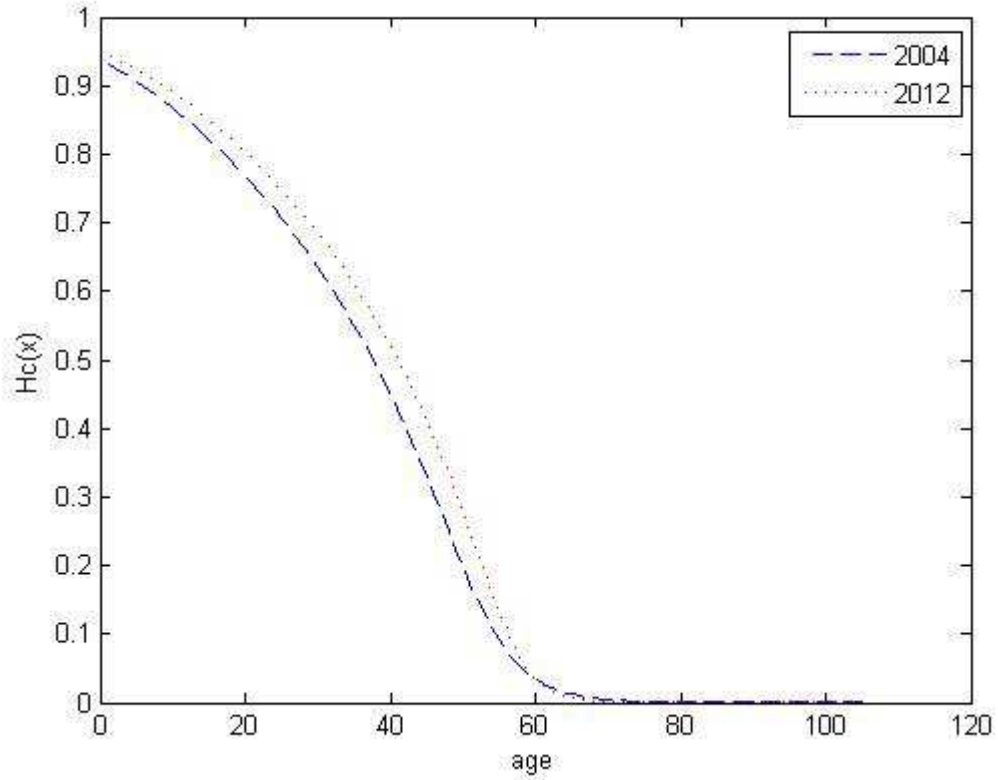

Figure 5.6. $H_c(x)$'s Probabilities to Male Lifetime for 2004 and 2012 years(*c=30*)

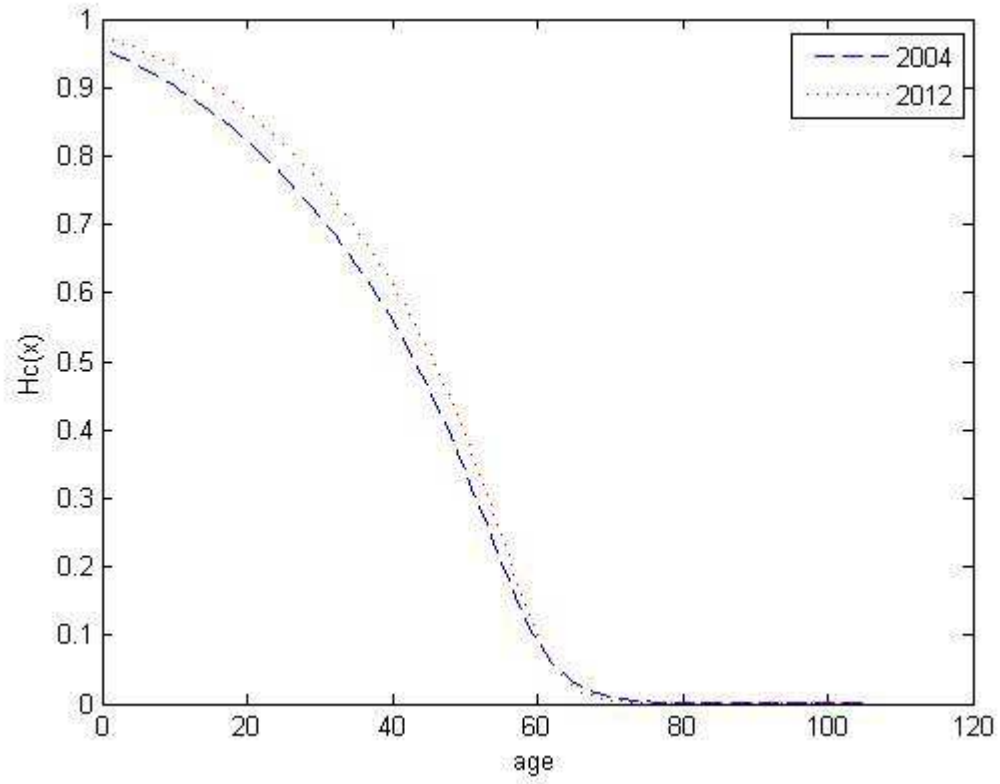

Figure 5.7. $H_c(x)$'s Probabilities to Female Lifetime for 2004 and 2012 years(*c=30*)

RESULTS

In this study, the lifetime of people in Turkey distributions were investigated. In this study between 2004-2012 years lifetime data were used. Lifetime was modelled by three parameters Dagum Distribution.

When the lifetime distributions were investigated, it is seen that average lifetime of the female was longer than of the male.

When the probability of a person who is known to have lived at least *c* year and then *c* at least living probability *x* year more is investigated, the below results are found out.

1) $H_c(x)$ probability of the female is higher than of the male,
2) In the following years an increase in $H_c(x)$ probability has been seen,
3) As long as the age goes on, $H_c(x)$ probability for males and females close to each other,
4) $X, Y \text{ and } Z$ random variables, respectively female, general and male will Show lifetime. In this case following stochastic inequalities are available, $Z < Y < X$

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