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EXPLORING BEARING ROOT MEAN SQUARE FIRST PASSAGE TIME BASED ON INVERSE GAUSSIAN DISTRIBUTION

Abstract:

Bearing becomes a critical rotational component in mechanical system, and its condition will affect the system. It is essential to predict bearing lifetime through acquisition and process degradation prediction. Vibration data contain information bearing degradation, and analysis based on this information is frequently applied in bearing prognostic. Proper models should be developed in order to find the relationship between degradation process and covariates. First passage time is a critical parameter in Brownian motion representing the time point when degradation curve passes through the failure for the first time, which equals to lifetime of the bearing. It is a random process that follows the inverse Gaussian distribution. This paper explores the application of first passage time of bearing vibration using bearing lifetime and operating condition as covariate. The lifetime data is extracted from bearing vibration data PHM Pronostia FEMTO database. The research methodology consist of inverse Gaussian parameter estimation, and interpretation of reliability of first passage analysis of operating condition.

Keywords:

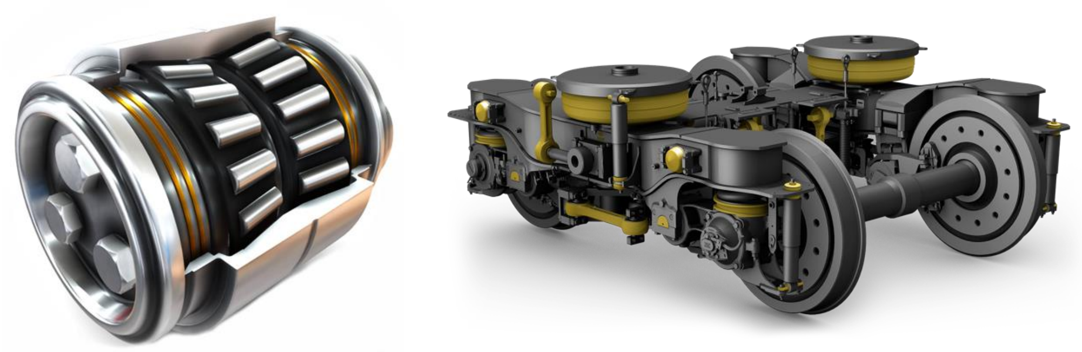
bearing lifetime, estimation inverse Gaussian, bearing operating condition, reliability first passage time

JEL Classification: C00, C80

1 Introduction

The bearing of bogie (Figure 1) is one of the key components of the locomotive, its role is to ensure the rotary motion of wheelsets. The reliability for bearings of bogie is designed based on model and parameter estimation. Its condition will affect the degradation state of the bearing. It is essential to predict bearing lifetime through degradation information. The first passage time is a critical parameter in Brownian motion representing the time point when degradation curve passes through the failure for the first time, which equals to lifetime of bearings. The application of first passage time on bearing of bogie lifetime is a challenging issue due to small number of bearing lifetime data. The lifetime data used in this study is based on root mean square extracted from Pronostia FEMTO database. It can show that the first passage time follows the inverse Gaussian distribution. The parameters of distribution are estimated using Pronostia dataset. The remaining life of bearing was estimated using reliability function of first passage time.

Figure 1: The rolling bearings of bogie in train play important role in train safety. The reliability for bearings of bogie is designed based on model and parameter estimation.



Source: internet

2 Review of literature

Let the stochastic process Brownian motion X_t be given by

$$X_0 = 0, X = \nu t + \sigma W_t \quad (1)$$

where W_t is a standard Brownian motion, X_t is a Brownian motion with drift $\nu > 0$. The first passage time for a fixed $a > 0$ is distributed as inverse Gaussian

$$T_a = \inf\{t > 0, X_t = a\} \sim IG\left(\frac{a}{\nu}, \left(\frac{a}{\sigma}\right)^2\right) = \frac{a}{\sigma\sqrt{2\pi x^3}} \exp\left(-\frac{(a - \nu x)^2}{2\sigma^2 x}\right) \quad (2)$$

The inverse Gaussian distribution is a reasonable distribution model for shelf life of a food product, fracture toughness's of welds, precipitation, runoff (Folks and Chhikara, 1978). This distribution derived as a model of the time a stock reaches a certain price for the first time (Bachelier, 1900). It was used as the time to first passage of a Brownian motion. It is used as the limiting distribution in a sequential probability ratio test. Suppose $p(x,t)$ denote the probability density function for the time when the process first hits some barrier α , known as the first passage time. The evolution of $p(x,t)$ is given by the Fokker-Plank differential equation

$$\frac{\partial p}{\partial t}+\nu\frac{\partial p}{\partial x}=\frac{\sigma^2}{2}\frac{\partial^2 p}{\partial x^2};p(0,x)=\delta(x-x_0),p(t,\alpha)=0 \tag{3}$$

where $\delta(.)$ is the Dirac delta function. The fundamental solution, denoted by $\psi(t,x)$ is

$$\psi(t,x)=\frac{1}{\sqrt{2\pi\sigma^2 t}}\exp\left[-\frac{(x-x_0-\nu t)^2}{2\sigma^2 t}\right] \tag{4}$$

The solution with initial condition $p(0,x)=\delta(x-x_0)-A\delta(x-m),m>\alpha$

$$p(t,x)=\frac{1}{\sqrt{2\pi\sigma^2 t}}\left\{\exp\left[-\frac{(x-x_0-\nu t)^2}{2\sigma^2 t}\right]-e^{2\nu(\alpha-x_0)/\sigma^2}\exp\left[-\frac{(x+x_0-2\alpha-\nu t)^2}{2\sigma^2 t}\right]\right\} \tag{5}$$

The reliability function computed as (Seshadri, 1999)

$$R(t)=\int_{-\infty}^{\alpha}p(t,x)dx=\Phi\left(\frac{\alpha-x_0-\nu t}{\sigma\sqrt{t}}\right)-e^{2\nu(\alpha-x_0)/\sigma^2}\Phi\left(\frac{-\alpha+x_0-\nu t}{\sigma\sqrt{t}}\right) \tag{6}$$

The first passage time distribution is obtained from $f(t)=-dR/dt$

$$f(t)=\frac{(\alpha-x_0)}{\sqrt{2\pi\sigma^2 t^3}}\exp\left(-\frac{(\alpha-x-\nu t)^2}{2\sigma^2 t}\right) \tag{7}$$

The first passage time T_α follows an inverse Gaussian

$$f(t) = \frac{\alpha}{\sqrt{2\pi\sigma^2 t^3}} \exp\left(-\frac{(\alpha - \nu t)^2}{2\sigma^2 t} \right) \sim IG\left(\frac{\alpha}{\nu}, \left(\frac{\alpha}{\sigma} \right)^2 \right) \quad (8)$$

Suppose $X_i \sim IG(\mu, \lambda w_i), i = 1, 2, \ldots, n$ and all X_i independent, the maximum likelihood estimates are

$$\hat{\mu} = \frac{\sum_{i=1}^{n} w_i X_i}{\sum_{i=1}^{n} w_i} = \bar{X}, \quad \frac{1}{\hat{\lambda}} = \frac{1}{n} \sum_{i=1}^{n} w_i \left(\frac{1}{X_i} - \frac{1}{\bar{X}} \right) \quad (9)$$

The graph of the inverse Gaussian density function is similar in shape to other skewed distribution such as log-normal, gamma, and Weibull. Its potential use as a model for distribution of lifetimes has been investigated (Chhikara and Folks, 1977).

Pronostia FEMTO provides real data related to bearings accelerated degradation test (Nectoux et al., 2012). Its dedicated to test and validate bearing fault detection, diagnostic and prognostic. Pronostia FEMTO is composed of three main parts: a rotating part, a degradation generation and a measurement part. The rotating part consist of an asynchronous motor and two shafts. The motor is the actuator that allows bearing to rotate through a system of gearing.

3 Methodology

The inverse Gaussian distribution is employed for bearing first passage time using lifetime data under three operating conditions as presented in Table 1. The bearing lifetimes are observed for three operating conditions of vertical and horizontal direction. The parameters of the distribution are estimated using maximum likelihood method. The result of parameter estimation is used to calculate the survival or the reliability function. The reliability functions are plotted as function of operating condition and the direction of the vibration. Its aim is to explore the significance effect of operating conditions and the direction of vibration.

Table 1: Lifetime of seventeen bearings from Pronostia experimental under three experimental conditions for vertical and horizontal direction

Operating condition: speed (rpm) and load (N)		
1800, 4000	1650, 4200	1500, 5000
1_1_V: 28030	2_1_V: 9110	3_1_V: 5150

1_1_H: 28030	2_1_H: 9110	3_1_H: 5150
1_2_V: 8710	2_2_V: 7970	3_2_V: 16370
1_2_H: 8710	2_2_H: 7970	3_2_H: 16370
1_3_V: 23750	2_3_V: 19550	3_3_V: 4340
1_3_H: 23750	2_3_H: 19550	3_3_H: 4340
1_4_V: 14280	2_4_V: 7510	
1_4_H: 14280	2_4_H: 7510	
1_5_V: 24630	2_5_V: 23110	
1_5_H: 24630	2_5_H: 23110	
1_6_V: 24480	2_6_V: 7010	
1_6_H: 24480	2_6_H: 7010	
1_7_V: 22590	2_7_V: 2300	
1_7_H: 22590 s	2_7_H: 2300 s	

Source: own data

4 Results and Discussion

The maximum likelihood was used to estimate the parameter of first passage time using inverse Gaussian distribution (Table 2). Code 0 was used for all dataset, code 1 for rpm 1800 loading 4000, code 1 for rpm 1650 loading 4200, and code 3 for rpm 1500 and loading 5000.

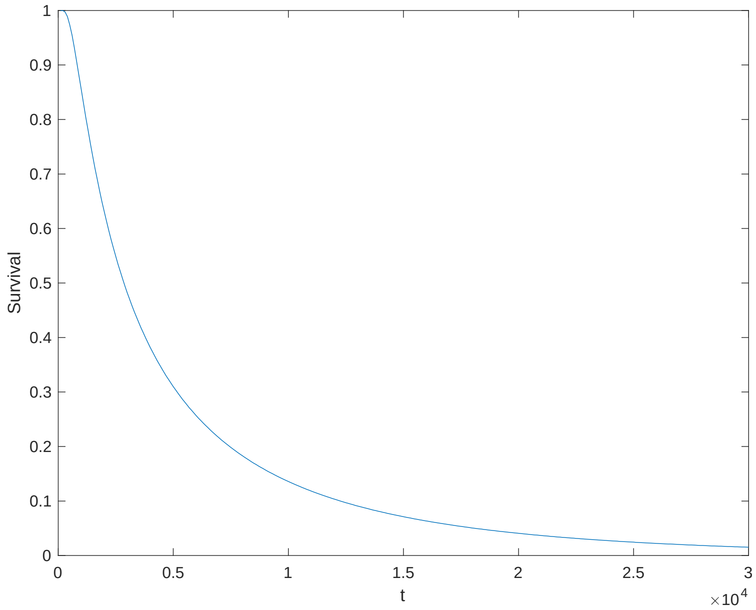
Table 2: The parameter estimates of first passage time using inverse Gaussian distribution for three operating conditions (1, 2, 3)

Parameter	0	1	2	3
n	.0003	.0002	.0004	.0005
s	.0313	.0130	.0350	.0318

Source: own calculation

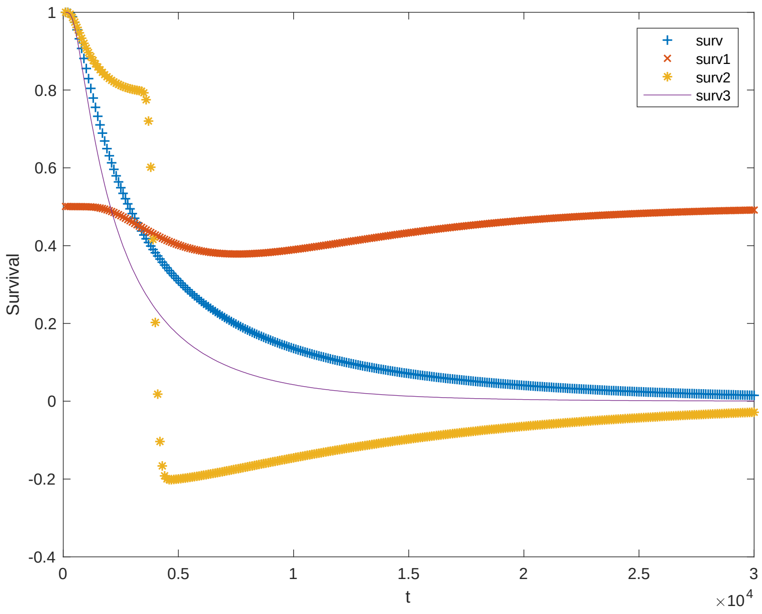
Figure 2 shows the reliability of bearing using all bearing lifetime data of Table 1. The plot is satisfactory, the first passage time inverse Gaussian is a reasonable model. The application to predicted bearing remaining life, the reliability is set as .3 and the point that reaches for .3 in the predicted reliability curve is $t = 5200$. Figure 2 shows the reliability curve for three speed and loading as well for all data. Compared to previous plot, the plot shows unsatisfactory results due to the variability of lifetime data. This research found difficulty to obtain a reliable bearing lifetime data. The simulation approach is one possibility to obtain bearing lifetime data. The values of barrier a and the initial x_0 play significant role in the computation of first passage time.

Figure 2: The reliability function of bearing lifetime data Table 1. Related to predicted remaining life, the reliability is set as .3 and the point that reaches for .3 in the predicted reliability is $t = 5200$. The predicted remaining life of the bearing after $t = 3000$ is 2200.



Source: own calculation

Figure 3: The predicted reliability function for bearing lifetime Table 1. The symbol + is for the lifetime data without operating condition, ´ is for operating condition1, * is for operating condition 2, and – is for operating condition 3. The plots are not totally satisfactory due to the variability of data for operating condition 1 and condition 2. For operating condition 3, the data very well describe by the inverse Gaussian.



Source: own calculation

5 Conclusions

The first passage time is used for modeling the reliability of bogie bearing lifetime. Its distribution has potential for providing the development of Brownian motion for bogie fault characterization. The reliability first passage time offers an attractive research area in domain safety of train transportation. The barrier and the initial state play significant role in the significance of first passage time.

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