A FUNDAMENTAL CAUSE OF ECONOMIC CRISIS

Abstract:

The purpose of this paper is to study a fundamental cause of the economic crisis which suffers the current capitalism economy. The basic approach we adopt is as follows. Firstly, we show that the capitalism economy cannot largely deviate from the balance which is defined by natural economy. Our attentions are focused on the balance between the real economic sector which produces GDP and the monetary sector which invests capital to the production of GDP. For the balance to be kept, there exists a rigorous range of the interest rate which the monetary sector can require from the real economic sector. The fundamental cause of economic crisis is the large deviation from this balance. Secondly, the capitalism economy is constructed by the economic agents who necessary accomplish their decision makings. Therefore, we construct our model by macro-economic game. Players of the game are the agents who are selected by the real economic sector and the monetary sector. Thirdly, we show that the deviation from the solution of the game is considered as the one from natural economy. Therefore, we conclude that the large deviation from the solution of the game is a fundamental cause of economic crisis. The game is defined by the macro-economic differential game with infinite horizon. In the usual cases, the solution of the game is defined by the stable steady point, or an equilibrium. However, our game has no stable steady point. Therefore, it is shown that the aid of financial policy is necessary for the game to have its solution. The aid is defined by controlling the distribution rate of GDP to the monetary sector. The financial policy is defined as the policy rule. This distribution rate has the rigorous restriction for the game to have its solution. It is shown that the solution of the game has the character of natural economy. The policy rule plays an important role to keep the balance of the economy and hence to prevent the economy from deviate from natural economy. In the final section, we investigate the actual transitions of distribution rate in USA, Euro area and Japan and analyze the fundamental cause of financial crisis.

Keywords:

Balance of real economic and monetary sector, natural distribution rate to monetary sector, deviation from natural economy financial policy rule, differential game

JEL Classification: E00, E10, E60
Introduction

The purpose of this paper is to analyze the fundamental cause of the economic crisis which suffers the current capitalism economy. The basic approach we adopt is as follows.

Firstly, the capitalism economy cannot largely deviate from a balance which we study as natural economy below. Our attention is focused on the balance between the real economic sector which produces GDP and the monetary sector which invests capital to the production of GDP and receives the interest rate. We show that there exists a rigorous limit of the interest rate for the balance to be kept. The fundamental cause of economic crisis is the large deviation from this balance which we define as natural economy. Secondly, the capitalism economy is constructed by the economic agents who necessary accomplish their decision makings. Therefore, we construct our model by the macro-economic dynamic-game. Players of the game are the agents who are selected by the economic sector and the monetary sector respectively. Thirdly, the deviation from the solution of the game is the one from the balance. Therefore, we conclude that the deviation should be recognized as a fundamental cause of economic crisis.

A remarkable trend in the capitalism economy from 1980s is the intumesce of world's financial assets and the rapid increase in the ratio of the world's financial assets and the world's GDP. Needless to say, this trend is the result of the current Keynesian policy. The enormous financial assets require the interest rates. There are two resources of them. One is the money created by credit creation and the other is the value produced by the real economic sector. However, the event that the interest rates are paid from the credit creation is the one within the monetary sector. Therefore, the problem to be analyzed is the latter case. This case should be considered as the event where the value produced by the real economic sector is absorbed to the monetary sector. The intumesce of money brings out the rapid increase in the absorption of value from the real economic sector. This is the fundamental cause of financial crisis, as is analyzed in this paper.

The intumesce of money should be considered as the destruction of the balance in economy. From the viewpoint of economic theory, this destruction is considered as the deviation from macro-economic equilibrium and therefore the one from natural economy. Of course, natural economy is not the synonym for the equilibrium of macro-economy. However, as we consider below, natural economy which is defined by various viewpoints premises the equilibrium of the economy. That is, natural economy involves the concept of equilibrium of macro-economy. Therefore, the deviation from the equilibrium is considered as the one from natural economy.

Then, it is not meaningless to study natural economy. As Smith, A. (1776) described, the marker price fluctuates around natural price. This fluctuation has two important implications. One is that natural economy constructs the basic structure of the economy. Therefore, to study natural economy means to analyze the basic law which manages and controls the economy.
The other is that the large deviation from it is very risky. The risk gives us an important alarm for controlling economy without rigorous rules. Human being cannot control the economy artificially ignoring natural economy.

When we study natural economy by Smith, A. (1778), we should also analyze natural interest rate. After Smith, many economists tried to define natural interest rate from viewpoint of macro-economy. Firstly, we should take out the definition by Wicksell, J.G.K. (1898). Wicksell defined natural interest rate as such a rate as it is neutral for the price level of the real market. More precisely, it is the rate at which the demand is equal to the supply in the real market and it seems as if the economy doesn't need the capital market. As is well known, this definition had an impact on Keynes. In his book, A Treatise on Money (1930), Keynes constructed the fundamental equation where natural interest rate was defined as such a rate as it made investment equal to savings. At the natural interest rate, price level is equal to the monetary income per output paid for the production factor. However, the event that investment equals savings implies the one that it depends on the shapes of investment and saving function. Therefore, for instance, if an innovation is expected to occur, the expectation of innovation will change the investment function and the natural interest rate will increase. In this case, no information with respect to what happens in the process of production is given. However, it is absolutely certain that the natural interest rate depends on the structure of the production process.

It is Pasinetti who proposed a different approach from Wicksell. Pasinetti concentrated on the structure of production process and innovation. His approach is based on the theory of labor. Firstly, he began with the condition of full employment of labor and capital stock using vertically integrated analysis which was defined by multi-sector model. Next, he introduced the concept of natural rate of profit and constructed natural economy\(^1\). Finally, he introduced the financial assets to the natural economy and proved the emergence of the rate of interest. After these preparations, Pasinetti defined the own-rate of interest for each commodity, rigorously analyzed the relation between the nominal rate of interest and the real rates of interest-the standard real rate of interest and finally reached at the natural rate of interest\(^2\).

The natural interest rates defined by Wicksell and Pasinnetti are the representative ones. It is important to note that both definitions are based on the equilibrium of macro-economy. However, macro-economy has manifold structures and therefore has multiple concepts of equilibrium. As is analyzed in section 3, the solution of the game in this paper is also an equilibrium in macro-economy and has a character of natural economy. Therefore, the large deviation from the equilibrium implies the one from natural economy.

Let us come back to the actual economy. The remarkable character we should point out is the rapid intumescence of monetary sector. If we look at the financial policies in USA, Euro

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\(^1\) Pasinetti (1981), chapter VII.
\(^2\) Pasinetti (1981), chapter VIII.
area and Japan, we recognize that the common feature of these areas is the rapid increase in monetary base. Corresponding to this increase, financial assets in the world intumesce and are 3.5 times as much as world’s GDP in 2006\(^3\). This unbalance implies the large deviation from the natural economy.

Finally, we show our approach. As is mentioned above, the actual economy is managed by many economic agents who accomplish their decision makings. This condition is modelled as ‘making expectation’ by many economic agents. However, each decision making has influence on each other. It is impossible for the economic agents to make decision independently. Therefore, we should adopt game theory to construct our model.

This approach is tried by many economists. The pioneer of this field is Lancaster(1971). Players in his model are capitalists and workers. The purpose of this study is to show the inefficiency of the capitalism, using the method of differential game. This study is developed by Basar,T.,Haurie,A.,and G.Ricci(1985) , Pohjola,M.(1985), Kaitala,V., and M. Pohjola(1990) , Benabou,Roland and J. Tirole(2006). These studies analyze the existence of equilibrium in infinite horizon and the inefficiency of the capitalism. Players are also capitalists and workers. However, these studies don’t involve the monetary sector. Therefore, we define the real economic sector which produces GDP and the monetary sector as the players of the game. That is, each sector is assumed to have its agent. We assume that the agents play the macro-economic game.

We construct the macro-economic game in section 2 and analyze it in section 3. We adopt differential game. However, the game has no stable steady point. We show that the aid of financial policy is inevitable for the game to have its solution. Here, we propose the rule of financial policy which control the rate with which the monetary sector absorbs the value produced by the real economic sector.

Finally, in section 4, we try to apply our game-model to the actual economy. The area we select is UAS, Euro area and Japan. Here, we prove that the actual economy largely deviate from the equilibrium of the game, hence from natural economy.

2. Model

Let us consider the economy which is constructed by the real economic sector which produces GDP and the monetary sector which holds financial assets and invests part of them to the real economic sector. The produced GDP is distributed among the real economic sector and the monetary one. We assume that each sector has its agent who makes the decision for the consumption and the investment to the production in the real economic sector.

We denote the production function of macro-economy by

\[ Y = F(K,L) = K^\theta L^{1-\theta}, \quad (0 < \theta < 1) \]  

where \( Y, K \) and \( L \) represent GDP, capital stock and labor input, respectively. \( \theta \) is a

\(^3\) www.meti.go.jp/report/tsuhaku2008/.../html/i1120000.html
constant. We also denote the price of capital stock and the wage rate by $p_k$ and $w$, respectively. They are constants. Then, we get

$$L = \frac{1-\theta}{\theta} \cdot \frac{p_w}{w} K$$

by the optimal condition for the input of factors; $\frac{F_k}{p_k} = \frac{F_L}{w}$ where $F_k$ and $F_L$ represent the partial derivatives of $F$ with respect to $K$ and $L$, respectively. Therefore, we get

$$Y = \left(1 - \frac{\theta}{\theta} \cdot \frac{p_w}{w}\right)^{1-\theta} K$$

from (1) and (2).

GDP is distributed to the real economic sector and the monetary sector. So, we get

$$Y = \pi K + wL + \gamma Y,$$

where $\pi$ denotes the profit rate of capital stock and $\gamma$ denotes the distribution rate of GDP to the monetary sector.

We represent the consumptions of Player R and Player M by

$$C_R = (1 - \alpha)\pi K,$$

$$C_M = (1 - \beta)\gamma Y$$

respectively where $\alpha$ is the consumption property of Player R and $\beta$ is the one of Player M. $\alpha$ and $\beta$ are the strategies of the players. The strategies $\alpha$ and $\beta$ have the constraints;

$$0 \leq \alpha \leq \overline{\alpha}, 0 \leq \beta \leq \overline{\beta}$$

where $\overline{\alpha}(<1)$ and $\overline{\beta}(<1)$ are constants. On the other hand, $\alpha\pi K$ and $\beta\gamma Y$ represent the investments of Player R and Player M to production, respectively. Therefore, we get

$$\dot{K} = \alpha\pi K + \beta\gamma Y - \delta K$$

where $\dot{\cdot}$ denotes the derivative with respect to time and the constant $\delta$ denotes the depreciation rate of capital stock.

Next, we assume that the distribution rate of GDP to Player M, $\gamma$, is determined by

$$\gamma = \frac{\dot{Y}}{Y} + \mu. \quad (-1 \leq \mu \leq 1)$$

where $\mu$ is a constant. That is, the distribution rate of Player M is determined by the sum of the growth rate and $\mu$. $\mu$ implies how much rate Player M can receive from GDP over the

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4 To construct the game, we assume $-1 \leq \mu \leq 1$. However, as is shown below, the same conditions for $\mu$ are required for the game to have its solution.
growth rate. As discussed below, we focus our attention on the range of $\mu$ which enables the game to have its solution.

Now, we define the problems of the player as follows;

$$\max_{\alpha} \int_0^\infty e^{-\rho} C_R dt, \quad \text{s.t.} \quad (6), \quad 0 \leq \alpha \leq \bar{\alpha},$$

$$\max_{\beta} \int_0^\infty e^{-\rho} C_M dt, \quad \text{s.t.} \quad (6), \quad 0 \leq \beta \leq \bar{\beta}.$$  \(\text{(8)}\)  \(\text{(9)}\)

where $\rho$ is the discount rate of time of the players.

We complete the construction of the model by (1)~(9). The nine unknowns are $Y, K, L, C_R, \alpha, \beta, \pi, \alpha, \beta$.

3. Analysis

3.1 Rearrangement of Model

We rearrange the model constructed in section 2. Rearranging (3) and (10) by using (1)', we get

$$\pi = \left(1 - \frac{\theta}{\theta}, \frac{p_k}{w}\right)^{1-\theta} - \frac{1-\theta}{\theta} p_k - \left(\frac{K}{K} + \mu\right) \left(\frac{1-\theta}{\theta}, \frac{p_k}{w}\right)^{1-\theta}.$$  \(\text{(10)}\)

Since $\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K}$ by (1)', equation (6) becomes

$$\dot{K} = MK$$  \(\text{(11)}\)

where

$$M = M(\alpha, \beta) = \frac{(1 - \mu) \alpha + \beta \left(\frac{1-\theta}{\theta}, \frac{p_k}{w}\right)^{1-\theta} - \frac{1-\theta}{\theta} p_k - \delta}{1 + (\alpha - \beta) \left(\frac{1-\theta}{\theta}, \frac{p_k}{w}\right)^{1-\theta}}.$$  \(\text{(12)}\)

$M = M(\alpha, \beta)$ implies that $M$ is the function of $\alpha$ and $\beta$. In (12), the condition that the denominator cannot be zero is required. That is, the combination of strategies $(\alpha, \beta)$ should satisfy

$$-\alpha \left(\frac{1-\theta}{\theta}, \frac{p_k}{w}\right)^{1-\theta} + \beta \left(\frac{1-\theta}{\theta}, \frac{p_k}{w}\right)^{1-\theta} \neq 1.$$  \(\text{G0}\)

Next, let us rearrange the problems of the players. Firstly, we define the Hamiltonian of Player $R, H_R$, as follows;

$$H_R = (1 - \alpha) \left(\frac{1-\theta}{\theta}, \frac{p_k}{w}\right)^{1-\theta} - \frac{1-\theta}{\theta} p_k - (M + \mu) \left(\frac{1-\theta}{\theta}, \frac{p_k}{w}\right)^{1-\theta} + \lambda^R M.$$  \(\text{(13)}\)
where \( \lambda_R \) is the auxiliary valuable of Player R and obeys the following equation;

\[
\dot{\lambda}_R = \rho \lambda_R - \frac{\partial H_R}{\partial K}.
\] (14)

Similarly, we define the Hamiltonian of Player M, \( H_M \), as follows;

\[
H_M = (1-\beta)(M + \mu \left( 1 - \frac{\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} + \lambda_M M,
\] (15)

where \( \lambda_M \) is the auxiliary valuable of Player M and obeys the following equation;

\[
\dot{\lambda}_M = \rho \lambda_M - \frac{\partial H_M}{\partial K}.
\] (16)

The problem of Player R is

\[
\max_{\alpha} H_R \quad \text{s.t. } (6),(14), \quad 0 \leq \alpha \leq \bar{\alpha}.
\] (17)

Gathering the terms which include \( \alpha \), we get

\[
\alpha \left[ \lambda_R - \left( 1 - \beta \left( 1 - \frac{\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} \right) \right] \left( 1 - \mu \left( 1 - \frac{\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - 1 - \frac{\theta}{\theta} p_k \right) - (\delta - \mu \beta) \left( 1 - \frac{\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} \right].
\]

Hamiltonian \( H_R \) is a linear function of \( \alpha \). Therefore, the strategy of Player R is expressed as follows;

\[
\begin{align*}
\dot{\lambda}_R & > V_R \rightarrow \alpha = \bar{\alpha}, \\
\dot{\lambda}_R & = V_R \rightarrow ???, \\
\dot{\lambda}_R & < V_R \rightarrow \alpha = 0.
\end{align*}
\] (18)

where

\[
V_R = V_k(\beta) = \frac{(\delta - \mu \beta) \left( 1 - \frac{\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta}}{\left( 1 - \mu \left( 1 - \frac{\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - 1 - \frac{\theta}{\theta} p_k \right)} + \left\{ 1 - \beta \left( 1 - \frac{\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} \right\}.
\] (19)

We call the point which value is equal to \( V_R \) the strategy exchange point of Player R.

Fig.1 illustrates the paths of the auxiliary valuable \( \dot{\lambda}_R \) which obeys (14).

**Figure 1** the path of the auxiliary valuable \( \dot{\lambda}_R \)

(a) the case of \( M < \rho \)

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\[
\lambda^*_R = \frac{(1-\alpha)N_R}{\rho - M}
\]

\[
N_R = \left( \frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - \frac{1-\theta}{\theta} p_k - (M + \mu) \left( \frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta}
\]

(b) the case of \( M > \rho \)

(c) the case of \( M = \rho \)
From Fig.1, \( M(\alpha, \beta) \) should satisfy the condition;
\[
M(\alpha, \beta) \geq \rho,
\]
(M1)
because \( M(\alpha, \beta) < \rho \) implies that the economy has no power for decreasing the value of the auxiliary valuable \( \lambda_R \) in the capital accumulation. Therefore, we assume that (M1) is satisfied in our game.

On the other hand, the problem of Player M is
\[
\max_{\beta} H_M \quad \text{s.t.} \quad (6), (16), \quad 0 \leq \beta \leq \bar{\beta}.
\]

Gathering the terms which include \( \beta \), we get
\[
\beta \left[ \lambda_M - \alpha \left( \left( \frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - \left( \frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right) \right) \right] - (\mu - \delta) \left( \frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta}
\]

Hamiltonian \( H_M \) is a linear function of \( \beta \). Therefore, the strategy of Player M is expressed as follows;
\[
\begin{align*}
& \lambda_M > V_M \rightarrow \beta = \bar{\beta}, \\
& \lambda_M = V_M \rightarrow ???, \\
& \hat{\lambda}_M < V_M \rightarrow \beta = 0.
\end{align*}
\] (21)

where
\[
V_M = V_M(\alpha) = \frac{\alpha \left( \frac{1-\theta}{\theta} \right)^{1-\theta} - \left( \frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)}{\mu} + 1
\] (22)

We call the point which value is equal to \( V_M \) the strategy exchange point of Player M.

Fig.2 illustrates the path of the auxiliary valuable \( \hat{\lambda}_M \) which obeys (16).

**Figure 2**  the path of the auxiliary valuable \( \hat{\lambda}_M \)

(a) the case of \( M < \rho \)
\[-(1 - \beta)(M + \mu \left( \frac{1 - \theta}{\theta} \cdot \frac{p_k}{w} \right)^{-\sigma} \]

\[\tilde{\lambda}_M = \frac{(1 - \beta)(M + \mu \left( \frac{1 - \theta}{\theta} \cdot \frac{p_k}{w} \right)^{-\sigma}}{\rho - M}\]

(b) the case of \( M > \rho \)

\[-(1 - \beta)(M + \mu \left( \frac{1 - \theta}{\theta} \cdot \frac{p_k}{w} \right)^{-\sigma} \]

(c) the case of \( M = \rho \)
3.2 Analysis of the Game

At this stage, we propose the condition for the game to have its solution. Firstly, we get the following proposition about $M = M(\alpha, \beta)$.

**Proposition 1** For the game to have its solution, the following conditions about $\mu$ are necessary. That is,

$$\mu \geq \delta,$$  \hspace{1cm} \text{(G1)}

$$\frac{\alpha}{1-\rho} \left(1-\frac{\theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta} - \frac{1-\theta}{\theta} \cdot \frac{p_k}{w} < \mu < \frac{\rho}{1-\beta} \left(1-\frac{\theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta} + \delta,$$  \hspace{1cm} \text{(G2)}

(pf.) (G1): If $\alpha = 0$ in (22), then $V_{\mu}(0) = \frac{\mu - \delta}{\mu}$. The condition $V_{\mu}(0) \geq 0$ is required.

(G2); (G2) is obtained by the following two conditions;

$$M(\overline{\alpha},0) < \rho,$$  \hspace{1cm} \text{(M2)}

$$M(0,\overline{\beta}) < \rho.$$  \hspace{1cm} \text{(M3)}

These are also the conditions for the game to have its solution. Let us consider (M3). If $M(0,\overline{\beta}) \geq \rho$, then Player R cannot change the situation of $M(\alpha,\overline{\beta}) \geq \rho$ for all $\alpha$. Therefore, Fig. 1 (b) or (c) occurs. If the value of $\lambda_R$ decreases enough and becomes smaller than the value of $V_k(\overline{\beta})$ or $\lambda_k(\overline{\alpha},0)$ which is the value of the steady state of (14), the value of $\lambda_k$ will become negative in the future and the game will become meaningless. That is, $M(0,\overline{\beta}) \geq \rho$ implies that Player M has the strong strategy which makes the strategy of Player R meaningless.

We can judge $M(\overline{\alpha},0) < \rho$ similarly.  \hspace{1cm} \text{(Q.E.D)}

With respect to $M(\overline{\alpha},0)$ and $M(0,\overline{\beta})$, we should consider whether the values of them are positive or not. For instance, $M(\overline{\alpha},0) \leq 0$ implies that Player R cannot make the direction of capital accumulation positive by his (or her) own efforts. Of course, even under such a condition, the game may have its solution which is constructed below for some restricted conditions. However, the condition where players cannot make the capital accumulation by his (or her) own efforts is game-theoretically meaningless. Therefore, we put following assumptions.

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Assumption

\[ M(\overline{\alpha}, 0) > 0, \]
\[ M(0, \overline{\beta}) > 0. \]  \hfill (M4)
\[ M(0, \overline{\beta}) > 0. \]  \hfill (M5)

From (M4) and (M5), we obtain

\[
\frac{\delta}{\beta \left( \frac{1 - \theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta}} < \mu < \frac{1 - \theta}{\theta} \cdot \frac{p_k - \delta}{\alpha}.
\]  \hfill (G3)

Next, we propose the following lemma for \( M = M(\alpha, \beta) \) which is necessary to guarantee the existence of the solution of the game.

**Fig.3** \( \mu \) which satisfies \( \chi^*_M(0, \beta) > V_M(0) \) and \( M(0, \beta) < 0 \)

**Lemma 1** For all \( \beta \in (0, \overline{\beta}) \), there exists a parameter \( \mu^*(\beta) \) such that for

\[ \mu \in \left( \delta, \mu^*(\beta) \right), \]  \hfill (G4)

it becomes

\[ \chi^*_M(0, \beta) > V_M(0), \]
Considering $\lambda^*_M$ as the function of $M$, that is,

$$\lambda^*_M = \frac{(1-\beta)(M + \mu\left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w}\right))^{1-\theta}}{\rho - M},$$

we can depict the graph of $\lambda^*_M$ as Figure 3.

The intercept of the vertical axis is $(1-\beta)\left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta} / \rho(>0)$. On the other hand, since

$V_M(0) = (\mu - \delta) / \mu$, we get $V_M(0) \to 0$ ($\mu \to \delta$). Then, the situation which is depicted in Fig. 3 occurs and lemma is guaranteed. (Q.E.D.)

Under the above preparations, let us study the solution of the game. As is shown in Fig. 1 and Fig. 2, the auxiliary equations (14) and (16) may have steady states in some cases. However, they are unstable. Therefore, we cannot adopt the ordinary method for stability analysis. To construct the solution of the game, we should solve the two problems.

(I) The first is to search for the strategy combination $(\alpha, \beta)$ which satisfies the (17) and (20).

(II) The second is to investigate the condition for strategy combination $(\alpha, \beta)$ to satisfy its constraints.

If they are usual optimal control problems, these problems are required to be solved together. However, the steady state in our game is unstable. Therefore, we analyze them one by one.

3.3 Instability of Capitalism Economy

Let us study the problem (I). As mentioned above, the game doesn’t the stable steady point. Therefore, we can guess that if there exists the solution of the game, players cannot but obey the following strategy rule.

**Strategy Rule**

The player can take the arbitrary strategy if the value of auxiliary variable is in the strategy exchange point (see (18) and (21)). Let us consider the time when it reaches at the strategy exchange point. If it is the Player R’s point, Player R takes the strategy $\hat{\alpha}$ such as $V_R(\beta) = \lambda^*_R(\hat{\alpha}, \beta)$ for a given $\beta$ and if it is the Player M’s point, Player M takes the strategy $\hat{\beta}$ such as $V_M(\alpha) = \lambda^*_M(\alpha, \hat{\beta})$ for a given $\alpha$. 

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Note that the strategy rule has its meanings, only when $M(\hat{a}, \hat{\beta}) \rho$ for a given $\beta$ and $M(\hat{a}, \hat{\tilde{\beta}}) \rho$ for a given $\alpha$. If these conditions are not satisfied, the auxiliary equations don’t have any their steady state points (see Fig.1(a) and Fig.(2)(a)). Therefore, we assume

$$M(\hat{a}, \hat{\beta}) \rho, \quad (M6)$$

$$M(\tilde{\alpha}, \hat{\tilde{\beta}}) \rho. \quad (M7)$$

The title of this section is “Instability of Capitalism Economy”. Now, let us study what happens in the economy if players obey the strategy rule. We analysis step by step.

**Step1.** Players take the strategies corresponding to (18) and (21) for the initial values of the auxiliary variables.

**Step 2.** The first player whose auxiliary variable reaches at the strategy exchange point obeys the strategy rule. Let us assume Player M reaches the strategy exchange point at first. At this moment, Player M takes the strategy which makes the strategy exchange point and the steady state of his (or her) auxiliary equation equal.

**Step 3.** By the exchange of the Player M’s strategy in step 2, the position of the strategy exchange point and that of the steady state point of the auxiliary equation of Player R change. However, when the value of the auxiliary variables of Player R reaches at his (or her) strategy exchange point, Player R takes the strategy by the strategy rule. That is, it makes the strategy exchange point and the steady state of his (or her) auxiliary equation.

**Step 4** By the exchange of the Player R’s strategy in Step 3, the positions of the strategy exchange point and that of the steady state point of the auxiliary equation of Player M which were set so as to coincide in step 2 change. However, when the value of the auxiliary variables of Player M reaches at his (or her) strategy exchange point, Player M takes the strategy by the strategy rule.

**Step 5** Each player continues to take the strategy shown in the above steps forever.

There is an important and serious problem in step 1-step 5. That is, when one player exchanges his (or her) strategy which satisfies the strategy rule, the strategy exchange point and the steady state point of the other player change. However, there is no guarantee that the value of auxiliary variable of the other player necessarily reaches at his (or her) strategy exchange point subsequently.

Let us analysis this point. For instance, the strategy exchange point and the steady state of Player R coincide in step 3. When Player M exchanges his (or her) strategy in step 4, the strategy exchange point and the steady state point of Player R change. In this case, the value of the auxiliary variable reaches at his (or her) strategy exchange point subsequently. This is because the strategy exchange point and the steady state point of Player R move toward
reverse directions each other reflecting Player M’s strategy change. See (19) and \( \lambda^*_R \) in Fig.1(a). This situation is depicted in Fig. 3. Therefore, the old strategy exchange point converges to the new one.

**Figure 3** the position change of Player R when \( \beta \) increases

\[
\begin{align*}
V_R(\beta') & = V_R(\beta) = \lambda^*_R(\alpha, \beta) \\
\lambda^*_R(\alpha, \beta') & \rightarrow \lambda^*_R(\alpha, \beta)
\end{align*}
\]

Next, let us analysis the step 3 and step 4. In step 3, Player R exchanges his(or her)strategy and takes the strategy by the strategy rule. By this exchange of strategy, the strategy exchange point and the steady stated point of the auxiliary equation which were coincided by Player M in step 2 change. However, they move toward the same direction. See (22) and \( \lambda^*_M \) in Fig.2(a).

This situation is depicted in Fig. 4. As is shown in Fig.4., the old strategy exchange point doesn’t converge to the new one and the value of the auxiliary variable become negative sooner or later. Therefore, players cannot continue playing the game and step 5 cannot be continued. We should point out that this phenomenon implies the instability of capitalism economy.

**Figure 4** the position change of Player M when \( \alpha \) increases

\[
\begin{align*}
V_M(\alpha) & = \lambda^*_M(\alpha, \beta) \\
\lambda^*_M(\alpha', \beta) & \rightarrow \lambda^*_M(\alpha', \beta)
\end{align*}
\]
3.4 Political Aid for the game to have its solution

Therefore, the problem to be solved is to construct the political aid to conquer this instability. In this section, we propose the political method which conquers our problem.

〈Policy Maker and Policy Rule〉

We assume that a policy maker exists and can control $\mu$ which represents how much rate Player M can receive over the growth rate (see (7))\(^5\).

Assume Player M’s strategy exchange point and the steady state point coincide at some time. Next, assume Player R exchanges his(or her) strategy. Then, Player M’s strategy exchange point and the steady state point move. As mentioned above, the two point move to the same direction. This fact occurs the instability. Therefore, a policy maker control $\mu$ such as Player M’s strategy exchange point moves to the different direction against the direction of the movement of the steady state point. That is, if Player R increases the value of his (or her) strategy $\alpha$, the policy maker should decrease the value of $\mu$ enough and vice versa(see (22)).

Under the existence of policy maker and his(or her) policy rule, we propose the following proposition.

Proposition 2 (the solution of optimal control)

For the strategy combination $(\alpha, \beta)$ and the parameter $\mu$, we assume that (G1),(G2),(G3) and (G4) $\mu \in [\delta, \mu'(\beta)]$ (see lemma 1) are satisfied. We also assume the existence of policy maker. He (or she) can control $\mu$ adopting the policy rule under (G1),(G2),(G3) and $\mu \in [\delta, \mu'(\beta)]$.

For initial conditions of the auxiliary variable $\lambda^0_\alpha$ and $\lambda^0_\beta$, we assume the following conditions; (C1)-(C4). Then there exists a strategy pair $(\alpha(t), \beta(t))$ which satisfies (17) and

---

\(^5\) As is shown in section 4, the policy maker is considered to be a central bank. Therefore, its method is financial policy, especially open market operation which controls the quantity of money and interest rate. See (28) in section 4.
\[ \lambda^0_R \geq \lambda^0_R (0,0), \quad (C1) \]
\[ \lambda^0_M \geq \lambda^0_M (\overline{\alpha},0), \quad (C2) \]
\[ \lambda^0_R \leq \lambda^0_R (\overline{\alpha}, \overline{\beta}), \quad (C3) \]
\[ \lambda^0_M \leq \lambda^0_M (0, \overline{\beta}), \quad (C4) \]

where the strategy \( \overline{\beta} \) satisfies \( V_M (\overline{\alpha}) = \lambda^\star_R (\overline{\alpha}, \overline{\beta}) \).

Let us consider the condition (C1), using Fig.5. Notice \( \lambda^\star_R (0,0) < V_R (0) \), because \( \alpha = 0 \).

If (C1) isn’t satisfied and Player M takes the strategy \( \beta = 0 \) notice \( \frac{\partial \lambda^\star_R (\alpha, \beta)}{\partial \beta} < 0 \), then Player R cannot exchange their strategies and the value of \( \lambda^\star_R \) becomes minus or notice \(-\infty\). In this case, the game doesn’t have its solution. Therefore, (C1) should be required. We can analyze (C2) similarly (notice \( \frac{\partial \lambda^\star_M (\alpha, \beta)}{\partial \alpha} > 0 \)).

Next, we consider (C3). Notice \( \lambda^\star_R (\overline{\alpha}, \overline{\beta}) > V_M (\overline{\alpha}) \), because \( \alpha = \overline{\alpha} \). If (C3) is not held and Player M takes the strategy \( \beta = \overline{\beta} \) Then, Player R cannot exchange his (or her) strategy and the value of \( \lambda^\star_R \) becomes \(+\infty\). In this case, the game doesn’t have its solution. Therefore, (C3) is required. We assume (C4) with the same reason (notice \( \frac{\partial \lambda^\star_M (\alpha, \beta)}{\partial \alpha} > 0 \)).

**Fig.5** the exchange points of the strategy of Player R and his/her strategy

<table>
<thead>
<tr>
<th>( (\alpha, \beta) )</th>
<th>Domain R1</th>
<th>Domain R2</th>
<th>Domain R3</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0,0), (0, \overline{\beta}))</td>
<td>( M(0,0) &lt; 0 )</td>
<td>( M(\overline{\alpha}, \overline{\beta}) &gt; \rho )</td>
<td>( M(\overline{\alpha}, \overline{\beta}) &gt; \rho )</td>
</tr>
<tr>
<td>(0 &lt; M(0, \overline{\beta}) &lt; \rho)</td>
<td>( M(\overline{\alpha}, \overline{\beta}) &gt; \rho )</td>
<td>( M(0,0) &lt; 0 )</td>
<td>( 0 &lt; M(\overline{\alpha}, 0) &lt; \rho )</td>
</tr>
</tbody>
</table>

\[ 0 \quad V_R (\overline{\beta}) \quad V_R (0) \]

**Fig.6** exchange points of the strategy of Player M and his/her strategy
Now let us construct the solution of the game by taking some examples. This construction seems to be arbitrary. However, under complete information, players should search for the strategy combination \((\alpha, \beta)\) which satisfies the (17) and (19). Under complete information, players recognize that they should obey the strategy rule, if not so they cannot construct the optimal strategies.

Firstly, we consider the case where the initial values of the auxiliary variables are in the domain R3 and M3. Since Player R and Player M choose \(\alpha = \overline{\alpha}\) and \(\beta = \overline{\beta}\) respectively, it happens \(M(\overline{\alpha}, \overline{\beta}) > \rho\) and Fig.1 (b) and Fig.2 (b) are chosen. Therefore, the values of \(\lambda_\alpha\) and \(\lambda_\beta\) decrease. In this case, one of the value of \(\lambda_\alpha\) or \(\lambda_\beta\) reaches at the strategy exchange point at first or at the same time. In the former case, if it is the point of Player R, he (or she) takes the strategy \(\hat{\alpha}\) which satisfies \(V_\alpha(\overline{\beta}) = \lambda_\alpha^*(\hat{\alpha}, \overline{\beta})\), or if it is the point of Player M, he (or she) takes the strategy \(\hat{\beta}\) which satisfies \(V_\beta(\overline{\alpha}) = \lambda_\beta^*(\overline{\alpha}, \hat{\beta})\). Notice that the auxiliary equations have the steady state points by (M6) and (M7).

Let us analyze the case where Player M reaches his (or her) strategy exchange point at first. We denote the time when Player M reaches his (or her) strategy exchange point by \(\hat{t}\). At time \(\hat{t}\), Player M exchanges his (or her) strategy from \(\beta = \overline{\beta}\) to \(\beta = \hat{\beta}(\leq \overline{\beta})\). Therefore, the strategy exchange point of Player R moves from \(V_\alpha(\overline{\beta})\) to \(V_\alpha(\hat{\beta}) (\geq V_\alpha(\overline{\beta}))\) (see (19)). Then, for the relative positional relation between \(V_\alpha(\hat{\beta})\) and \(\lambda_\alpha^*(\hat{t})\), the following three cases could happen. That is, (i) \(V_\alpha(\hat{\beta}) < \lambda_\alpha^*(\hat{t})\), (ii) \(V_\alpha(\hat{\beta}) > \lambda_\alpha^*(\hat{t})\) and (iii) \(V_\alpha(\hat{\beta}) = \lambda_\alpha^*(\hat{t})\).

In the case of (i), Player R continues to take the strategy \(\alpha = \overline{\alpha}\). By (M6), it happens \(M(\overline{\alpha}, \hat{\beta}) < \rho\). In addition, by (C3), it happens \(V_\alpha(\hat{\beta}) < \lambda_\alpha^*(\hat{t}) < \lambda_\alpha^*(\overline{\alpha}, \hat{\beta})\). Therefore, the value of the auxiliary variable \(\lambda_\alpha\) decreases and will reach at \(V_\alpha(\hat{\beta})\). At the moment when the value of

![Diagram](http://proceedings.iises.net/index.php?action=proceedingsIndexConference&id=7)
\( \lambda_R \) reaches at \( V_R(\hat{\beta}) \), Player R takes the strategy \( \alpha' \) which satisfies \( V_R(\hat{\beta}) = \lambda^*_R(\alpha', \hat{\beta}) \) according to the strategy rule.

At this moment, Player R exchanges his(or her) strategy from \( \alpha \) to \( \alpha'(<\alpha) \). Therefore, the values of \( V_M(\alpha) \) and \( \lambda^*_M(\alpha, \hat{\beta}) \) decrease to \( V_M(\alpha') \) and \( \lambda^*_M(\alpha', \hat{\beta}) \) respectively. We cannot judge which value is greater, \( V_M(\alpha') \) or \( \lambda^*_M(\alpha', \hat{\beta}) \). However, since \( V_M(\alpha') \) and \( \lambda^*_M(\alpha', \hat{\beta}) \) is smaller than \( V_M(\alpha) = (\lambda_M(t)) \), it happens \( \lambda_M(t) \rightarrow \infty(t \rightarrow \infty) \) and the game is destroyed. Therefore, the policy maker should control \( \mu \) by the policy rule. That is, he (or she) control \( \mu \) so as for \( V_M(\alpha') \) to increase. Then, the value of \( \lambda_M(t) \) moves to \( V_M(\alpha') \). At the moment when the values of \( \lambda_M(t) \) and \( V_M(\alpha') \) coincide, Player M obeys the strategy rule.

The players have only to repeat the above process. Notice that the value of the auxiliary variables are confined in a finite domain. Of course, the case may happen where players reach at the strategy \((\alpha', \beta')\) which satisfies

\[
\begin{align*}
V_R(\beta') &= \lambda^*_R(\alpha', \beta'), \\
V_M(\alpha') &= \lambda^*_M(\alpha', \beta')
\end{align*}
\]

In either case, the transversal conditions are satisfied.

We have analyzed the case where the initial values of the auxiliary variables are in the domain R3 and M3. However, even if the initial values of the auxiliary variables are in other domains, we can construct the strategy pair which satisfies the optimal problems.

(QED)

Next, let us analyze the problem (II). The strategy constructed in proposition 2 should satisfy the following conditions:

\[
\begin{align*}
0 &< \alpha^* < 1, \\
0 &< \beta^* < 1, \\
0 &< M(\alpha^*, \beta^*) < \rho.
\end{align*}
\]

With respect to (24), we can propose the following proposition.

**Proposition 3** If \( \mu \) satisfies

\[
\begin{align*}
\rho \left[ \alpha \left( \frac{1 - \theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - \frac{1 - \theta}{\theta} \cdot \frac{p_k}{w} \right] - \delta \\
(1 - \beta)M(\alpha^*, \beta^*) + \mu \left( \frac{1 - \theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - \rho
\end{align*}
\]

(G5)

http://proceedings.iises.net/index.php?action=proceedingsIndexConference&id=7
and the solutions of (23), \(\alpha^*\) and \(\beta^*\), take positive values, then the strategy combination \((a, \beta)\) constructed in proposition 2 satisfies (24).

(pf.) Firstly, we write out the terms in (23) concretely.

\[
V_R(\beta) = \frac{\left(1 - \frac{\theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta} (\delta - \beta \mu)}{(1 - \mu)\left(\frac{1 - \theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta} - \frac{1 - \theta}{\theta} \cdot p_k} + \left\{1 - \beta \left(\frac{1 - \theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta}\right\}
\]

\[
\lambda^*_R(\alpha, \beta) = \frac{(1 - \alpha)\left[\left(\frac{1 - \theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta} - \frac{1 - \theta}{\theta} \cdot p_k\right] + \mu}{\rho - M(\alpha, \beta)}
\]

\[
V_M(\alpha) = \frac{\left(\frac{1 - \theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta} - \frac{1 - \theta}{\theta} \cdot p_k}{\mu} + (\mu - \delta)
\]

\[
\lambda^*_M(\alpha, \beta) = \frac{(1 - \beta)\left[\left(M(\alpha, \beta) + \mu\right)\left(\frac{1 - \theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta}\right]}{\rho - M(\alpha, \beta)}
\]

Except \(1 + (\alpha - \beta)\left(\frac{1 - \theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta} = 0\) (see (G0)), the function \(M(\alpha, \beta)\) is continuous. In addition, we assume (M6) and (M7). Therefore, the above four functions are continuous at the domain of definition we consider. So, it is apparent that the equation system (23) has its solution.

Let us analyze the condition (23). Since \(V_M(\alpha) > 0\), we get \(M(\alpha^*, \beta^*) < \rho\) by the second equation of (23). In addition, we get \(M(\alpha^*, \beta^*) > 0\) by (G5).

On the other hand, we get \(\alpha^* < 1\) and \(\beta^* < 1\) by \(V_M(\beta^*) = \lambda^*_M(\alpha^*, \beta^*) > 0\) and \(V_M(\alpha^*) = \lambda^*_M(\alpha^*, \beta^*) > 0\).

The remained problem is the positivity of \(\alpha^*\) and \(\beta^*\). It depends on the value of the parameter \(\mu\), as is the main problem of our paper. At this point, we can insist that it is apparent that if we put an additional condition, the upper-limit of \(\mu\), as (G6), we can guarantee the positivity of non-negativity \(\alpha^*\) and \(\beta^*\). (Q.E.D.)

We can guarantee the existence of the solution of the game by proposition 2 and proposition
Finally, we should insist two points. The first is that since the auxiliary equation of our game has no stable steady point, we cannot adopt the usual method for analyzing optimal problem. Under the complete information, players know these conditions and construct their strategies for the optimal conditions to be satisfied.

The second point is that our game needs the aid of financial policy which is shown in the policy rule. This implies that the capitalism economy is unstable by itself and needs the proper control of $\mu$.

### 3.5 Definition of Natural Interest Rate

Let us consider the equations (23). It means that the strategy exchange point of each player coincides with the steady state point. As is shown in (18) and (21), players invest all of the rest of consumption or nothing except the strategy exchange points. They want to improve the condition of the economy. However, players think that the strategy exchange points are neutral, where they feel no necessity for improving their condition. The word 'neutral' means that consumption and investment are *indifference* for the players. In addition, the savings of players are equal to the investment which contributes to the capital accumulation (see (6)). Furthermore, the rest of the part of GDP which is invested to production is accurately equal to the consumption of the players. These conditions seemed to construct the character of *natural economy*. Therefore, we call the strategy combination $(\alpha^*, \beta^*)$ *natural strategy* and the growth rate of economy which is accomplished by the natural strategy *natural growth rate*. For the natural strategy, the economy is considered to be in the stable steady point.

On the other hand, we define *natural distribution rate* of Player $M$, $\gamma^*$, by

$$\gamma^* = M(\alpha^*, \beta^*) + \mu^*, \quad (25)$$

where $\mu^*$ satisfies $(G1)$–$(G6)$.

Next, let us put

$$rM = \gamma Y, \quad (26)$$

where $M$ and $r$ denote the amount of monetary assets and its interest rate respectively. We define *natural nominal interest rate* $r^*$ by

$$r^*M = \gamma^*Y^* \quad (27)$$

where $Y^*$ denotes the GDP which is produced by the natural strategy. Since we define the natural nominal interest rate which depends on the amount of monetary assets, we also call the natural distribution rate, $\gamma^*$, natural real interest rate. By definition, if $M = Y$, then the natural nominal interest rate is equal to the natural distribution rate (the natural real interest rate).

As is mentioned in the introduction, there are some definitions for natural economy. However, it is important to recognize that the natural economy is an absolute criterion from which
economy cannot deviate over a certain range. We should recognize that the large deviation from natural rate gives us an important alarm.

4. The fundamental Cause of Financial Crisis

In section 3, we analyzed the condition for the game to have its solution. By this analysis, we get the result that there are rigorous conditions which $\mu$ should satisfy for the capitalism economy to be stable.

In this section, we investigate the actual condition with respect to $\mu$. We adopt three areas to be investigated, US, Euro Area and Japan. Our purpose is to show what happens about $\mu$ and to prove that the unbalance of the real economic sector and the monetary sector causes the economic crisis.

4.1 Preparation for the Simulation

To introduce the structure of our game to the real economy, let us put

$$M = aY,$$

where the constant $a(>0)$ denotes the ratio of the amount of monetary assets and GDP.

From (26), (28) and (7), we get

$$\mu = ar - \frac{\dot{Y}}{Y}. \tag{29}$$

Now, we try the simulation of (29).

4.2 Current Trends and our Model

Firstly, we should confirm the current trend of the actual economy. The remarkable trend in the current economy is the rapid increase in monetary base. This trend is common in US, Euro Area and Japan. As is well known, this is the result of Keynesian fiscal and financial policies. As a result, the financial assets rapidly intumesce and the scale of the world’s financial assets becomes three times as much as that of the world’s GDP. We cannot but say that the figure of ‘3.5’ implies the large deviation from natural economy. Therefore we should investigate what happened about $\mu$ behind the financial crisis such as dot-com bubble, the Lehman crash or PIIGS.

Now we show the transition of $\mu$ and the basis data used by calculating $\mu$ for US, Euro Area and Japan.

In the Euro area, the ratio of monetary base-GDP keeps high level and the scale of monetary base is about 27 times as much as that of GDP in 2013. The level seems to be dangerous. Compare with the other two areas. In Fig.7(c), $\mu$ has two peaks. The increases in $\mu$ during the period of 1999-2000 corresponds to the dot-com bubble and that of 2005-2007 corresponds

---

to the Lehman crash. These increases imply that the monetary sector absorbed the value from the real economic sector at the high rate. As a result, \( \mu \) decreased rapidly. These decreases correspond to the recession of the economy (see Fig.7(a)). Further decrease in 2010 corresponds to the big recession, or PIIGS.

### Table 1 \( \mu \) in Euro Area

<table>
<thead>
<tr>
<th>Year</th>
<th>Growth Rate of GDP(%)</th>
<th>( a=M/Y )</th>
<th>( ar )</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>2.9</td>
<td>14.12568928</td>
<td>0.423771</td>
<td>0.394771</td>
</tr>
<tr>
<td>2000</td>
<td>3.81</td>
<td>15.0034893</td>
<td>0.712666</td>
<td>0.674566</td>
</tr>
<tr>
<td>2001</td>
<td>1.73</td>
<td>15.28847263</td>
<td>0.496875</td>
<td>0.479575</td>
</tr>
<tr>
<td>2002</td>
<td>1.15</td>
<td>15.43035691</td>
<td>0.41662</td>
<td>0.40512</td>
</tr>
<tr>
<td>2003</td>
<td>0.53</td>
<td>16.00202686</td>
<td>0.320041</td>
<td>0.314741</td>
</tr>
<tr>
<td>2004</td>
<td>2.15</td>
<td>17.02344694</td>
<td>0.340469</td>
<td>0.318969</td>
</tr>
<tr>
<td>2005</td>
<td>1.88</td>
<td>18.64130773</td>
<td>0.419429</td>
<td>0.400629</td>
</tr>
<tr>
<td>2006</td>
<td>3.37</td>
<td>20.36590368</td>
<td>0.712807</td>
<td>0.679107</td>
</tr>
<tr>
<td>2007</td>
<td>2.97</td>
<td>22.15376957</td>
<td>0.886151</td>
<td>0.856451</td>
</tr>
<tr>
<td>2008</td>
<td>-0.02</td>
<td>23.27430814</td>
<td>0.640043</td>
<td>0.640243</td>
</tr>
<tr>
<td>2009</td>
<td>-5.35</td>
<td>24.7946816</td>
<td>0.247947</td>
<td>0.301447</td>
</tr>
<tr>
<td>2010</td>
<td>-4.41</td>
<td>25.73965035</td>
<td>0.257397</td>
<td>0.301497</td>
</tr>
<tr>
<td>2011</td>
<td>1.43</td>
<td>25.86537428</td>
<td>0.258654</td>
<td>0.244354</td>
</tr>
<tr>
<td>2012</td>
<td>-0.73</td>
<td>27.67888397</td>
<td>0.207592</td>
<td>0.214892</td>
</tr>
<tr>
<td>2013</td>
<td>-0.36</td>
<td>27.601707</td>
<td>0.069004</td>
<td>0.072604</td>
</tr>
</tbody>
</table>

Next, let us investigate the data in USA. The graph of $\mu$ in USA also has two peaks as that of:

-6
-4
-2
0
2
4
6


Fig. 7(a) Growth Rate of GDP in Euro Area

Fig. 7(b) $a=M/Y$ in Euro

Fig. 7(c) $\mu$ in Euro

http://proceedings.iises.net/index.php?action=proceedingsIndexConference&id=7
Euro area. These peaks are sharper than those in Euro area. However, the transitions of data in USA and that in Euro area have the same trend.

Table 2  μ in USA

<table>
<thead>
<tr>
<th>Year</th>
<th>Growth Rate of GDP(%)</th>
<th>a=M/Y</th>
<th>ar</th>
<th>μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>5.691544885</td>
<td>3.024098</td>
<td>0.15937</td>
<td>0.102455</td>
</tr>
<tr>
<td>1997</td>
<td>6.2751537</td>
<td>3.18922</td>
<td>0.177321</td>
<td>0.114569</td>
</tr>
<tr>
<td>1998</td>
<td>5.582854156</td>
<td>3.403109</td>
<td>0.189213</td>
<td>0.133384</td>
</tr>
<tr>
<td>1999</td>
<td>6.343862429</td>
<td>3.61053</td>
<td>0.171861</td>
<td>0.108423</td>
</tr>
<tr>
<td>2000</td>
<td>6.455817996</td>
<td>3.604031</td>
<td>0.235343</td>
<td>0.170785</td>
</tr>
<tr>
<td>2001</td>
<td>3.261513941</td>
<td>3.704366</td>
<td>0.147063</td>
<td>0.114448</td>
</tr>
<tr>
<td>2002</td>
<td>3.340140984</td>
<td>3.714313</td>
<td>0.065</td>
<td>0.031599</td>
</tr>
<tr>
<td>2003</td>
<td>4.845084789</td>
<td>3.916306</td>
<td>0.047779</td>
<td>-0.00067</td>
</tr>
<tr>
<td>2004</td>
<td>6.643387016</td>
<td>4.03531</td>
<td>0.041564</td>
<td>-0.02487</td>
</tr>
<tr>
<td>2005</td>
<td>6.666123646</td>
<td>4.104502</td>
<td>0.124777</td>
<td>0.058116</td>
</tr>
<tr>
<td>2006</td>
<td>5.822655284</td>
<td>4.29026</td>
<td>0.214084</td>
<td>0.155857</td>
</tr>
<tr>
<td>2007</td>
<td>4.491300991</td>
<td>4.508443</td>
<td>0.236693</td>
<td>0.19178</td>
</tr>
<tr>
<td>2008</td>
<td>1.657424225</td>
<td>4.45351</td>
<td>0.08907</td>
<td>0.072496</td>
</tr>
<tr>
<td>2009</td>
<td>-2.054305958</td>
<td>4.646086</td>
<td>0.009757</td>
<td>0.0303</td>
</tr>
<tr>
<td>2010</td>
<td>3.748118658</td>
<td>4.544032</td>
<td>0.008179</td>
<td>-0.0293</td>
</tr>
<tr>
<td>2011</td>
<td>3.847362334</td>
<td>4.464381</td>
<td>0.004018</td>
<td>-0.03446</td>
</tr>
<tr>
<td>2012</td>
<td>0</td>
<td>4.73501</td>
<td>0.007576</td>
<td>0.007576</td>
</tr>
</tbody>
</table>

Source; http://www.federalreserve.gov/
### Table 3: μ in Japan

<table>
<thead>
<tr>
<th>Year</th>
<th>Growth Rate of GDP (%)</th>
<th>a=M/Y</th>
<th>ar</th>
<th>μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981</td>
<td>4.17684392</td>
<td>2.99706671</td>
<td>0.18731667</td>
<td>-0.04176844</td>
</tr>
<tr>
<td>1982</td>
<td>3.37660646</td>
<td>3.24407638</td>
<td>0.1784242</td>
<td>0.144658136</td>
</tr>
<tr>
<td>1983</td>
<td>3.06073847</td>
<td>3.55714537</td>
<td>0.195643</td>
<td>0.16503561</td>
</tr>
<tr>
<td>1984</td>
<td>4.4638997</td>
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**Fig. 8(b) a=M/Y in USA**

**Fig. 8(c) μ in US**

in Japan
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<th>Value 4</th>
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Source: http://www.boj.or.jp/

Finally, let us investigate Japan. As is shown in Fig.9(c), the graph of $\mu$ has a sharp and high peak in 1991. This peak corresponds to the collapse of babble in Japan. The low level of $\mu$ during about 20 years corresponds to so called ‘two lost decade of Japan’. During this period, the monetary sector had little power to absorb the value from real economic sector because there was no room for the real economic sector to produce enough GDP to be absorbed. More precisely, Japanese firms were starting up activities overseas to pursue the low cost production and cultivate the markets.
Finally, let us investigate US, Euro Area and Japan totally. Concentrating our attentions on \( \mu \), the graphs of \( \mu \) have sharp peaks which corresponds to the financial crashes. This trend is common to the three areas. At the aspect of rising, the monetary sector absorbed the value from the real economic sector with the background of the intumescence of monetary base. After the peaks, the value of \( \mu \) decreased rapidly. This decrease has an essential reason. That is, it is the result of the rapid decrease of growth rate, or economic collapse.

The rapid increase and decrease implies the large deviation from natural real interest rate (natural distribution rate of monetary sector). Therefore we should result that the most important and fundamental cause of financial crisis is the intumescence of monetary base and the deviation from natural economy.

If the economy enters into economic collapse, the government achieves its big fiscal policy by issuing the big amount of national bonds. In addition, the central bank achieves the enormous buying operations and supply the enormous amount of money. This money absorbs the value from the real economic sector. As a result, the economy is obliged to experience more severe collapse. The world economy falls in the vicious cycle. The vicious cycle deepens the problem.

4.3 The Difficulty of Controlling \( \mu \)

Can we escape from the vicious cycle? This is depends on the capability of controlling \( \mu \). Let us investigate (29) again. From the viewpoint of financial policy, if central bank wants to lower the interest rate, it should increase the monetary base (or the money supply) and vice vasa. The increase (decrease) in monetary base brings out the increase (decrease) the parameter \( a(>0) \) in (29). However, for instance, if the central bank increases the monetary base, the interest rate will decrease. That is, in \( ar \) in (29), \( a \) and \( r \) move the opposite direction when financial policy is achieved. This implies that the central bank faces on a contradiction in controlling \( \mu \). The difficulty of controlling \( \mu \) also means the difficulty of escaping from the vicious cycle.

Conclusion

Finally, let us review our paper from the viewpoint of theoretical economics. Needless to say, it is Keynes who built up the macro-economics. The representative feature of Keynes’ economics is to inspire uncertainty to the economics. The uncertainty brings out the expectation among the economic agents. However, the agents in the Keynes’ economics doesn’t make decisions corresponding to the strategy made by the other agents. Therefore, it is necessary to inspire the game theory to macro-economy which is constructed by the real economic sector and the monetary sector.

This approach proposes three important theoretical results.

1. The Parameter \( \mu \) which represents how much rate Player M can receive over the growth
rate has a rigorous range for the game to have its solution.

(2) The aid of financial policy is inevitable for the game to have its solution. In other word, capitalism economy becomes unstable without the policy rule which is shown in section 3.

(3) We define the natural economy which is realized under natural strategies and natural distribution rate of monetary sector (see section 3.3). As is mentioned above, the importance of natural economy is that it represents the rigorous bench mark around which the real economy could fluctuate. The large deviation from this bench mark gives us an articulate and critical alarm. Our model is Keynesian-game theoretic model. Therefore, the natural economy proposes an earnest criterion which the policy should obey. It also implies there is a rigorous balance between the real economic sector and the monetary sector, or a balance between GDP and monetary base for capitalism economy to survive.

Conferences


http://www.boj.or.jp/


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