DOI: 10.20472/EFC.2020.014.011

JINGWEI PAN

Department of Empirical Economic Research, Technical University of Darmstadt, Germany

EVALUATING CORRELATION FORECASTS UNDER ASYMMETRIC LOSS

Abstract:

Correlation indicates the strength of the linear relationship between two random variables and is therefore relevant for asset pricing, portfolio choice and risk management. In addition, forecasts of correlation dynamics allow for a better evaluation of the systemic risk and may give an initial signal about potential crises (Engle (2009)).

This paper aims to evaluate daily correlation forecasts. For the calculation of the correlation forecasts, the BEKK model of Engle and Kroner (1995) and the DCC model of Engle (2002) are applied. Since there is no clear suggestion regarding the sampling scheme to estimate the realized correlations from intraday data (Andersen et al. (2006)), several experimental schemes with different sampling intervals are examined. Following Komunjer and Owyang (2012), a multivariate loss function which may be asymmetric is used to measure the distance between model correlation forecasts and realized correlations.

The data sample contains intraday high-frequency and closing prices of the three major US indices: S&P 500, NASDAQ 100 and Russell 2000. Based on the results obtained so far, the following conclusions can be drawn: (i) Both models better predict correlations for the pair S&P 500 and NASDAQ 100 than for Russell 2000 and other two indices. (ii) The DCC model performs better than the BEKK model applying the symmetric loss function. (iii) On the basis of the correlation pairs between the Russell 2000 index and other two indices, the optimal degrees of asymmetry are negative for the BEKK forecast errors and positive for the DCC forecast errors in most cases. (iv) The degrees of asymmetry depend on the choice of sampling schemes for calculating the realized correlations. (v) Both models are unable to capture the sudden decrease of correlations during the crisis period.

Keywords:

Correlation forecasting, BEKK, DCC, asymmetric loss function

JEL Classification: C10, C52, G17

1 Introduction

Financial markets quickly respond to the future expectation of different market participants, resulting in a continuous change of asset prices. Dynamic correlations measure the degree to which two or more assets move together in time. On the one hand, correlations are critical inputs for the common tasks of financial management. For instance, the valuation of financial products such as quanto option, correlation swap, CDS and CDO depends on the anticipated future correlations. On the other hand, correlation risk, referred to the adverse movements in correlations, is widely used to measure systematic risk (Engle (2009)). A sharp increase in correlations from September 2008 to March 2009 implies large unexpected contagion losses across the global economy.

From econometric literature, one of the easiest model for estimating and forecasting covariance and correlation over time is the Exponentially Weighted Moving Average (EWMA) Approach proposed by RiskMetrics (see J.P. Morgan and Reuters (1996)). However, the assumption of a single decay factor for all assets is not convincing. In comparison, the multivariate GARCH models such as the VECH model of Bollerslev et al. (1988) and the BEKK model of Engle and Kroner (1995) estimate the optimal weighting parameters based on the sample data. For practical applications, the number of parameters explodes as the number of assets increases, causing estimation and implementation problems. Commonly applied alternatives to forecast the large-scale time-varying correlation matrices are the Dynamic Conditional Correlation (DCC) model of Engle (2002) and the Varying Conditional Correlation (VCC) model of Tse and Tsui (2002). Moreover, Cappiello et al. (2006) present an asymmetric specification of conditional correlations while Hafner and Frances (2006) generalize the DCC model by allowing asset-specific correlation sensitivities.

The choice of a particular forecasting model leads to the theoretical discussion about flexibility and feasibility (see De Almenda (2018)). In this paper, we choose the first-order BEKK model as a possible flexible model and compare its forecast performance with the more restrictive first-order DCC model in a forecast evaluation framework. Using the available empirical data, the following three research questions will be studied:

- Do we need both the BEKK and the DCC model?
- How do both models perform over time, especially during the crisis period of 2008/9?
- Does a specific model choice reflect the asymmetric user preference over a defined forecasting period?

2 Methodological Considerations

In this paper, the bold symbol in lowercase represents a vector and the capitalized bold symbol represents a matrix while the light symbol refers to the data observation.

2.1 Multivariate GARCH Models

Let $\boldsymbol{r}_t$ be a vector containing n asset returns. The multivariate decomposition can be written as

$$\boldsymbol{r}_t = \boldsymbol{M}_t + \boldsymbol{\epsilon}_t, \quad \boldsymbol{\epsilon}_t = \boldsymbol{H}_t^{1/2}\boldsymbol{Z}_t, \quad \boldsymbol{Z}_t \sim iid\; N(\boldsymbol{0}, \boldsymbol{I}), \quad \boldsymbol{\epsilon}_t |\, F_{t-1} \sim N(\boldsymbol{0}, \boldsymbol{H}_t) \qquad (1)$$

where $\boldsymbol{M}_t$ and $\boldsymbol{H}_t$ denote the conditional mean vector and the conditional covariance matrix for $\boldsymbol{r}_t$. Because this paper focuses on the daily data, it is plausible to assume $\boldsymbol{M}_t = E(\boldsymbol{r}_t | F_{t-1}) = \boldsymbol{0}$ and $\boldsymbol{r}_t = \boldsymbol{\epsilon}_t$.

The $BEKK(1,1,1)$ model of Engle and Kroner (1995) specifies the covariance matrix

$$\boldsymbol{H}_t = \boldsymbol{CC}' + \boldsymbol{A}\boldsymbol{r}_{t-1}\boldsymbol{r}'_{t-1}\boldsymbol{A}' + \boldsymbol{B}\boldsymbol{H}_{t-1}\boldsymbol{B}' \tag{2}$$

with $0.5n(n+1)+2n^2$ parameters and guarantees positive semidefinteness under very weak regularity conditions. However, even the simple BEKK model suffers from the curse of dimensionality and the estimation is computationally infeasible for a larger system with $n > 5$, see Ledoit et al. (2003).

The $DCC(1,1)$ model of Engle (2002) shows an alternative decomposition of the covariance matrix

$$\boldsymbol{H}_t = \boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t \tag{3}$$

where $\boldsymbol{D}_t$ and $\boldsymbol{R}_t$ represent the diagonal matrix of the conditional standard deviations and the conditional correlation matrix, respectively. The DCC model can be estimated in two steps at lower computational costs. In the first step, the univariate volatilities of n individual assets are estimated in a $GARCH(1,1)$ framework, resulting in an estimate of $\widehat{\boldsymbol{D}}_t$. Then, the vector of volatility-adjusted returns $\hat{\boldsymbol{\epsilon}}_t = \widehat{\boldsymbol{D}}_t^{-1}\boldsymbol{r}_t$ and the corresponding long-run correlation matrix $\widehat{\boldsymbol{R}} = \frac{1}{T_{in}}\sum_{t=1}^{T_{in}} \hat{\boldsymbol{\epsilon}}_t\,\hat{\boldsymbol{\epsilon}}'_t$ are calculated with the in-sample length T_{in}. In the second step, the symmetric positive definite quasi-correlation matrix is given by

$$\boldsymbol{Q}_t = (1-\alpha-\beta)\cdot\widehat{\boldsymbol{R}} + \alpha\cdot\hat{\boldsymbol{\epsilon}}_{t-1}\hat{\boldsymbol{\epsilon}}'_{t-1} + \beta\cdot\boldsymbol{Q}_{t-1} \tag{4}$$

The persistence factor $\alpha+\beta$ defines the speed of mean-reversion. Subsequently, both parameters are separately estimated using the maximum likelihood method for the correlation part. The estimated $\widehat{\boldsymbol{Q}}_t$ is re-scaled

$$\widehat{\boldsymbol{R}}_t = \mathrm{diag}\{\widehat{\boldsymbol{Q}}_t\}^{-1/2}\widehat{\boldsymbol{Q}}_t\mathrm{diag}\{\widehat{\boldsymbol{Q}}_t\}^{-1/2} \tag{5}$$

to ensure that the diagonal elements are one and the off-diagonal elements are between minus one and one.

2.2 Evaluation of Correlation Forecasts

Denoting the full set of the possible correlation matrices and of the possible candidate models by $\dot{\boldsymbol{R}}$ and $\dot{M}$, the optimal model correlation matrix on the day t is

$$\boldsymbol{R}^*_{i,t} = \underset{\boldsymbol{R}_{i,t}\in\dot{\boldsymbol{R}}}{\arg\min}\, L(\boldsymbol{P}_t, \boldsymbol{R}_{i,t}), \quad \forall i \in \dot{M} \tag{6}$$

where L and $\boldsymbol{P}_t$ indicate the loss function of a unknown forecaster and the true conditional correlation matrix, respectively.

Since the $\boldsymbol{P}_t$ is latent, a benchmark proxy matrix is needed for the evaluation framework. Therefore, the feasible realized correlation matrix of Andersen et al. (2000) will be used, given the availability of high-frequency data.

Using an equidistant calendar-time sampling scheme of Hansen and Lunde (2006), the daily interval $[a,b]$ is equally divided into m subintervals with $a = t_0(m) < \cdots < t_m(m) = b$. Let $\boldsymbol{p}$ be the $nx1$ price vector, the vector of intraday continuously compounded returns over the ith subinterval can be formulated as

$$\boldsymbol{r}_{t_i(m)} = \log(\boldsymbol{p}_{t_i(m)}) - \log(\boldsymbol{p}_{t_{i-1}(m)}). \tag{7}$$

The realized correlation matrix $\widehat{\boldsymbol{P}}_t$ is

$$\widehat{\boldsymbol{P}}_t = \mathrm{diag}\left\{\sum_{i=1}^{m} \boldsymbol{r}_{t_i(m)} \boldsymbol{r}'_{t_i(m)}\right\}^{-0.5} \sum_{i=1}^{m} \boldsymbol{r}_{t_i(m)} \boldsymbol{r}'_{t_i(m)} \, \mathrm{diag}\left\{\sum_{i=1}^{m} \boldsymbol{r}_{t_i(m)} \boldsymbol{r}'_{t_i(m)}\right\}^{-0.5}. \quad (8)$$

To gain more information efficiency, the robust averaging and subsampling technique of Zhang et al. (2005) is applied for calculating the feasible subsampled realized correlation matrix.

According to Laurent et al. (2013), the consistency of ordering based on proxy matrices is ensured when the underlying loss function is well defined. However, the robust family of loss functions in Patton and Sheppard (2009) does not include the possibility to separately model the asset-specific user preferences, whereas the approach of Komunjer and Owyang (2012) does.

Given a forecast error vector[1] containing l correlation pairs

$$\boldsymbol{e}_{t+1} = \begin{pmatrix} e_{1,t+1} \\ \vdots \\ e_{l,t+1} \end{pmatrix}, \quad e_{i,t+1} = \hat{\rho}_{i,t+1} - \hat{f}_{i,t+1}, \quad i = 1, \dots, l = \frac{n(n-1)}{2} \quad (9)$$

the l-variante loss on the forecast day $t+1$ is defined by Komunjer and Owyang (2012) as

$$L_p(\boldsymbol{\tau},\, \boldsymbol{e}_{t+1}) = \left(\|\boldsymbol{e}_{t+1}\|_p + \boldsymbol{\tau}'\boldsymbol{e}_{t+1}\right)\|\boldsymbol{e}_{t+1}\|_p^{p-1}, \quad \|\boldsymbol{e}_{t+1}\|_p = \left(\left|e_{1,\,t+1}\right|^p + \dots + \left|e_{l,\,t+1}\right|^p\right)^{1/p} \quad (10)$$

with $\boldsymbol{\tau} = (\tau_1, \dots, \tau_l)'$ as the vector for asymmetric preference and $p \geq 1$ as the degree of curvature. For all $i = 1, \dots, l$, the condition $-1 \leq \tau_i \leq 1$ holds.

2.3 Optimal Preference of Forecasters

The orthogonal moment condition combines the first-order derivative of Equation (10) and the vector of k instrumental variables $\boldsymbol{w}_t = (1, w_{1,t}, \dots, w_{k,t})'$ with the Kronecker product:

$$\boldsymbol{g}_p(\boldsymbol{\tau},\, \boldsymbol{e}_{t+1}, \boldsymbol{w}_t) = \left[p\boldsymbol{v}_t(\boldsymbol{e}_{t+1}) + \boldsymbol{\tau}\|\boldsymbol{e}_{t+1}\|_p^{p-1} + (p-1)\boldsymbol{\tau}'\boldsymbol{e}_{t+1}\|\boldsymbol{e}_{t+1}\|_p^{-1}\boldsymbol{v}_t(\boldsymbol{e}_{t+1})\right] \otimes \boldsymbol{w}_{\mathrm{t}}. \quad (11)$$

Over a defined forecast period, the optimal preference is calculated

$$\hat{\boldsymbol{\tau}}^* = \min_{\boldsymbol{\tau} \in \mathbb{R}^n \text{ with } -1 \leq \tau_i \leq 1} \left[T_{out}^{-1} \sum_{t=T_{in}}^{T_{in}+T_{out}-1} \boldsymbol{g}_p(\boldsymbol{\tau}, \hat{\boldsymbol{e}}_{t+1}, \boldsymbol{w}_t)\right]' \widehat{\boldsymbol{S}}^{-1} \left[T_{out}^{-1} \sum_{t=T_{in}}^{T_{in}+T_{out}-1} \boldsymbol{g}_p(\boldsymbol{\tau}, \hat{\boldsymbol{e}}_{t+1}, \boldsymbol{w}_t)\right] \quad (12)$$

using the GMM method of Hansen (1982), where T_{out} represents the length of the out-of-sample forecast period and $\widehat{\boldsymbol{S}}$ is a consistent estimator of

$$\boldsymbol{S} = E\left[\boldsymbol{g}_p(\boldsymbol{\tau}_0,\, \boldsymbol{e}_{t+1}, \boldsymbol{w}_t)\boldsymbol{g}_p(\boldsymbol{\tau}_0,\, \boldsymbol{e}_{t+1}, \boldsymbol{w}_t)'\right] \quad (13)$$

with an initial asymmetry vector $\boldsymbol{\tau}_0$.

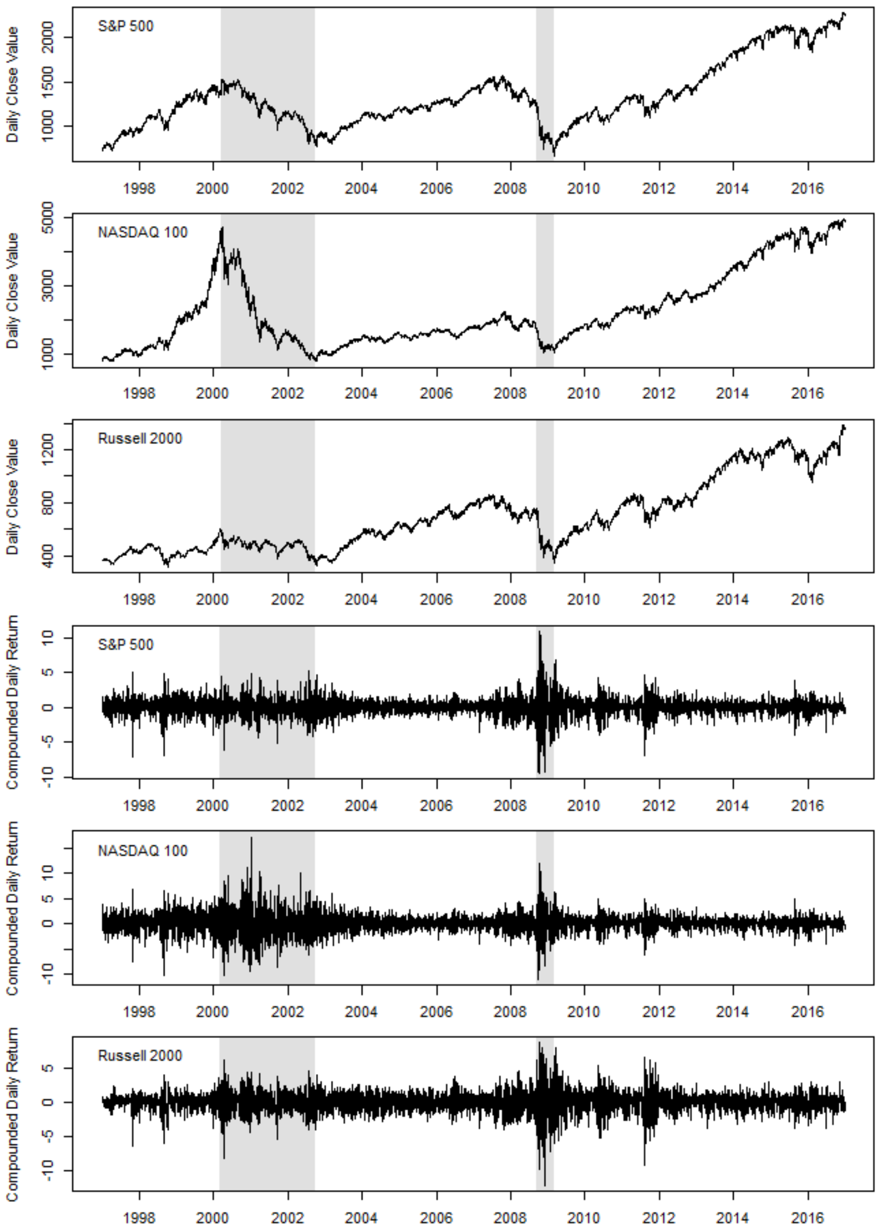
3 Data Description

The historical intraday split and dividend-adjusted index values of S&P 500, NASDAQ 100 and Russell 2000 from the sample period 1997-2016 are gathered from the PiTrading Database. All these values are provided in one-minute time intervals, covering the full trading day starting from 9:30 to 15:30 EST and including opening, closing, highest and lowest values.

[1] $\hat{\rho}_{i,t+1}$ denotes the ith correlation pair from the ex-post subsampled proxy matrix $\widehat{\boldsymbol{P}}_{t+1}$, while $\hat{f}_{i,t+1}$ denotes the ith correlation pair from forecasted model correlation matrix $\widehat{\boldsymbol{R}}_{t+1}$.

To better illustrate the stock market developments over time, we sample for each day the last closing value, calculate the corresponding returns and present the daily patterns in Figure 1.

Figure 1: Sample Paths on Daily Close Index Value and Compounded Return



Note: The compounded daily returns are reported in percent.

Following the sample patterns of these three indices and the important events of stock market history (see e.g. Schiller (2005)), we divide the full time period into five different sub-periods:

- P1 (1997/01/02 – 2000/03/10): Millennium Boom
- P2 (2000/03/13 – 2002/10/07): Burst of Dot-Com Bubble

- P3 (2002/10/08 – 2008/09/12): Recovery and Uncertainty
- P4 (2008/09/15 – 2009/03/09): Intense Stage of Global Financial Crisis
- P5 (2009/03/10 – 2016/12/30): Post-Crisis and New-Normal Boom

The grey shades highlight the two crisis sub-periods with high volatility while the white shades mark the three normal sub-periods with low volatility.

To better understand the co-dynamics among different market segments, we apply the estimation methods of EWMA, BEKK and DCC and show three correlation pairs in Figure 2.

Figure 2: Correlation Estimates

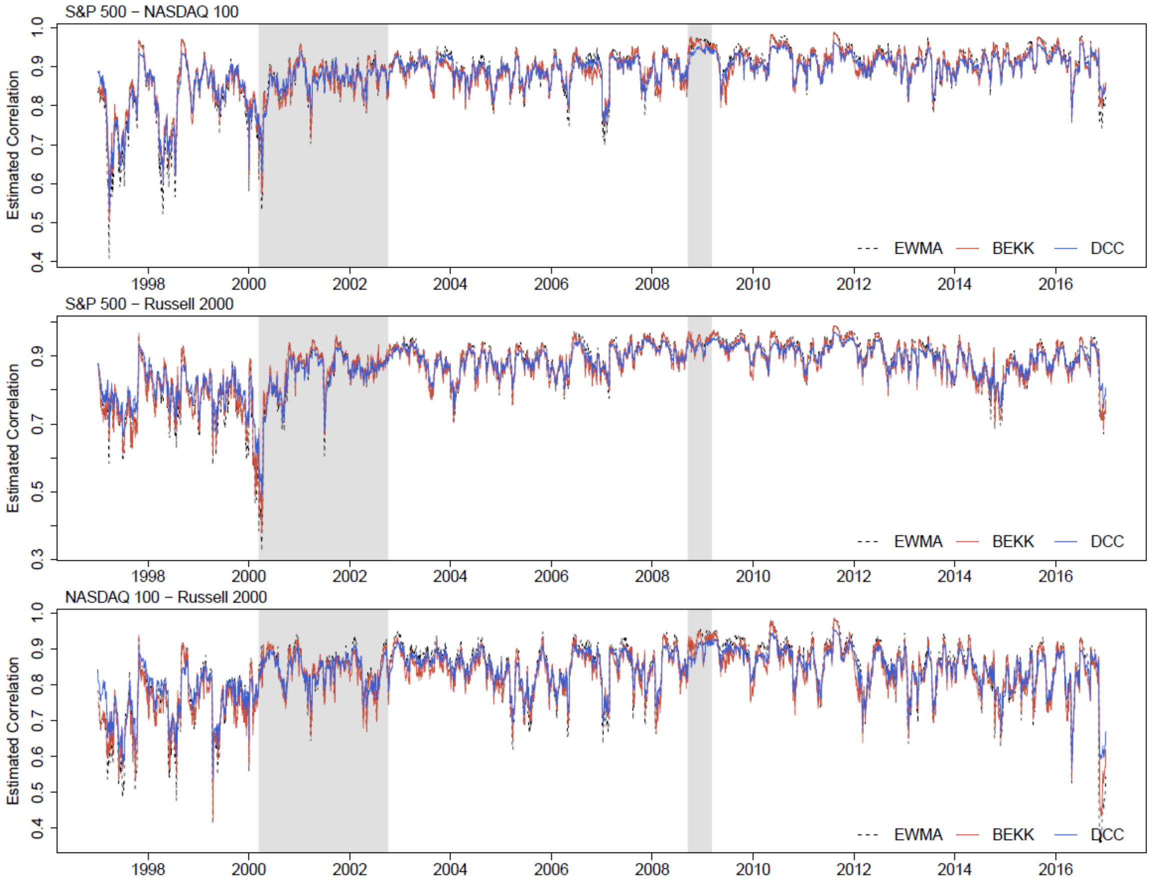


Figure 2 gives rise to three findings: First, all the methods show similar correlation paths over time. Second, the burst of speculative dot-com mania in sub-period P2 affects the high-tech and small-cap companies more than the big-cap companies, causing a sudden correlation decrease for the index pairs S&P 500 – NASDAQ 100 and S&P 500 – Russell 2000 after March 10, 2000. Third, because the same macroeconomic shocks influence almost all industries in the same way during sub-period P4, high correlations for all three index pairs are observed.

4 Forecast Performance Evaluation

In this section, we compare the one-step ahead forecast performance of BEKK and DCC models over the full evaluation period and all the sub-periods. In general, we construct an initial estimation sample from January 2, 1997 to December 31, 1998 and an evaluation sample from January 4, 1999 to December 30, 2016. Then, we forecast with the expanding (E) and rolling window (R) estimation strategies. Based on the Equation (9), we calculate the forecast error vectors applying the realized correlation matrices. Subsequently, the model ranks are determined with regard to the choice of different evaluation measures.

4.1 Symmetric Loss

In Figure 3, we show the evaluation results for the index pair S&P 500 – NASDAQ 100. The first subplot presents the selected benchmark proxies and both model forecasts over time. It is obvious that the 5-minute subsampled realized correlation proxies (RCOR-5.S) in black dashed line exhibit a dynamic structure with the sudden increase and decrease of correlations. This finding is consistent with the Epps effect in the literature, when the sampling frequency is increased (see Epps (1979)). Besides, the BEKK forecasts show a volatile structure while the process of DCC forecasts is relatively smooth and tends to overestimate the RCOR-5.S. The remaining subplots present the patterns of the forecast errors and the patterns of the evaluation measures. For calculating the Mean Error (ME), the Mean Absolute Error (MAE) and the Root Mean Squared Error (RMSE), we define an initial evaluation window with the first 50 forecast errors and expand it over time. Regarding the reported evaluation measurements in Table 1 during the full period, the BEKK model has a lower absolute ME than the DCC model corresponding to greater MAE and RMSE. A reasonable explanation is that the BEKK forecasts depend on the estimation of a larger number of unknown parameters, leading to a higher accuracy and more sensitivity in the evaluation period. However, this bias variance trade-off is partially observed during the crisis sub-period P4.

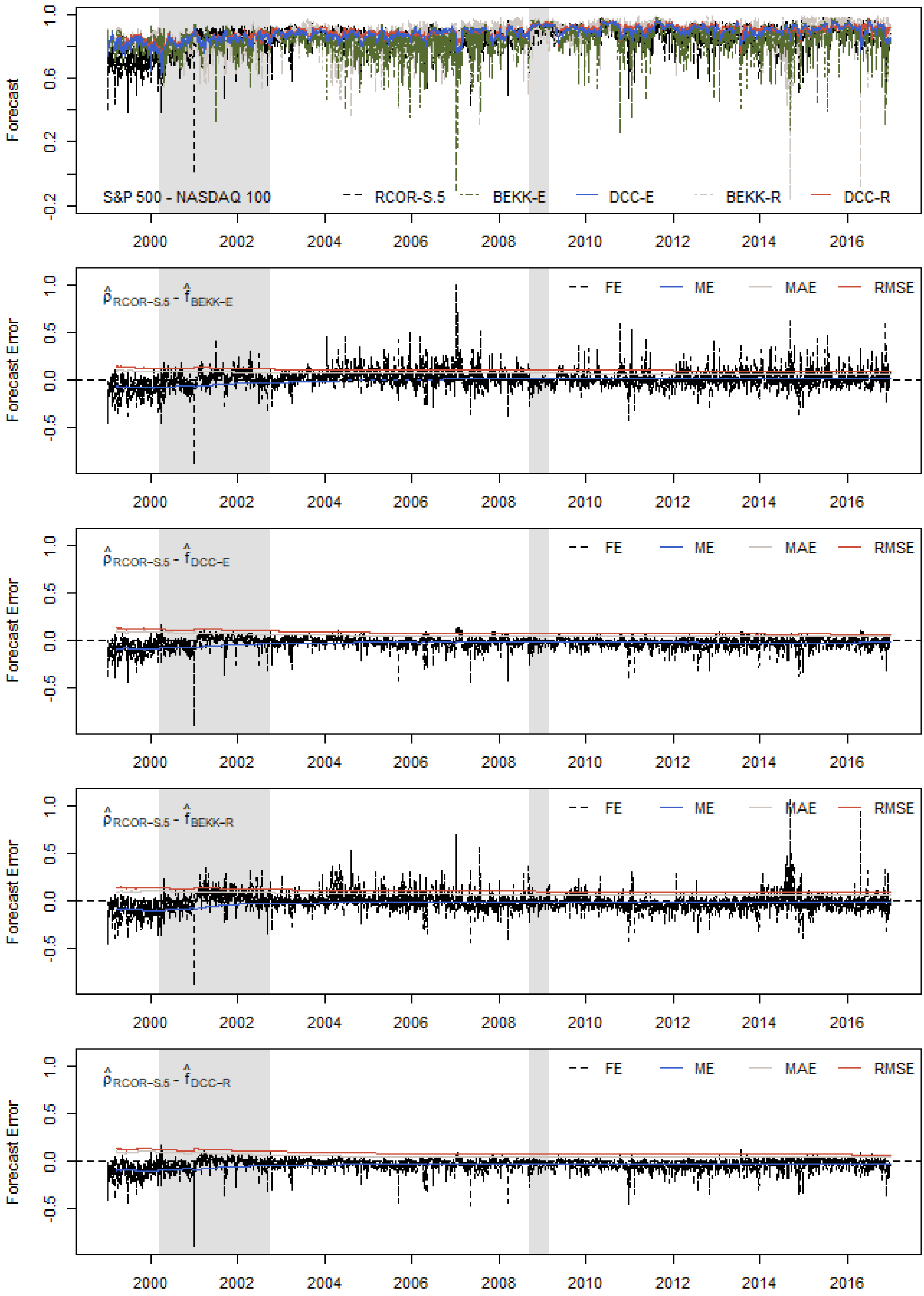
Table 1: Model Forecast Performance for S&P 500 – NASDAQ 100

	Full Evaluation Period			Sub-Period P4		
	ME	MAE	RMSE	ME	MAE	RMSE
BEKK–E	**0.012**	0.066	0.097	-0.030	0.036	0.056
DCC–E	-0.021	**0.045**	**0.069**	-0.022	**0.031**	**0.052**
BEKK–R	-0.015	0.064	0.094	**-0.016**	0.041	0.059
DCC–R	-0.030	0.047	0.072	-0.026	0.033	0.054

Note: For each column, the lowest measurement values are highlighted in bold.

Although the use of a moderate five-minute sampling interval is highly recommended for the user of realized volatility (see Liu et al. (2015)), there has been little discussion on the preferred sampling interval for realized correlations (see Andersen et al. (2006)). Hence, we use 30 equally spaced sampling intervals for calculating the 1min – 30min subsampled realized correlation matrices and repeat the analysis for the daily forecast horizon.

Figure 3: Forecast Error Evaluation for S&P 500 – NASDAQ 100



For the index pair S&P500 – NASDAQ 100, Figure 4 shows the empirical rankings of both correlation models estimated with the expanding and rolling window schemes. In most cases, the BEKK model outperforms with a high sampling interval of one-minute while the DCC model outperforms with moderate and low sampling intervals. This is not particularly surprising since the BEKK forecasts better describe the noisy realized correlations with a higher sampling interval. During the crisis sub-period P4, the proxy correlations are relatively high and the fluctuations of these proxy correlations are relatively low. As expected, the smooth DCC model shows a strong forecast performance regardless of the choice of sampling intervals.

For the next step, we present the evaluation results of the remaining two index pairs in Figure 5 and Figure 6. For the sub-period of dot-com bubble burst P2, the correlation forecasts for the index pair S&P 500 – NASDAQ 100 clearly outperform the correlation forecasts between Russell 2000 and other two indices. This is because the bankruptcy of the listed small-cap companies leads to the change of the index composition, thus making the forecast of co-movements with Russell 2000 more difficult. Another possible reason for this is that the choice of five-minute sampling interval for S&P 500 – Russell 2000 and NASDAQ 100 – Russell 2000 distorts the estimation of latent correlations by inducing too much microstructure noise. Therefore, subsampled realized correlations with lower sampling intervals may be more appropriate for this empirical evaluation.

4.2 Asymmetric Loss

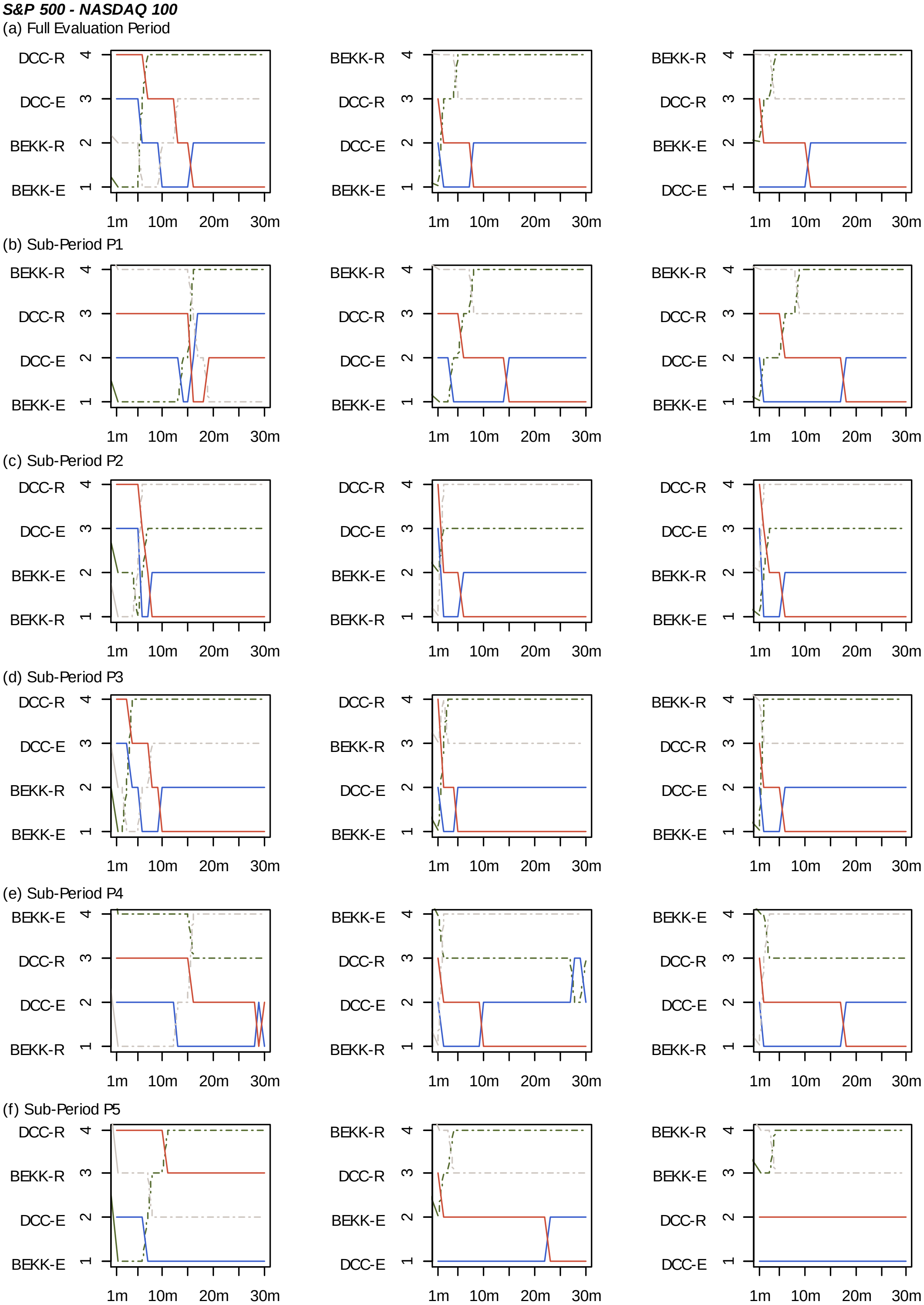
An alternative evaluation approach uses the aggregated loss function of Komunjer and Owyang (2012). Setting the curvature parameter of Equation (10) to $p = 2$, we consider three types of forecasters and present the aggregated average losses in Table 2 with four representative sampling intervals.

Table 2: Aggregated Model Forecast Performance based on Komunjer and Owyang (2012)

Assumption	$\boldsymbol{\tau} = (0,0,0)'$				$\boldsymbol{\tau} = (0.5,0.5,0.5)'$				$\boldsymbol{\tau} = (-0.5,-0.5,-0.5)'$			
Sampling Interval	1min	5min	10min	20min	1min	5min	10min	20min	1min	5min	10min	20min
$L_2(\boldsymbol{\tau}, \boldsymbol{e}_{t+1})$	Full Evaluation Period											
BEKK-E	**0.241**	0.080	0.055	0.049	0.058	0.040	0.036	0.042	**0.425**	0.144	**0.073**	**0.056**
DCC-E	0.255	**0.074**	**0.045**	**0.036**	**0.051**	**0.021**	**0.015**	0.016	0.458	**0.128**	0.074	**0.056**
BEKK-R	0.267	0.089	0.060	0.052	0.060	0.035	0.032	0.035	0.474	0.143	0.088	0.069
DCC-R	0.274	0.083	0.050	0.039	0.054	0.022	**0.015**	**0.015**	0.494	0.144	0.085	0.063
$L_2(\boldsymbol{\tau}, \boldsymbol{e}_{t+1})$	Sub-Period P4											
BEKK-E	0.086	0.023	0.016	0.014	0.017	0.006	0.005	0.006	0.156	0.041	0.027	0.023
DCC-E	**0.080**	**0.020**	**0.014**	**0.013**	**0.016**	**0.005**	**0.004**	**0.005**	**0.145**	**0.035**	**0.024**	**0.020**
BEKK-R	0.091	0,029	0.023	0.023	0.021	0.013	0.014	0.018	0.162	0.048	0.031	0.028
DCC-R	0.088	0.023	0.016	0.014	0.017	**0.005**	**0.004**	**0.005**	0.160	0.041	0.027	0.023

Note: For each column, the lowest measurement values are highlighted in bold.

Figure 4: Model Rankings implied by *ME* (left), *MAE* (middle) and *RMSE* (right)



Note: Ranks from 1 (best) to 4 (worst) for the index pair S&P 500 – NASDAQ 100

Figure 5: Forecast Error Evaluation for S&P 500 – Russell 2000

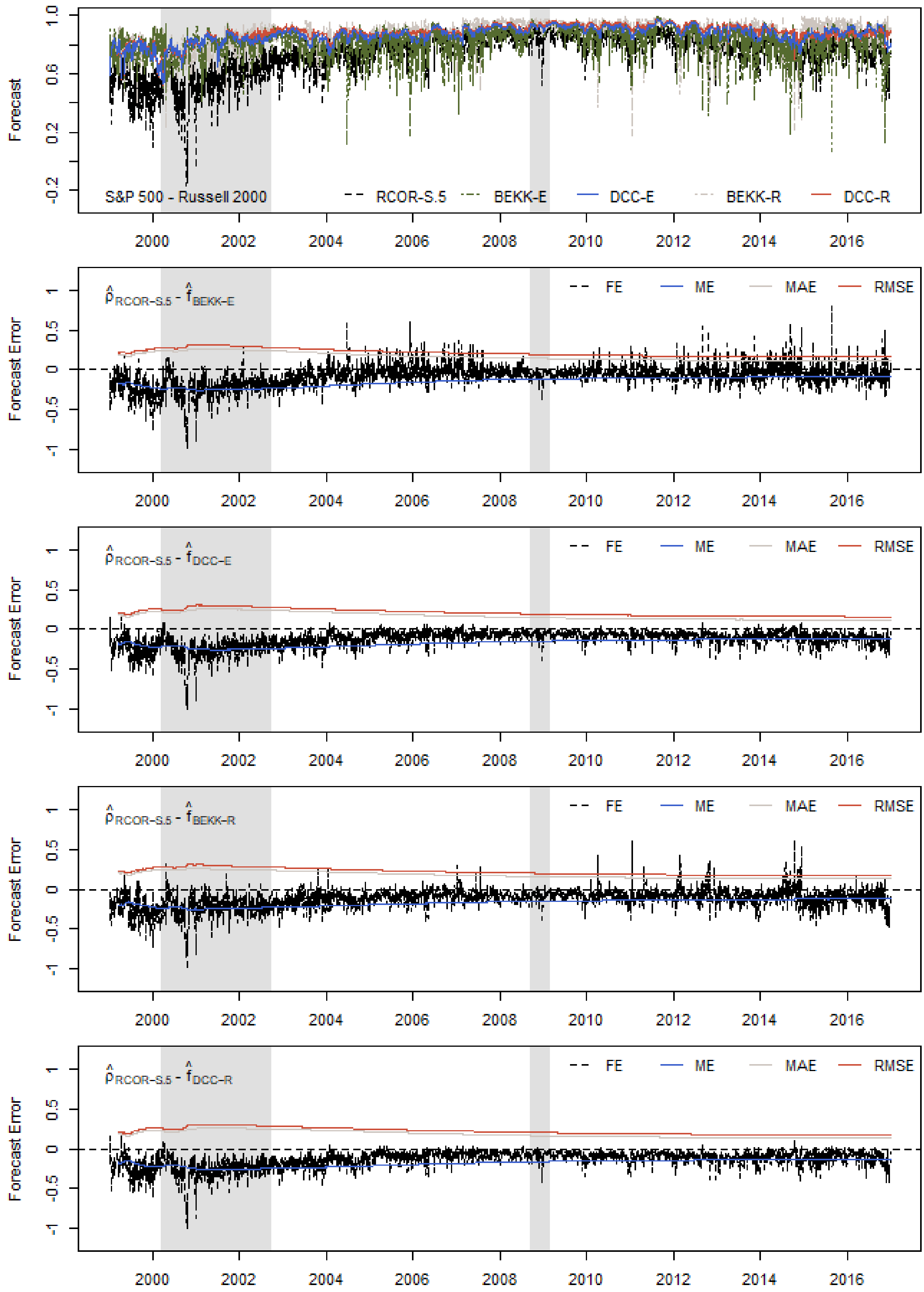
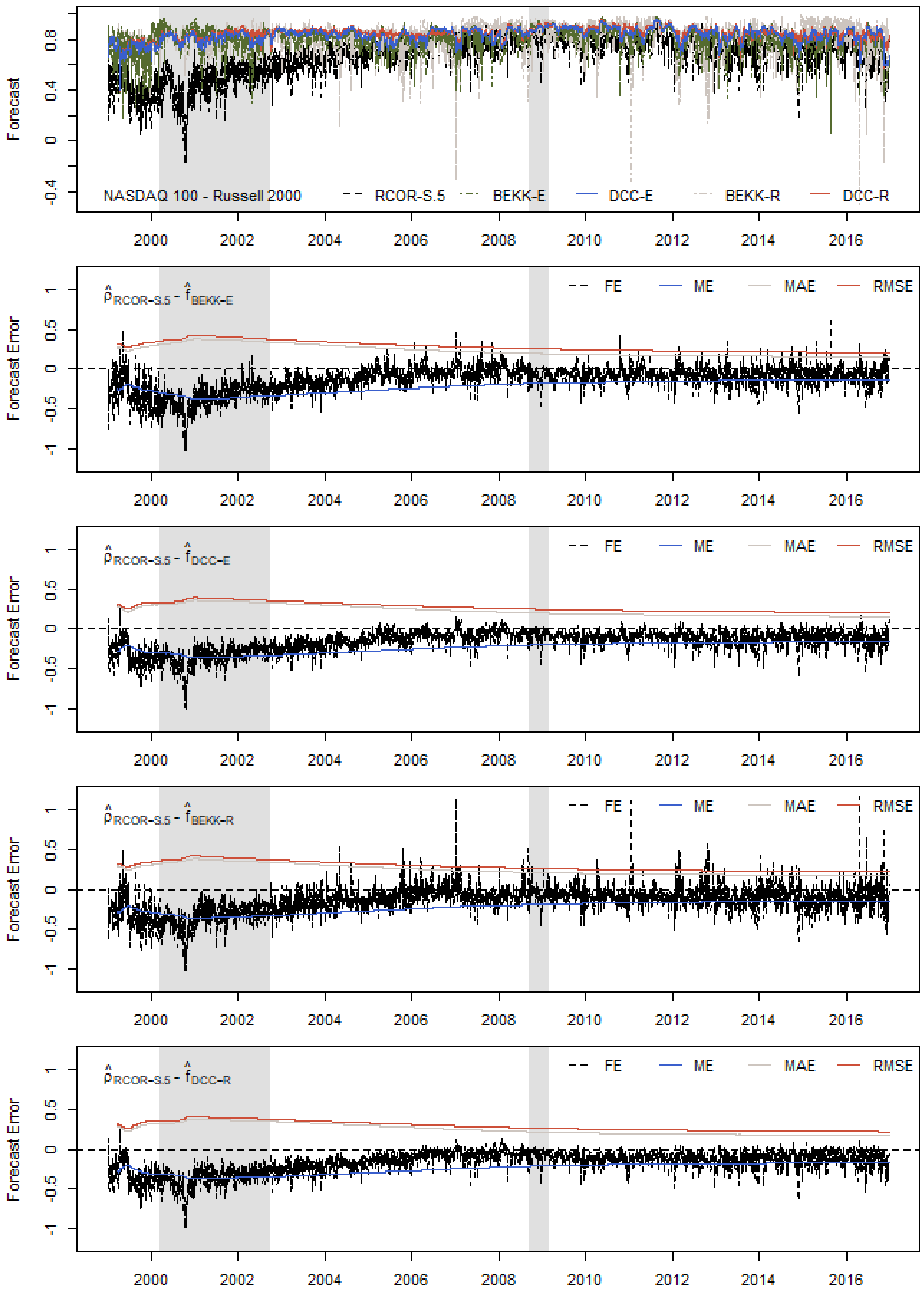


Figure 6: Forecast Error Evaluation for NASDAQ 100 – Russell 2000



In Table 2, the first forecaster is assumed to be risk neutral. On average, the BEKK model is preferred for the one-minute sampling interval while the DCC model is preferred for the other three sampling intervals. For the sub-period P4, the DCC model has lower average aggregated losses than the BEKK model which is consistent with the earlier finding in Figure 4.

The underlying loss functions of the second and third forecaster are assumed to be asymmetric. Regarding to all index pairs, the second forecaster suffers more from the positive forecast errors (underestimation) than from the negative forecast errors of the same magnitude (overestimation) while the third forecaster penalizes overestimation more than underestimation. Given the lower aggregated average losses, the results of Table 2 show that both forecasting models are more suitable to the second forecaster.

4.3 Optimized Asymmetric Loss

This subsection discusses the asymmetric preferences of forecasters. Figure 7 describes the relationship between the optimal GMM estimates in Equation (12) and the sampling intervals without instrumental variables (IV) except constants. Considering the following sets of IV A and B

$$\begin{gathered} \boldsymbol{w}_{At} = \left(1, r_{S\&P500,t-1}, r_{NASDAQ100,t-1}, r_{Russell2000,t-1}\right)', \\ \boldsymbol{w}_{Bt} = \left(1, r^2_{S\&P500,t}, r^2_{NASDAQ100,t}, r^2_{Russell2000,t}\right)' \end{gathered} \quad (14)$$

we are able to estimate the asymmetry parameters and test the forecast efficiency simultaneously. However, the GMM results in Figure 8 are similar to the estimates in Figure 7 and could be therefore interpreted by the same way.

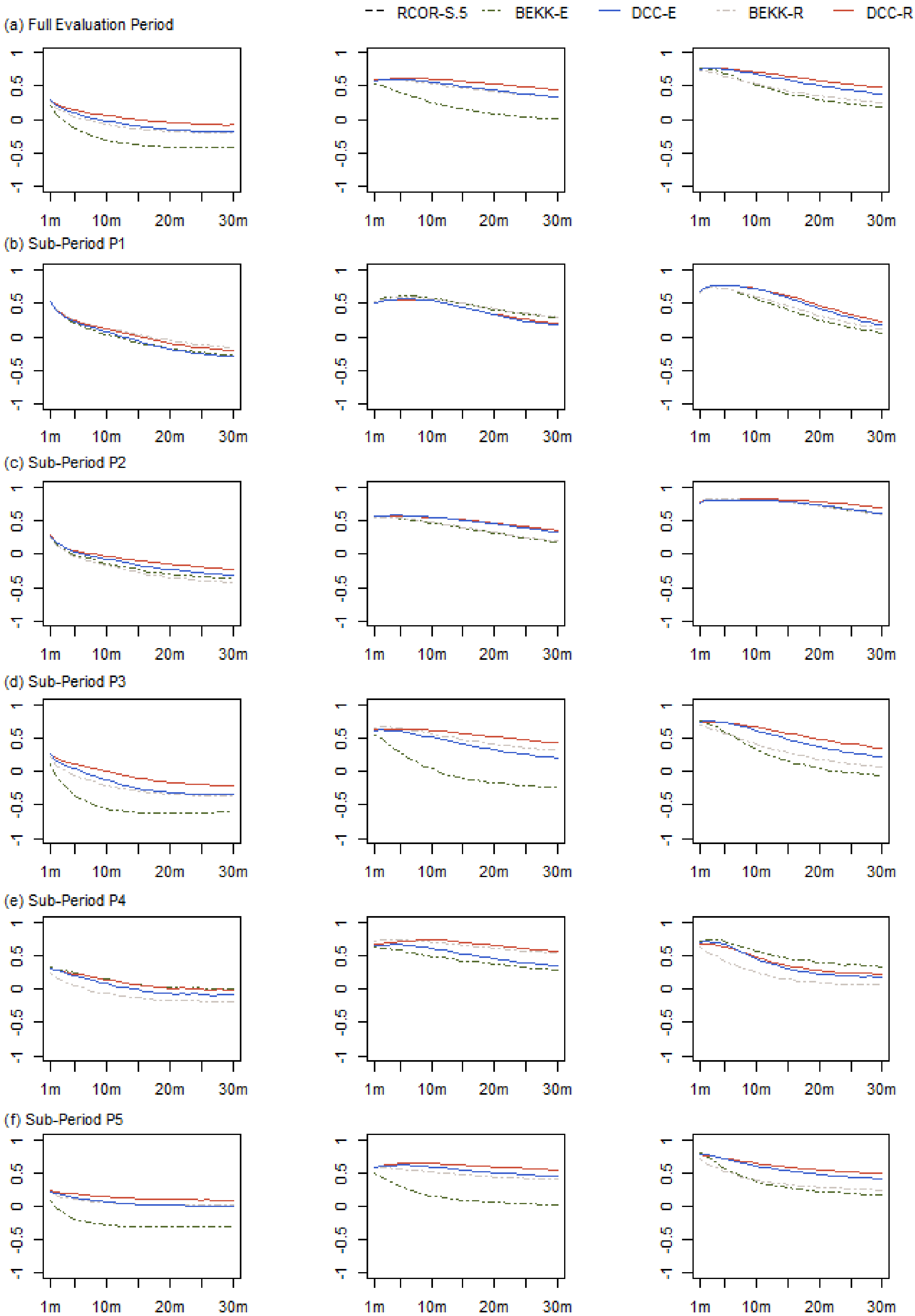
Turning to Figure 7, we predominantly observe positive $\hat{\tau}_2^*$ and $\hat{\tau}_3^*$. Consequently, loss appears to be larger with the underestimation of correlation proxies than with the overestimation by the same size for the index pairs S&P 500 – Russell 2000 and NASDAQ 100 – Russell 2000. In particular, the DCC users show higher degrees of asymmetry than the BEKK users, implying a larger tendency to overestimate. Further, the rolling window estimation scheme leads to higher degrees of asymmetry than the expanding window estimation scheme. Regarding $\hat{\tau}_1^*$, the DCC–E users seem to have negative degrees of asymmetry in most cases while the optimal preferences of the remaining users depend on the sampling schemes for calculating the subsampled realized correlations.

5 Conclusion

This paper evaluates the forecast performance of two widely used models for forecasting time-varying correlation matrices. Analyzing 20 years of intraday data on the three major U.S. equity indices, we summarize our main findings as follows.

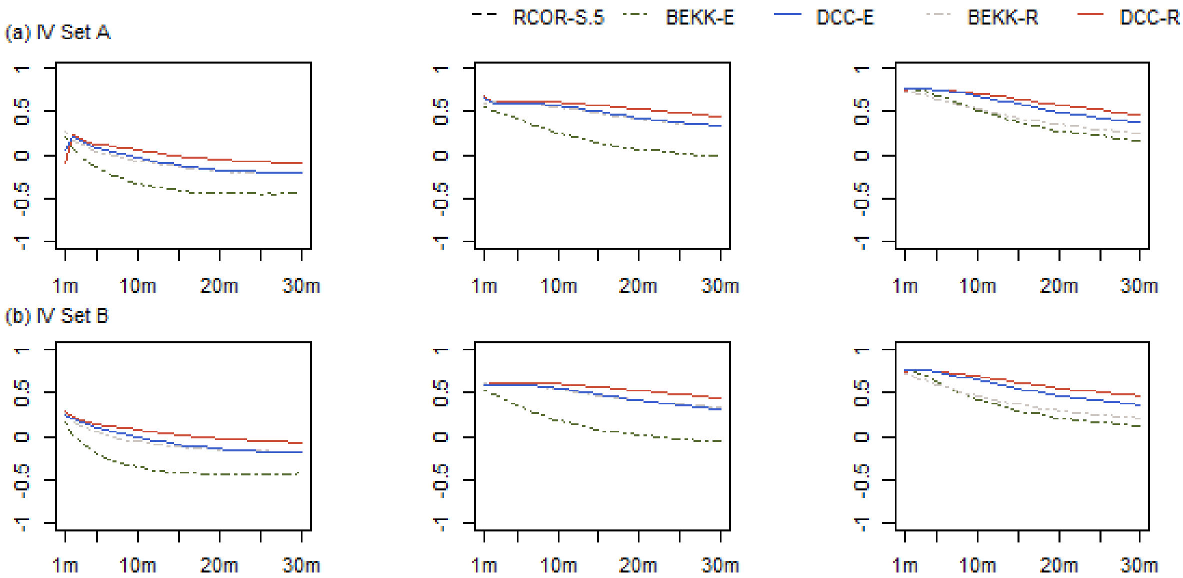
Firstly, we observe the empirical trade-off between the flexibility and feasibility. Secondly, the evaluation sample performance of BEKK model improves when the sampling interval for realized correlations increases. Thirdly, both BEKK and DCC models perform better during the crisis sub-period P4 than during the whole evaluation period. In particular, the DCC model demonstrates a strong forecast performance in sub-period P4. Moreover, both models better predict correlations for the index pair S&P 500 – NASDAQ 100 than for Russell 2000 and other two indices. Finally, the specific model choice reflects the asymmetric user preference. For the index pairs S&P 500 – Russell 2000 and NASDAQ 100 – Russell 2000, the DCC users demonstrate higher positive degrees of asymmetry than the BEKK users.

Figure 7: Optimal Preferences without Instruments



Note: Reported are the GMM estimates $\hat{\boldsymbol{\tau}}_1^*$ (left), $\hat{\boldsymbol{\tau}}_2^*$ (middle) and $\hat{\boldsymbol{\tau}}_3^*$ (right)

Figure 8: Optimal Preferences using IV Sets A and B



Note: Reported are the GMM estimates $\hat{\tau}_1^*$ (left), $\hat{\tau}_2^*$ (middle) and $\hat{\tau}_3^*$ (right) over the full evaluation period

The research in the field of forecasting and evaluating correlation matrices is still ongoing. Discussions about consistent multivariate loss functions (see Laurent et al. (2013)), appropriate sampling intervals and further instrumental variable sets are interesting avenues for future analysis.

References

Anderson, T.G., Bollerslev, T., Christoffersen, P.F., Diebold, F.X. (2006). Volatility and Correlation Forecasting, in G. Elliott, C. W. Granger and A. Timmermann (Ed.), Handbook of Economic Forecasting, Vol. 1, North-Holland, Chapter 15, pp. 777-878.

Anderson, T.G., Bollerslev, T., Diebold, F.X., Labys, P. (2000). Great Realizations, Risk 13(March): 105-108

Bollerslev, T., Engle R.F., Wooldridge J.M. (1988), A Capital Asset Pricing Model With Time-Varying Covariance, Journal of Political Economy, 96(1): 116-131.

Capiello, L., Engle, R.F., Sheppard, K. (2006). Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns, Journal of Financial Econometrics 4(4): 537-572.

De Almeida, D., Hotta, L., Ruiz, E. (2018). MGARCH Models: Trade-off between Feasibility and Flexibility, International Journal of Forecasting 34(1): 45-63.

Engle, R.F., Kroner, K.F. (1995). Multivariate Simultaneous Generalized ARCH, Econometric Theory 11(1): 122-150.

Engle, R.F. (2002). Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroscedasticity Models, Journal of Business & Economic Statistics 20(3): 339-350.

Engle, R.F. (2009). Anticipating Correlations: A New Paradigm for Risk Management, Princeton University Press.

Epps, T.W. (1979). Comovements in Stock Prices in the Very Short Run, Journal of the American Statistical Association 74(366), 291-298.

Hafner, C.M., Franses, P.H. (2009). A Generalized Dynamic Conditional Correlation Model: Simulation and Application to Many Assets, Econometric Reviews 28(6): 612-631.

Hansen, L.P., Heaton, J., Yaron, A. (1996). Finite Sample Properties of Some Alternative GMM Estimators, Journal of Business and Economic Statistics 14(3): 262–280.

J.P. Morgan, Reuters (1996). RiskMetrics: Technical Document, Fourth Edition, New York.

Komunjer, I., Owyang, M.T. (2012). Multivariate Forecast Evaluation and Rationality Testing, Review of Economics and Statistics 94(4): 1066-1080.

Laurent, S., Rombouts, J., Violante, F. (2013). On Loss Functions and Ranking Forecasting Performances of Multivariate Volatility Models, Journal of Econometrics 173(1): 1-10.

Ledoit, O., Santa-Clara, P., Wolf, M. (2003). Flexible Multivariate GARCH Modeling with an Application to International Stock Markets, Review of Economics and Statistics 85(3): 735-747.

Liu, L.Y., Patton, A.J., Sheppard, K. (2015). Does Anything Beat 5-Minute RV? A Comparison of Realized Measures across Multiple Asset Classes, Journal of Econometrics 187(1): 293-311.

Shiller, R.J. (2015). The Stock Market in Historical Perspective, J.R. Schiller (Ed.), Irrational Exuberance, Princeton University Press, pp. 1-12.

Tse, Y.K., Tsu, A.K.C. (2002). A Multivariate Generalized Autoregressive Conditional Heteroscedasticity Model with Time-Varying Correlations, Journal of Business and Economic Statistics 20(3): 351-362.

Zhang, L., Mykland, P.A., Aït-Sahalia, Y. (2005). A Tale of Two Time Scales: Determining Integrated Volatility with Noisy High-Frequency Data, Journal of the American Statistical Association 100(472): 1394-1411.