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NEWTON´S LAWS OF MOTION AND PRICE THEORY

Abstract:

The paper focuses on factual research in bibliographic and biographical databases showing that representatives of the Czech School of Economics took a leading role in the methodological use of applied and theoretical physics in the basic economic research, especially in the second half of the twentieth century.

The linear and non-linear analytical structures of theoretical physics are compared with the analytical structures of commodity price theory in a market with nearly perfect competition. Newton´s equations of motion for the non-relativistic speed of instantaneous relative depreciation and instantaneous relative commodity prices over time are analyzed. Assuming that the market value of a commodity is fully determined exclusively by the value of the instantaneous commodity price, the price jerk equation acquires a form corresponding to the non-relativistic equation for jerk in mechanics, following from Newton´s second law of motion. In this paper price jounce and price crackle are defined.

Keywords:

Instantaneous relative depreciation, Newton´s laws of motion, market value, nearly perfect competition, price jerk, price jounce, price crackle

JEL Classification: A12, C65

Introduction

The results of applying statistical mechanics in microeconomics suggest that the time is ripe for using the experimental, theoretical and mathematical methodologies of physics to model economic systems beyond the framework of traditional economic modeling. The question remains whether the import of concepts from physics to economics is merely an applied metaphor, or actually a modification of the analytical structure of economics. If we are to find at least a partial answer to this question, we must compare the linear and non-linear analytical structure of physics with the analytical structure of economics. To enable such a comparison, work has begun on a principle of correspondence between economic variables and the physical variables used in one of the most highly developed disciplines of classical physics, classical non-relativistic mechanics (Zeithamer, 2012 a, p.547). The final form of this principle of correspondence will to a certain degree be reflected in the methodological foundations used to teach economics itself, for example, in the training of appraisers at the university level.

The application of methods of classical non-relativistic mechanics in microeconomics presented in this work aims to derive a single motion equation for price which describes non-chaotic as well as chaotic fluctuations of price on a market with nearly perfect competition.

During the past four decades, great efforts have been made to understand chaotic dynamics in greater detail. Both the geometric theory of dynamics and its numeric counterpart, have proven to be powerful tools on the road to this success. For three-dimensional non-linear dynamic systems, the minimal functional forms required to generate a chaotic flow have been found and tested (Sprott, 1994, p.647).

Minimal chaotic dynamics have also been investigated from the viewpoint of jerky dynamics (Sprott, 1997, p.271; Eichhorn, Linz and Hänggi, 1998, p.7151; Linz, 1998, p.1109; Munmuangsaen, B., Srisuchinwong, B. and Sprott, 2011, p.1445). Jerky dynamics should also be able to investigate nonchaotic as well as chaotic development over time (Eichhorn, Linz and Hänggi, 1998, p.7152). Elementary jerky dynamics can also be found in economics, as shown in this paper and in the paper of professor Jiří Pospíšil (Pospíšil, 2013, p.2071).

Let us briefly consider at a market with nearly perfect competition: a) in each market there are a large number of buyers and sellers, none of which are strong enough to influence the price or output of a sector; b) all goods are homogeneous; c) there is free entry to and exit from market; d) all manufacturers and consumers have perfect information about prices and quantities traded on the market; e) companies attempt to maximize profit and consumers attempt to maximize utility; f) companies and consumers have free access to information about technologies. This set of assumptions is further specified by the specific quantitative expression of the degree of understanding of information about technologies: companies and consumers understand only a part $\delta(t)$ of the available amount of information about technologies at time *t*, where $0 < \delta(t) < 1$ for $t \in \langle t_0, +\infty)$, t_0 is the initial time of monitoring the commodity state.

The methodology of qualitative and quantitative physical research of any system strives to achieve one basic goal, namely that the signal to noise ratio be much greater than one. If it is possible to deliberately increase the output signal from an inanimate system above the background noise, this brings to the forefront the natural relations which are common to different systems investigated (Roehner, 2007, p.97; Štroner and Pospíšil, 2011, p.731). Of course there are other systems which do not permit the researcher to amplify the level of output. In such case, there is another way to increase the signal to noise ratio. Here, it is necessary to continually decrease the background noise to the lowest possible level. A classic current example requiring such noise reduction is the detection of gravitational waves, the existence of which was predicted by prof. A. Einstein in his work from 1916 "Näherungweise Integration der Feldgleichungen der Gravitation" (Einstein, 1916, p.688). Outside the solar system, the theory predicts a number of "stellar" sources of gravitational waves, which could be detected in the event they reached Earth. For the Sun, a typical class G main-spectrum star, it has not yet been possible to theoretically determine such mechanisms which would be responsible for detectable levels of gravitational radiation (Křivský and Zeithamer, 1982, p.309; Karmakar and Borah, 2013, p.516). Efforts similar to the detection of gravitational waves can be seen in numerous other multi-disciplinary fields, explored in publications such as: Physics of the Earth´s Magnetosphere, Heliometeorology and Helioclimatology, Biophysics of the Sun – Earth Relations.

A situation similar to the physical research of inanimate systems arises in the physical research of economic systems. In economic systems, one of the main reasons that the signal to noise ratio is close to one is the high degree of self-organization and self-improvement.

1 The incorporation of physics into economics in the framework of the Czech School of Economics

At the Czech School of Economics during the 19th century, no reliable sources have yet been found indicating such an interdisciplinary approach or related original work. In the second half of the twentieth century however, we do find economists at the Czech School of Economics whose works represent applications of physics in economics, i.e. in econophysics in broader sense, i.e. in physical economics. Einstein's special theory of relativity was applied by professor Pavel Hrubý (*5. 5. 1914 - †25. 6. 1994) in order to use economic spacetime for more precise economic analysis and prognosis (Hrubý and Kálal, 1974, p.21). Another Czech economist, who represents the Czech School of Economics in econophysics in broader sense, is professor František Drozen (*30. 5. 1949), whose results were inspired by the work of German railway engineer August Wöhler (*22. 6. 1819 – †21. 3. 1914). František Drozen constructed an analogy between the process of fatigue crack growth in axles of railway wagons and the process of price reduction for goods. This approach to modeling the process of falling prices for goods can be found in its final form in Drozen´s work (Drozen, 2008, p.1033).

2 Linear motion equation of commodity state without inflexion

In this paper it is assumed that the market value of a commodity is quantifiably determined only by the market price *n* of the commodity on the market with nearly perfect competition. We now make the generalizing assumption that the instantaneous acceleration of reduction of the market value is directly proportional to the instantaneous rate of reduction of the market value (Zeithamer, 2013, p.1597). Then the deterministic differential equation of price which expresses this model is

$$\frac{d^2n}{dt^2}(t) = -A\frac{dn}{dt}(t) \ , \tag{1}$$

where $A > 0$ is the proportionality constant, and a negative sign is used to indicate that n, the market value of goods, i.e. a price, is decreasing and the acceleration of reduction of the market value decreases over time. The initial conditions now are that over time $t = t_0$ the market value is $n(t_0) = n_0$ and $\frac{dn}{dt}(t_0) = r_0 < 0$, where t_0 is the initial time of monitoring the commodity price, $[A] = s^{-1}$; *s* – designates the basic time unit, seconds.

3 Nonlinear motion equation of commodity state with inflexion and price jerk

In this section of our work, we again presume the following conditions to be met: (1) the commodity is on one of the markets of a model of market structure with nearly perfect competition at initial time t_0; (2) at time t_0 the commodity is found in its initial state, which is uniquely determined by the magnitude of instantaneous commodity depreciation $w(t_0) = w_0$.

Let the acceleration $\frac{d^2n}{dt^2}$ of the instantaneous commodity price be the sum of two components, i.e.

$$\frac{d^2n}{dt^2} = \left(\frac{d^2n}{dt^2}\right)_1 + \left(\frac{d^2n}{dt^2}\right)_2 . \tag{2}$$

The first component of acceleration is a consequence of physical and chemical processes, which cause the first component of the instantaneous acceleration to increase in direct proportion to the magnitudes of rate of change of the instantaneous commodity price *n*, i.e.

$$\left(\frac{d^2n}{dt^2}(t)\right)_1 = B\frac{dn}{dt}(t), \tag{3}$$

where B is the proportionality constant, $B > 0$, $[B] = s^{-1}$, s – designates the basic time unit, seconds and $t \in \langle t_0, +\infty)$. The second component of acceleration results from socio-psychological processes, which cause the second component of the instantaneous price acceleration to be directly proportional to the product of the magnitude of rate of change of the instantaneous price $\frac{dn}{dt}(t)$ and the magnitude of instantaneous price $n(t)$, while the proportionality constant is negative, thus

$$\left(\frac{d^2n}{dt^2}(t)\right)_2 = -A\frac{dn}{dt}(t)\cdot n(t), \tag{4}$$

where $(-A)$ is the proportionality constant, $A > 0$, $[A] = (c.u.)^{-1}s^{-1}$, $c.u.$ – designates the basic currency unit, s – designates the basic time unit, seconds, $t \in \langle t_0, +\infty)$.

By substituting relations (3) and (4) into equation (2), we obtain the following motion equation for the acceleration of instantaneous commodity price n

$$\frac{d^2n}{dt^2}(t) = B\frac{dn}{dt}(t) - A\frac{dn}{dt}(t)\cdot n(t). \tag{5}$$

A similar equation holds for commodity relative depreciation RD (Zeithamer, 2012 b, p.445; 2013, p.1598)

$$\frac{d^2RD}{dt^2} = E\frac{dRD}{dt}(t) - F\frac{dRD}{dt}(t)\cdot RD(t), \tag{6}$$

where $F > 0$, $E > 0$ are the proportionality constants, $[F] = [E] = s^{-1}$, $t \in \langle t_0, +\infty)$.

For the motion of a solid body through space in which the magnitude of the force F of resistance in that space against the movement of the body is directly proportional to the velocity v of the body, i.e. $F = -kv$ ($k > 0$ is the constant of proportionality), the magnitude of jerk j is expressed by the following equation (Pospíšil, 2013, p.2077),

$$j = \frac{d^3s}{dt^3}(t) = -\frac{k}{m}\frac{d^2s}{dt^2}(t), \tag{7}$$

where s is the path traveled by the body, m is the mass of the body, t is time, and j is the magnitude of jerk in units m/s^3. From the equation of motion for instantaneous price (1) we get the following equation for the magnitude of price jerk j_P in units $c.u./s^3$

$$j_P = \frac{d^3n}{dt^3}(t) = -A\frac{d^2n}{dt^2}(t), \tag{8}$$

where $n(t)$ is the instantaneous price of the commodity and t is the physical time. Equations (7) and (8) are the first basic step in constructing a principle of correspondence between economic variables and physical variables of classical nonrelativistic mechanics: the path s traveled by a solid body through space with a force of resistance against this movement is directly proportional to the velocity, which corresponds ($\leftrightarrow$) to the instantaneous price n of a commodity in a market structure with nearly perfect competition i.e. $s \leftrightarrow n$. Equations (7) and (8) are also a second basic step in deriving a complete principle of correspondence between economic variables and physical variables: for the motion of a solid body through space, where the force of resistance against this movement is directly proportional to the velocity v, jerk j corresponds ($\leftrightarrow$) to price jerk j_P for a commodity in a market structure with nearly perfect competition, i.e. $j \leftrightarrow j_P$.

The price jerk function $j_P(t)$ for a non-linear motion equation of commodity state with inflexion (5) may be derived in the following manner. By taking the derivative of equation (5) with respect to time t and substituting into the right side of the resulting equation for $\frac{d^2n}{dt^2}(t)$ from equation (5), we get the price jerk equation in the form

$$\frac{d^3n}{dt^3}(t) = (A\,n(t) - B)^2 \frac{dn}{dt}(t) - A\left(\frac{dn}{dt}(t)\right)^2 . \tag{9}$$

The price jerk function $j_P(t)$ on the right side of equation (9) may be expressed by a derivative of function $G(t)$ with respect to time t in the form

$$j_P(t) = (A\,n(t) - B)^2 \frac{dn}{dt}(t) - A\left(\frac{dn}{dt}(t)\right)^2 = \frac{dG}{dt}(t), \tag{10}$$

where

$$G(t) = \frac{1}{3A}(A\,n(t) - B)^3 + A\int_0^t \left(\frac{dn}{dt}(u)\right)^2 du + const., \tag{11}$$

while constants of proportionality A and B from equation (5) are expressed in the following units $[A] = (c.u.)^{-1}s^{-1}$, $[B] = s^{-1}$; $c.u.$ – designates the basic currency unit, s – designates the basic time unit, seconds. Then the price jerk equation (9) acquires the form

$$\frac{d^3n}{dt^3}(t) = \frac{dG}{dt}(t). \tag{12}$$

Equation (12) corresponds to the non-relativistic equation for mechanical jerk, following from Newton’s second law of motion.

Let us define price jounce as the change in price jerk over time in units $c.u./s^4$, i.e.

$$\frac{d^4 n}{dt^4}(t) = \frac{d\ j_P}{dt}(t) = \frac{d^2 G}{dt^2}(t) \tag{13}$$

where

$$\frac{d^2 G}{dt^2} = (An(t) - B)\frac{dn}{dt}(t)\left[4A\frac{dn}{dt}(t) - (An(t) - B)^2\right]. \tag{14}$$

Equation (13) corresponds to the non-relativistic equation for mechanical jounce, following from Newton's second law of motion.

Let us define price crackle as the change in price jounce over time in units $c.u./s^5$, i.e.

$$\frac{d^5 n}{dt^5}(t) = \frac{d^2 j_P}{dt^2}(t) = \frac{d^3 G}{dt^3}(t), \tag{15}$$

where

$$\frac{d^3 G}{dt^3}(t) = \left[2A\frac{dn}{dt}(t) - (An(t) - B)^2\right]^2 \frac{dn}{dt}(t) - 7A\left[\frac{dn}{dt}(t)\right]^2 (An(t) - B)^2 . \tag{16}$$

Equation (15) corresponds to the non-relativistic equation for mechanical crackle, following from Newton's second law of motion.

The instantaneous commodity relative depreciation *RD* is defined by a non-linear deterministic differential equation (6) of the form

$$\frac{d^2 RD}{dt^2}(t) = E\frac{dRD}{dt}(t) - F\frac{dRD}{dt}(t)RD(t), \tag{17}$$

where $E > 0, F > 0$ are constants of proportionality and t is time, $t \in \langle t_0, t_e \rangle$, where t_0 – is the initial time of instantaneous commodity price monitoring and t_e – is the time at which we cease monitoring the level of instantaneous commodity price (Zeithamer, 2012b, p.447). If we express equation (17) in accordance with analogous equation (9) above, we get an equation of motion expressed through the quantity of "jerk" in the form

$$\frac{d^3 RD}{dt^3}(t) = (F\ .\ RD(t) - E)^2 \frac{dRD}{dt}(t) - F\left(\frac{dRD}{dt}(t)\right)^2, \tag{18}$$

where $\frac{dRD}{dt}(t)$ corresponds to the velocity v of instaneous relative depreciation.

The jerk equation for instantaneous relative depreciation may be expressed in the form

$$\frac{d^3RD}{dt^3}(t) = \frac{dG}{dt}(t), \tag{19}$$

where

$$G(t) = \frac{1}{3F}(F\ .\ RD(t) - E)^3 + F\int_0^t \left(\frac{dRD}{dt}(u)\ du\right)^2 + const.,, \tag{20}$$

while E and F are constants of proportionality.

A more detailed approach to modeling the process of falling prices with acceleration can be found in the following works (Zeithamer, 2012a, p.547; 2012b, p.445; 2013, p.1597; 2014, p.1016).

Conclusion

The main objective of this paper is to verify methods of deriving equations of motion for commodity pricing theory in a market with nearly perfect competition (Zeithamer, 2014, p.1016) in comparison to the equations of motion of non-relativistic mechanics based on Newton´s laws of motion. A secondary objective of this paper is to outline the possible future development and use of equations derived from classical Newtonian mechanics in theoretical economics. From the analysis of equations of motion for the motion of a rigid body with constant mass and variable mass, it is shown that the method proposed for deriving equations of motion for the instantaneous price of a commodity as well as the instantaneous relative depreciation of the commodity and instantaneous relative price of the commodity is in accordance with the conclusions drawn by non-relativistic mechanics and is a new direction for basic economic and physical research founded upon the causal mechanisms of change in the market value of commodities.

Assuming that the market value of the commodity at time t is fully determined exclusively by the value of the instantaneous commodity price $n(t)$, methodological procedures taken from theoretical physics are used to construct motion equations for a commodity's instantaneous price $n(t)$ and instantaneous relative depreciation $RD(t)$. Motion equation (5) for instantaneous commodity price with inflexion as well as motion equation (6) for instantaneous relative depreciation with inflexion are non-linear differential equations of the second order with constant coefficients. These motion equations were derived for a sequence of markets with nearly perfect competition. The principle of correspondence takes the following form:

(A) $s \leftrightarrow n$, (B) $j \leftrightarrow j_P$, (C) $\frac{d^3s}{dt^3}(t) = -\frac{k}{m}\frac{d(ds/dt)}{dt}(t) \leftrightarrow \frac{d^3n}{dt^3}(t) = \frac{dG}{dt}(t)$

(D) $\frac{d^4s}{dt^4}(t) = -\frac{k}{m}\frac{d^2(ds/dt)}{dt^2}(t) \leftrightarrow \frac{d^4n}{dt^4}(t) = \frac{d^2G}{dt^2}(t)$, i.e. jounce $\leftrightarrow$ price jounce,

(E) $\frac{d^5 s}{dt^5}(t) = -\frac{k}{m}\frac{d^3(ds/dt)}{dt^3}(t) \leftrightarrow \frac{d^5 n}{dt^5}(t) = \frac{d^3 G}{dt^3}(t)$, i.e. crackle $\leftrightarrow$ price crackle.

These five correspondences concluding the work present the basis for constructing a principle of correspondence between economic variables and kinematic variables of classical nonrelativistic mechanics.

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