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PORTFOLIO OPTIMIZATION OF GLOBAL REITS RETURNS: HIGH-DIMENSIONAL COPULA-BASED APPROACH

Abstract:

The objectives of this paper are to investigate the optimum portfolio of REIT index return of Asia – Pacific, Europe, USA, and emerging markets with multivariate t copula based on GARCH model, and to measure portfolio risk with value at risk (VaR) and component VaR (CVaR). The 1,454 REIT price index return observations were collected from 1 Dec 2009 to 29 June 2015 and calculated based on a continuous compound basis. The empirical results showed that the estimated equations of USA, Europe and emerging REIT index returns were ARMA(2,2)-GARCH(1,1), while ASIA-Pacific was ARMA(3,3)-GARCH(1,1). The coefficients of t distribution of these equations were also statistically significant at 1%, meaning the assumption of t distribution for ARMA-GARCH estimation was reasonable. Then, the multivariate t copula was used to construct an optimized portfolio for high dimensional risk management. The Monte Carlo simulation was applied in order to construct the optimized portfolio by using the mean-CVaR model at the given significance level of 5% and to obtain the efficient frontier of the portfolio under different expected returns. Finally, the optimal weights of the portfolio were obtained with the various expected returns in frontier.

Keywords:

REITs, Portfolio Optimization, Multivariate t Copula, CVaR

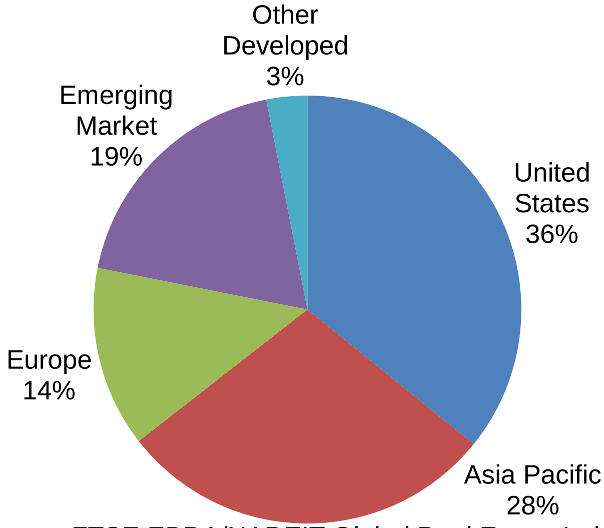
JEL Classification: G11, C32, C58

1. Introduction

REITs are investable securities of real estate companies that own (and often operate) baskets of income-producing properties. The basket may include several groups of real estate such as office buildings, shopping malls, hotels, and warehouses. Since REITs are traded on stock exchanges just like equities, the REIT structure allows small investors to invest in commercial properties managed by professional real estate operators. Therefore, REITs are liquid assets related to real asset investment. The values of REIT around the world have continued to grow and emerge in many countries. Until now, many countries had legislation that supported the establishment of a REIT in more than 40 countries (that number is steadily raising as more nations pass REIT laws).

Although the REIT was affected by the subprime crisis and the global economic crisis in 2007 – 2008, and REIT prices fell sharply in line with global equities, after 2009 the global REIT market recovered back to good returns. In 2014, the total global REITs had market capitalization of approximately 1.4 trillion US dollars, or about 2.5% of the total world equity market capitalization. The largest of the REIT market is the US, followed by Japan, Australia, England, Singapore, and Hong Kong. Figure 1 shows the global real estate securities market classified by region in terms of market capitalization in US$ billions. The market share of U.S. REITs accounts for 36% of the global market, followed by the Asia-Pacific and Europe with 29% and 14%, respectively.

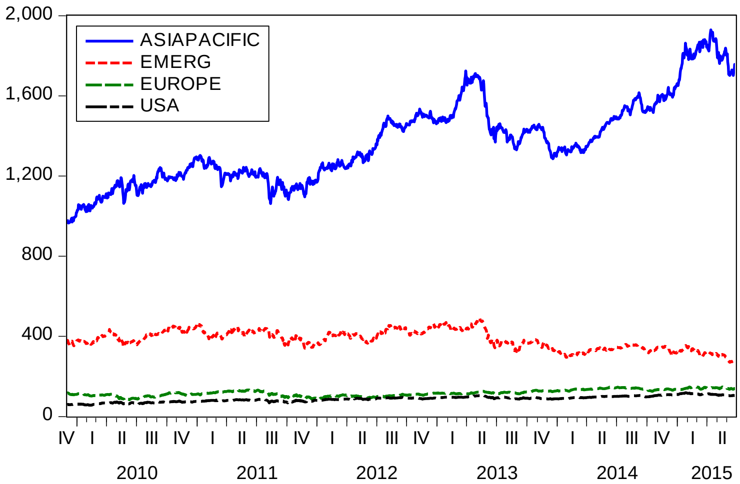
Fig 1 Global Real Estate Securities Market by Region of 2014



Source: FTSE EPRA/NAREIT Global Real Estate Index.

Figure 1 presents the plot of synchronous REITs price index obtained from Datastream database since December 2009 to June 2015. After the subprime crisis, the Asia-Pacific REITs, European REITs, and US REITs price index had a positive trend, while emerging market REITs price index had a negative trend. In addition, the correlations among REITs price indices were relatively weak and had a moderate positive relationship except for the correlation between European REITs, emerging market REITs and European REITs and US REITs had a strong positive relationship (see Table 2).

Fig 2 Asia-pacific REITs, Emerging Market REITs, European REITs and US REITs Price Index



Source: Datastream, 2015.

Table 2 Correlation Matrix of Daily REITs Price Index among Group of Country

REIT	ASIAPACIFIC	EMERG	EUROPE	USA
ASIAPACIFIC	1.0000	0.1778	0.1244	0.0867
EMERG	0.1778	1.0000	0.5194	0.3045
EUROPE	0.1244	0.5194	1.0000	0.4977
USA	0.0867	0.3045	0.4977	1.0000

Source: Calculation

The fundamental goal of portfolio optimization for quantitative investment managers and risk managers is to find the proportions of various assets to be held in a portfolio with the highest returns. According to Markowitz (1959), the portfolio construction requires dependence between financial returns which explains by correlation coefficient. However, for the financial analysis, correlation may be appropriate because most of financial return series exhibits fatter tail compared to normal distribution and often displays volatility cluster. Consequently, the copula function is an appropriate tool for dependency modeling (see Sklar, 1973). In addition, the model of financial returns should be composed of a GARCH model and a copula function depending on heavy-tailed marginal distributions.

In order to optimize a portfolio, Value at risk (VaR) has important role and a widespread measure of risk. However, McKay and Keefer (1996) and Mauser and Rosen (1999) showed that VaR has some drawbacks. Therefore, Conditional value-at-risk (CVaR), known as mean excess loss, mean shortfall, or tail VaR, is an alternative risk gauge and had more preferable properties than VaR (see Artzner et al., 1997, Embrechts et al. 1999, and Pflug, 2000).

In this study we investigated the dependence among groups of REITs by using multivariate *t* copula based on GARCH with student's *t* distribution approach in order to calculate VaR and CVaR, and to construct the optimal portfolio. The utilized dataset consisted solely of the REITs price index returns from 4 groups of REITs, namely Asia – Pacific REITs, European REITs, US REITs and emerging market REITs.

Empirical results showed that the GARCH with student's *t* distribution is appropriate to estimate parameter and conditional volatility. Then, the estimated VaR and CVaR were calculated based on 10%, 5%, 1% levels, respectively. The Monte Carlo simulation was applied in order to estimate the expected shortfall of an optimal weighted portfolio. At given significant level of 5%, the efficient frontier of the portfolio was created with the optimized portfolio based on mean-CVaR model. Eventually, the optimal weights of the portfolio were obtained with the various expected returns in frontier.

In the literature, a number of studies examined the behavior and volatility of REIT price and return. Najand & Lin (2004) applied both a GARCH and GARCH-M model in their analysis of daily REIT volatility, and indicated that the volatility shocks were persistent. Winniford (2003) focused on seasonality in REIT volatility and found that volatility in Equity REITs does vary on a seasonal basis. In long memory studies, Liow (2009), Zhou (2011) and Pavlova, et al. (2014) focused mostly on examining the long memory features of REIT volatility. Wei-ming and Zhong-fu (2012) applied GARCH, ARFIMA, and Markov swiching model to analyze three REITs from the Hong Kongs Hang Seng market.

However, only a few of the studies have examined the relationship, correlation or spillover of REITs across markets. Cotter (2005) studied volatility and volatility linkage between REIT sub-sectors and the influence of other US equity series by BEKK process in both daily and monthly data, and found the linkage both within the REIT sector and between RETIs and related sectors. Lee (2009) examined the volatility spillover in Australian REITs futures, and showed that the equity market is more influential than REITs in affecting the volatility of REITs futures. Stevenson (2015) examined a causal relationship between Equity REITs to the other REIT sectors with GARCH and EGARCH, and found that the REIT sector was generally influenced much strongly by volatility in small cap stocks and in firms classified as value stocks.

In case of VaR assessment in RETIs, Devaney (2001) employed GARCH-M model to examine REITs risk premium and discovered different REITs own different risk premium. Lu, et al. (2009) calculated the VaR of twelve REITs portfolios and found that VaR varies among individual portfolios, and the largest VaR was the Hotel REITs. Zhou (2015) compared the performance of various common used methods in forecasting Value at Risk (VaR) and Expected Shortfall (ES) and found that EGARCH skewed *t*, GARCH *t*, and GARCH based extreme value were the most reliable forecasts while GARCH normal and RiskMetrics were the worst performers.

Peng and Lee (2013) studied the relationship between US REITs and Japan REITs with ARMAX-GJR-GARCH copula models and time-varying dynamic copula models. The results showed that the kendall tau was lower before the submortgage crisis. The contagion effect test exhibits the US submortgate crisis would affect Japan REITs. In addition, no matter the large, middle or small scale positive and negative shock, the contagion probability during the crisis was larger than pre-submortgage crisis.

This paper is organized as follows. Section 2 presents a brief review of the research methodology. Section 3 explains the data, descriptive statistics, and unit root tests. Section 4 describes the empirical estimates, some diagnostic tests, VaR and CVaR, and optimal portfolio. Section 5 presents the economic implications for optimal hedge ratios and optimal portfolio weights. Section 5 provides some concluding remarks.

2. Methodology

2.1 GARCH

In 1986, Bollerslev (1986) propose the Generalized Autoregressive Conditional Heteroskedasticity (GARCH), which relaxed the unrealistic assumption that the innovations ε_t have constant variance overtime, because the lagged effects of the conditional variance or GARCH can capture the volatility of price movement in the market. For a log return series (r_t), the ARMA(*p,q*) and GARCH (*l,k*) can be formulated as follows,

$$r_t = \mu + \sum_{i=1}^{p} \phi_i r_{t-i} + \sum_{i=1}^{q} \psi_i \varepsilon_{t-i} + \varepsilon_t \tag{1}$$

$$\varepsilon_t = \sigma_t v_t \tag{2}$$

$$\sigma_t^2 = \varpi + \sum_{i=1}^{k} \alpha_i \varepsilon_{i-t}^2 + \sum_{i=1}^{l} \beta_i \sigma_{t-i}^2 \tag{3}$$

where ε_t is the innovation at time *t*. v_t is a sequence of *iid* random variables with mean 0 and variance 1. $\varpi > 0$, $\alpha_i, \beta_i > 0$ and $\sum_{i=1}^{k} \alpha_i + \sum_{i=1}^{l} \beta_i \le 1$. In this case, v_t is assumed to be student's *t* distribution because financial data usually have a heavy tail distribution. The α_i and β_i are known as ARCH and GARCH parameters, respectively.

2.2 Multivariate *t* copula

A copula is the joint distribution of random variables, $x_1,...,x_n$, and each of which is marginally uniformly distributed as $x(0,1)$. By Sklar's Theorem, for any random variable $X = (x_1,...,x_n)$ with joint cumulative distribution function is defined by

$$F(x_1, x_2, ..., x_n) = C(u_1, u_2, ..., u_n) \tag{4}$$

where $u_i = F_{X_i}(x_i)$, $i = 1, 2, ..., n$, and $F_{X_i}(\cdot)$ is marginal cumulative distribution function. If each $F_{X_i}(\cdot)$ for all $i = 1, 2, ..., n$ are continuous, the n-copula function C is unique. The high dimensional copula means a high dimensional distribution function which is uniquely determined on $[0,1]^n$. If $F(\cdot)$ is a multivariate *t* distribution with n-dimensional random vector with degree of freedom $v = n-1$, mean vector μ, and positive definite dispersion matrix Σ or $X \sim t_n(v, \mu, \Sigma)$, and its density is given by

$$f(x) = \frac{\Gamma\left(\frac{v+n}{2}\right)\left(1 + \frac{1}{v}(x-\mu)\Sigma^{-1}(x-\mu)^T\right)^{-\frac{v+n}{2}}}{\Gamma\left(\frac{v}{2}\right)\sqrt{(v\pi)^n |\Sigma|}} \tag{5}$$

then $C(\cdot)$ is a *t*-copula. Σ is a correlation matrix and is defined by

$$\Sigma = \begin{pmatrix} 1 & \rho_{u_1,u_2} & \cdots & \rho_{u_1,u_2} \\ \rho_{u_2,u_1} & 1 & & \rho_{u_1,u_2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{u_n,u_1} & \rho_{u_n,u_2} & \cdots & 1 \end{pmatrix}$$, where $\rho_{i,j} \in [-1,1]$ and $i, j = \{1,...,n\}$

where $\Gamma : \alpha > 0 \to \Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$. Since the multivariate *t* belongs to the class of multivariate normal variance mixtures and has the representation as

$$X = \mu + \frac{\sqrt{v}}{W} Z \tag{6}$$

where $W \sim \chi^2(v)$, $Z \sim N_n(0,\Sigma)$ and W and Z are independent. Since the copula invariant under a standardization of the marginal distribution meaning that the unique *t*-copulas, $t_n(v,\mu,\Sigma)$, is given by

$$C_{v,\Sigma}^{t} = \int_{-\infty}^{T_v^{-1}(u_1)} \cdots \int_{-\infty}^{T_v^{-1}(u_n)} \frac{\Gamma\left(\frac{v+n}{2}\right)\left(1+\frac{1}{v}(x-\mu)\Sigma^{-1}(x-\mu)^T\right)^{-\frac{v+n}{2}}}{\Gamma\left(\frac{v}{2}\right)\sqrt{(v\pi)^n\left|\Sigma\right|}} dx \tag{7}$$

where T_v^{-1} is the quantile function of standard univariate T_v function. According to Demarta and McNeil (2005), the density of high dimensional is

$$c_{v,\Sigma}^{n}\left(t_1(x_1),...,t_v(x_v)\right) = \left|\Sigma\right|^{-1/2} \frac{\Gamma\left(\frac{v+2}{2}\right)}{\Gamma\left(\frac{v}{2}\right)} \left[\frac{\Gamma\left(\frac{v}{2}\right)}{\Gamma\left(\frac{v+1}{2}\right)}\right]^n \frac{\left(1+\frac{\zeta\Sigma^{-1}\zeta}{v}\right)^{-\frac{v+n}{2}}}{\prod_{i=1}^{n}\left(1+\frac{\zeta_i^2}{2}\right)^{-\frac{v+1}{2}}}$$

where $\zeta = \left(T_v^{-1}(u_1),\cdots,T_v^{-1}(u_n)\right)$ is the *t*-student univariate vector inverse distribution functions.

2.3 VaR and CVaR

To calculate the empirical VaR and CVaR of an eqully weighted portfolio with 4 assets. The equations are follow:

$$\text{Min} \qquad ES = E[r \mid r \le r_\alpha]$$

$$\text{Subject to} \qquad r = w\left[r_{(1,t+1)} + r_{(2,t+1)} + r_{(3,t+1)} + r_{(4,t+1)}\right]$$

$$w_1 = w_2 = w_3 = w_4 = \frac{1}{4}$$

$$0 \le w_i \le 1,\ i = 1,2,3,4$$

where r_α is the lower α - quantile and $r_{i,t+1}$ is the return on individual asset at time $t+1$

2.4 Optimal portfolio with minimum risk via *t* copula

In order to estimate the optimal weighted portfolio, the Monte Carlo simulation with estimated multivariate *t* copula is adopted to generate N sample size. Then, the optimal portfolio weights of the selected asset are constructed under minimize expected shortfalls with respect to maximize returns given by:

$$\text{Min } ES = E[r \mid r \leq r_\alpha]$$

$$\text{Subject to } r = w_1 r_{(1,t+1)} + w_2 r_{(2,t+1)} + w_3 r_{(3,t+1)} + w_4 r_{(4,t+1)}$$

$$w_1 + w_2 + w_3 + w_4 = 1$$

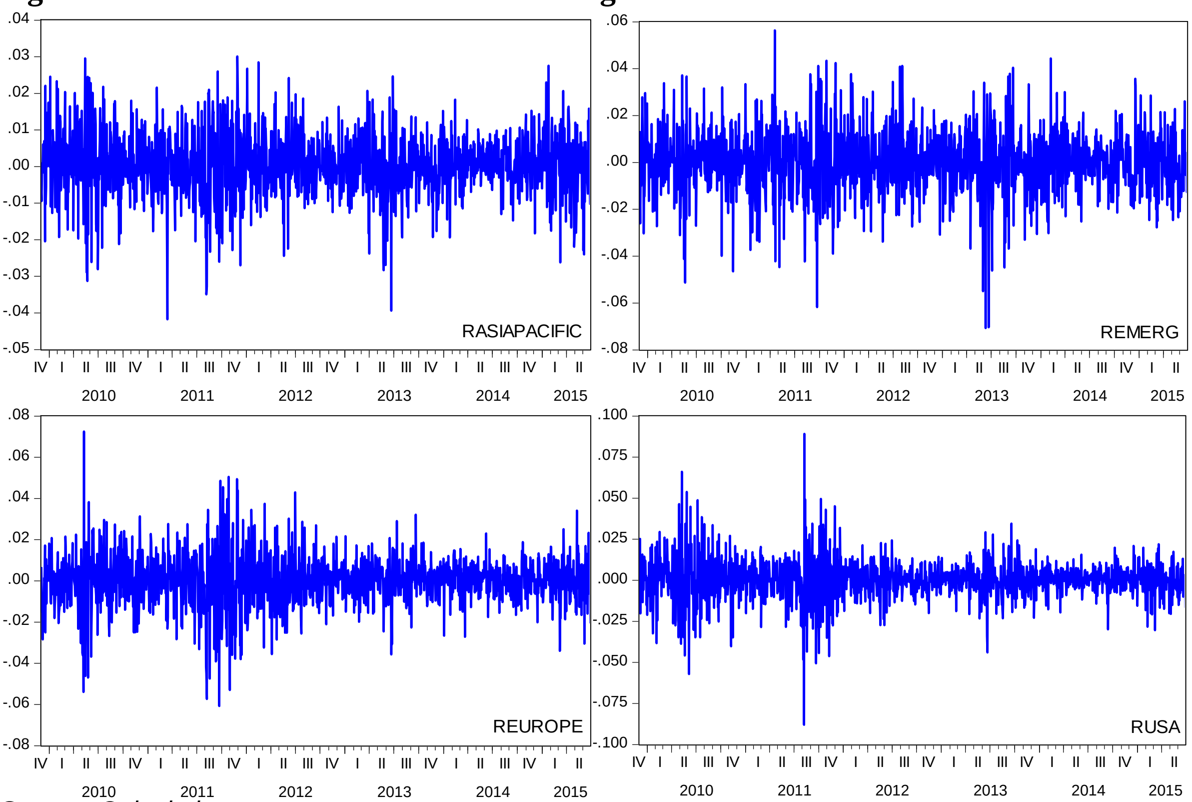
$$0 \leq w_i \leq 1,\ i = 1,2,3,4$$

where r_α is the lower α - quantile and $r_{i,t+1}$ is the return on individual asset at time $t+1$

3. Data

In this paper, we used the REIT price index returns from 4 groups of countries, namely Asia-Pacific, emerging market, Europe, and United State. All the daily data are obtained from Datastream database. The 1,455 price index observations are corrected from 1 December 2009 to 29 June 2015. The returns of REIT price index of market *i* at time *t* are calculated based on a continuous compound basis as $r_{i,t} = \log(P_{i,t}/P_{i,t-1})$, where $P_{i,t}$ and $P_{i,t-1}$ are the REIT price index in region *i* for time and *t* and *t* – 1, respectively. The daily return of each REIT index is given in Figure 1. The plots of returns in respective markets move in a similar fashion and present volatility clustering.

Figure 2 Return of REIT Price Index for Regional REIT



Source: Calculation

The descriptive statistics for the REIT price index returns represent in Table 2. The average returns of REIT price index are low and positive value, except for emerging market return is negative value. Based on the standard deviation, REIT price index return of emerging country has the highest historical volatility, while REIT price index return of USA has the lowest. In addition, even though the skewnesses are quite low and close to zero, kurtosis for each return series is greater than 3, so these series may be non-normally distribution. Therefore, under the normality distribution of Jarque-Bera (JB) test, these test statistics confirm that they are not normally distributed. Consequently, the GARCH models are estimate based on *t* distribution. Before to estimating the conditional mean and variance, the Augmented Dickey-Fuller (ADF) and Phillips and Perron (PP) tests are applied to check unit roots in the series. They all reject the null hypothesis of a nonstationary series at 1% level of significance in all cases.

Table 2: Descriptive Statistics and Unit Root Tests for REITs Price Index Returns

	RASIAPACIFIC	**REMERG**	**REUROPE**	**RUSA**
Mean	0.0004	-0.0002	0.0001	0.0004
Maximum	0.0301	0.0563	0.0724	0.0891
Minimum	-0.0417	-0.0701	-0.0607	-0.0879
SD.	0.0089	0.0137	0.0125	0.0124
Skewness	-0.2731	-0.2329	-0.1945	-0.0678
Kurtosis	4.4127	5.0313	6.1258	9.0139
JB-test	138.99***	263.12***	601.10***	2,192.23***
ADF-Test				
None	-35.641***	-37.145***	-35.938***	-23.339***
Constant	-35.690***	-37.141***	-35.927***	-23.375***
Constant and Trend	-35.683***	-37.154***	-35.923***	-23.388***
PP-test				
None	-35.586***	-37.154***	-35.957***	-42.941***
Constant	-35.629***	-37.150***	-35.947***	-43.134***
Constant and Trend	-35.621***	-37.162***	-35.912***	43.186***

*Note: ***,**,* denote significant at the 1%, 5% and 10%, respectively.*

4. Empirical Results

Table 3 shows the coefficient for the ARMA(p,q)-GARCH(1,1) with student' *t* distribution specification for each REITs price index return series. The optimum lag for ARMA(p,q) was selected by the minimum value of Akaike information criteria (AIC) information criteria. The estimated equation of emerging market, European and USA are ARMA(2,2)-GARCH(1,1), while Asia-Pacific was ARMA(3,3)-GARCH(1,1). In addition, most of the estimated coefficients were statistically significant at 1% in most cases. The coefficient of *t* distribution for each equation was also statistically significant at 1%, meaning that the assumption of *t* distribution for ARMA-GARCH estimation is reasonable.

Table 3: Estimate ARMA-GARCH(1,1)

	RASIAPACIFIC	REMERG	REUROPE	RUSA
μ	-0.0006***	-0.0004*	-0.0001***	0.0007***
$AR(1)$	-1.5760***	-0.1599***	1.3920*	-1.1530***
$AR(2)$	-0.2699***	-0.9359***	-0.8187**	-0.6624***
$AR(3)$	0.3601***			
$MA(1)$	1.6019***	0.1842***	-1.3782***	1.1091***
$MA(2)$	0.2869***	0.9621***	0.8096***	0.6482***
$MA(3)$	-0.3890***			
ω	1.00E-06***	1.00E-06***	1.00E-05***	1.00E-06***
α	0.0600***	0.0483***	0.0074***	0.1105***
β	0.9298***	0.9439***	0.8725***	0.8833***
K (t-coefficeint)	8.4028***	7.7721***	6.3380***	6.2988***
BERK-test (prob)	(0.6983)	(0.7883)	(0.8941)	(0.8537)
$Q^2(5)$ (prob)	(0.8887)	(0.9005)	(0.9363)	(0.8189)
$Q^2(10)$ (prob)	(0.9722)	(0.5758)	(0.7716)	(0.9715)
$Q^2(15)$ (prob)	(0.9927)	(0.7992)	(0.9952)	(0.6120)
$Q^2(20)$ (prob)	(0.8553)	(0.9058)	(0.9800)	(0.5489)
AIC	-6.767	-6.1738	-5.8393	-6.3496

*Note: ***,**,* denotes significance at 1%, 5% and 10%, respectively.*

The estimated residuals of ARMA-GARCH process for each REIT index return were transformed to marginal. The Berkowit test (Berkowit, 2001) was applied in order to test whether the transformed marginals have uniform distribution in (0,1). Under the null hypothesis of Berkowit test that the transformed marginals are as uniform distribution, at 1% significance level, we cannot reject the null hypothesis (see table 3). In order to guarantee that residuals are independent and identically distributed random variable (i.i.d), the Ljung-Box Q statistic is used at given 5, 10, 15, and 20 lag with 1% significance level, we failed to reject the null hypothesis (see table 3). The estimated multivariate *t* copula parameters were shown in table 4. These values were utilized to construct efficient portfolio and found optimal portfolio for the best expected returns with respect to minimum loss.

Table 4: Empirical *t* copulas parameters ($\hat{\rho}$)

	Asia-Pacific	Europe	Emerging	USA
Asia-Pacific	1.00000	0.05607	0.12908	0.05384
Europe	0.05607	1.00000	0.50783	0.44591
Emerging	0.12908	0.50783	1.00000	0.29780
USA	0.05384	0.44591	0.29780	1.00000

$\hat{v} = 8.0794$ *Source: Calculation*

Under the equally weighted assumption, table 5 shows the estimated VaR and CVaR or expected shortfall at level of 1%, 5% and 10%, respectively. In period $t+1$, the estimated CVaR were higher than VaR and converged to -1.49, -1.88 and 2.87 at 10%, 5% and 1% level, respectively.

Table 5: Value at risk equally weighted portfolios

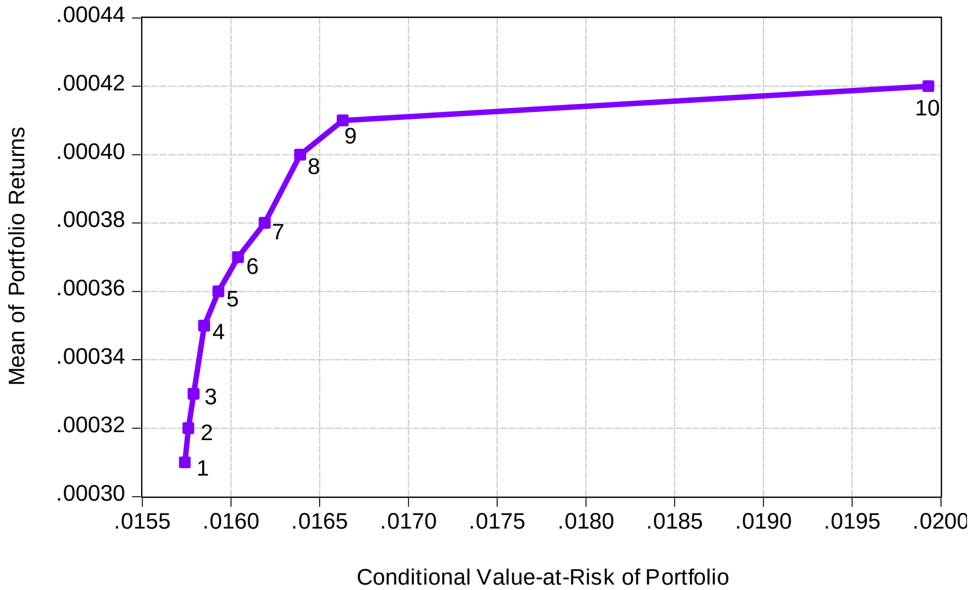
Portfolio	Expected Value		
	1%	5%	10%
VaR	-2.22%	-1.30%	-0.95%
CVaR	-2.87%	-1.88%	-1.49%

Source: Calculation

Next, the Monte Carlo simulation was applied in order to simulate a set of 100,000 samples and to estimate the expected shortfall of an optimal weighted portfolio. At given significance level of 5%, the optimized portfolio based on mean-CVaR model provided the efficient frontier of the portfolio under different expected return as shown in Figure 3.

Finally, under minimized expected shortfall with respect to maximized returns we obtained the optimal weight of the portfolios. Table 6 showed some of the results of optimal weight with the expected returns in the frontier. The optimal portfolio weight suggests to invest in the largest proportion in Asia-Pacific REITs, followed by European REITs, emerging market REITs, and US REITs, respectively.

Figure 3 The efficient frontiers of CVaR under mean



Source: Calculation

Table 6 Optimal Weighted portfolios for ES5%

Portfolios	Asia-Pacific	Europe	Emerging	USA	Returns
1	0.495	0.322	0.100	0.083	0.031%
2	0.495	0.321	0.101	0.083	0.032%
3	0.496	0.321	0.100	0.083	0.033%
4	0.496	0.321	0.100	0.083	0.035%
5	0.496	0.320	0.101	0.083	0.036%
6	0.497	0.320	0.101	0.083	0.037%
7	0.497	0.320	0.101	0.083	0.038%
8	0.497	0.319	0.101	0.083	0.040%
9	0.497	0.319	0.101	0.083	0.041%
10	0.498	0.319	0.101	0.083	0.042%

Source: Calculation

5. Conclusions

In this paper, we investigated the optimum portfolio of REIT index return of Asia – Pacific, Europe, USA, and emerging markets with multivariate *t* copula based on GARCH-*t*, multivariate *t* copula, and component VaR (CVaR) models. Empirical results showed that the ARMA-GARCH with student's *t* distribution was appropriate to estimate parameter and conditional volatility. Then, the dependency structure among Global REITs was obtained from multivariate *t* copula. The estimated VaR and CVaR were calculated based on 10%, 5%, 1% levels, respectively. Finally, the optimal portfolio weight suggest to invest the largest proportion in Asia-Pacific REITs, followed by European REITs, emerging market REITs, and US REITs.

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Reference

Anderson, R. I., Chen, Y-C. and Wang, L-M. (2015) A Range-Based Volatility Approach to Measuring Volatility Contagion in Securitized Real Estate Markets. *Economic Modelling*. 45: 223-235.

Artzner, P., Delbaen, F., Eber, J.M. and Heath, D. (1997). Thinking Coherently. *Risk*, 10, 11: 68 – 71.

Berkowitz, J. (2001). Testing Density Forecasts, with Applications to Risk Management. *Journal of Business and Economic Statistics*, **19**(4), 465–474.

Chang, G-D. and Chen, C-S. (2014). Evidence of Contagion in Global REITs Investment. *International Review of Economics and Finance*. 31: 148-158.

Chiou, S.C. and Tsay, R.S. (2008). A copula-based approach to option pricing and risk assessment. *Journal of Data Science*, 6, 3: 273-301.

Cotter, J. (2005). Multivariate Modeling of Daily REIT Volatility. Centre for Financial Markets, Department of Banking & Finance, University College Dublin, Ireland.

Devaney, M. (2001). Time Varying Risk Remain for Real Estate Investment Trust: A GARCH-M model. *The Quarterly Review of Economics and Finance*, 41, 3: 335-346.

Embrechts, P., Resnick, S. I., and Samorodnitsky, G. (1999). Extreme Value Theory as Risk a Management Tool. *North American Actuarial Journal*, 3, 2: 32 - 41.

Lee, C-C., Chien, M-S. and Lin, T.C. (2012). Dynamic Modelling of Real Estate Investment Trusts and Stock Markets. *Economic Modelling*. 29: 395-407.

Liow, K. H. (2009). Long-term Memory in Volatility: Some Evidence from International Securitized Real Estate Markets. *The Journal of Real Estate Finance and Economics*, 39, 415–438.

Lu, C., Wu, S-C. and Ho, L-C. (2009). Applying VaR to REITs: A Comparison of Alternative Methods. *Review of Financial Economics*, 18, 2: 97 – 102.

McKay, R. and Keefer, T. E. (1996). Extreme Returns, Downside Risk, and Optimal Asset Allocation. *Journal or Portfolio Management*, 25, 1: 71-79.

Mausser, H. and Rosen, D. (1999). Beyond VaR: From Measuing Risk to Managing Risk. *ALGO Researh Quarterly*. 1, 2: 5-20.

Najand, M. & Lin, C. (2004). Time Varying Risk Premium for Equity REITs: Evidence from Daily Data, *Working Paper*, Old Dominion University.

Palaro, H.P. and Hotta, L.K. (2006). Using Conditional Copula to Estimate Value at Risk. *Journal of Data Science*, 4, 1: 93-115.

Pavlova, I., Cho, J. H., Parhizgari, A. M. and Hardin III, W. G. (2014). Long Memory in REIT Volatility and Changes in the Unconditional Mean: Modifies FIGARCH Approach. *Journal of Property Research*, 31, 4, 315 – 332.

Peng, M-Y. and Lee, W-C. (2013). The Correlation and Contagion Effect Between US REITS and Japan REITS - Based on the ARMAX-GJR-GARCH-Copula Model. *Asian Economic and Financial Review*. 3, 12, 1609 – 1619.

Rees, P. and Selcuk-Kestel, A. S. (2014). Analysis of Portfolio Diversification between REIT Assets. *Journal of Computational and Applied Mathematics*. 259: 425-433.

Stevenson, S. (2015). An Examination of Volatility Spillovers in REIT Returns. *Journal of Real Estate Portfolio Management*. 8, 3: 229-238.

Wei-ming, H. and Zhong-fu, L. (2012). A Comparison of Forecast Models of REIT Volatility GARCH Model, AFIMA Model, Markov Switching Model. 19th International Conference on Management Science & Engineering.

Winniford, M. (2003). Real Estate Investment Trusts and Seasonal Volatility: A Periodic GARCH Model, *Working Paper*, Duke University.

Zhou, J. (2011). Long Memory in REIT Volatility Revisited: Genuine or Spurious, and Self-similar? *Journal of Property Research*, 28, 213–232.

Zhou, J. (2015). Extreme Risk Measures for REITs: A Comparison among Alternative Method. *Applied Financial Economics*, 22, 2, 113-126.