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## **A COMPARATIVE STUDY OF STOCK PRICE FORECASTING USING NONLINEAR MODELS**

### **Abstract:**

This study compared the in-sample forecasting accuracy of three forecasting nonlinear models namely: the Smooth Transition Regression (STR) model, the Threshold Autoregressive (TAR) model and the Markov-switching Autoregressive (MS-AR) model. Data used was daily close stock prices of five banks in the South African banking sector and was obtained from the Johannesburg Stock Exchange (JSE). It covered the period from 2010 to 2012 with a total of 563 observations. Nonlinearity and nonstationarity tests used confirmed the validity of the assumptions of the study. The study used model selection criteria, SBC to select the optimal lag order and for the selection of appropriate models. The Mean Square Error (MSE), Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) served as the error measures in evaluating the forecasting ability of the models. The MS-AR models proved to perform well with lower error measures as compared to LSTR and TAR models in most cases.

### **Keywords:**

Stock price, nonlinear time series models, error metrics

**JEL Classification:** C10, C32, E32

## 1 Introduction

In recent years, modelling economic and financial data nonlinear time series has received great attention as opposed to linear time series models. This is due to the realization that linear models fail to describe the dynamics of financial time series (Ismail and Isa, 2006). According to Maponga (2013), linear time series analysis involves simple models that describe the behaviour of time series in terms of past values. Nonlinear time series are generated by nonlinear dynamic equations which show attributes that cannot be modeled by linear time series models. These attributes are time-changing variance, asymmetric cycles, higher-moment structures, thresholds and breaks data to mention a few.

A variety of nonlinear models have been considered as alternative to standard linear models. For instance, the parametric nonlinear models such as the autoregressive conditional heteroscedasticity (ARCH) developed by Engle (1982) and the generalized autoregressive conditional heteroscedasticity (GARCH) of Bollerslev (1986) are some of the alternative linear models. Recently, the application of novel regime switching nonlinear models in financial data analysis is receiving great attention (Franses and Dijk, 2000). Most analysts of financial and economic data have effectively used these models. Commonly used among these models are the Threshold Autoregressive (TAR) of Tong (1978), Smooth Transition Regressive (STR) of Teräsvirta and Anderson (1992) and Markov-Switching Autoregressive (MS-AR) of Hamilton (1989).

These three models differ from conventional linear econometric models due to the assumption of different regimes within which the time series may exhibit different behaviour. The current study sought to explore the possibility of developing empirical models capable of describing and forecasting the South African major banks' closing stock prices. In the main, the study intends to determine the predictive performance of each of the three models in modeling and forecasting stock prices. Forecast error metrics will be used to judge performance of the models. The study assumes that the data used satisfy the nonlinear properties so as to allow an efficient performance of the three suggested models.

The findings could empower stock market investors to make informed and accurate investment decisions. Again this may also boost the confidence of stakeholders in the financial industry to do more business with less risk exposure. Other beneficiaries of the study may be regulators and other financial institutions as well as researchers in academia.

The rest of the paper is organized as follows: Section 2 study provides a brief discussion of literature; in Section 3 study describes the methodology and results; Section 5 provides concluding remarks and recommendations.

## 2 Literature Review

There is much interest in modeling and forecasting the nonlinearity in a variety of macroeconomic and financial series, such as stock market, exchange rate and Gross Domestic Products (GDP). A number of nonlinear time series models have been suggested in literature, for instance the bilinear models developed by Granger and Andersen (1978), the TAR, STR and the MS-AR models. The studies reviewed herewith adopted these models.

Moolman (2004) used the idea of MS-AR model as a tool to provide evidence that the South Africa stock market returns depends on the state of the business cycle. McMillan (2005) employed the STAR model to examine nonlinear behavior in the international stock market. The study by Pérez-Rodríguez *et al.* (2005) concluded that the artificial neural network (ANN) and the Smooth Transition autoregressive (STAR) models in the Spanish market outperform the ARMA and the random-walk models. On the other hand, Cheung and Lam (2010) compared profitability in the US stock market using the self-exciting threshold autoregressive (SETAR) and linear models. In their studies, Ismail and Isa (2006) and Yarmohammadi *et al.* (2012) evaluated the performance of MS-AR model and six different time series modelling approaches to model Iranian exchange rate series. The study found MS-AR to be a useful model with the best-fit for modeling fluctuations of exchange rates.

Wasim and Band (2011) employed a two state MS-AR to identify the existence of bull and bear regimes in the Indian stock market. The model appropriately showed that Indian stock market will remain under bull regime compared to bear regime. Amiri (2012) have compared the forecasting performance of linear and nonlinear univariate time series models for GDP growth. The evaluation of the forecasting performance of their set of nonlinear models using real time data proved that the nonlinear models are able to capture the underlying processes of GDP as opposed to linear models. Cruz and Mapa (2013) also contributed to the literature by developing an early warning system for predicting the occurrence of high inflation in the Philippines with MS model. The study successfully managed to identify episodes of high and low inflation with this model.

### 3 Methodology and results

This section discusses the data and methods used and provide the results of the study

#### 3.1 Preliminary analysis

The study employed the 563 daily South African stock prices collected for the period 2010-2012 from <http://www.jse.com>. After using purposive sampling technique, five (5) banks from a population of twenty-one (21) banks were sampled. The banks that responded were ABSA Bank (ABSA), Capitec Bank (CAPB), First National Bank (FIRB), Nedbank (NEDB) and Standard Bank (STDB). A time series plot for these stock prices is shown a Figure 1.

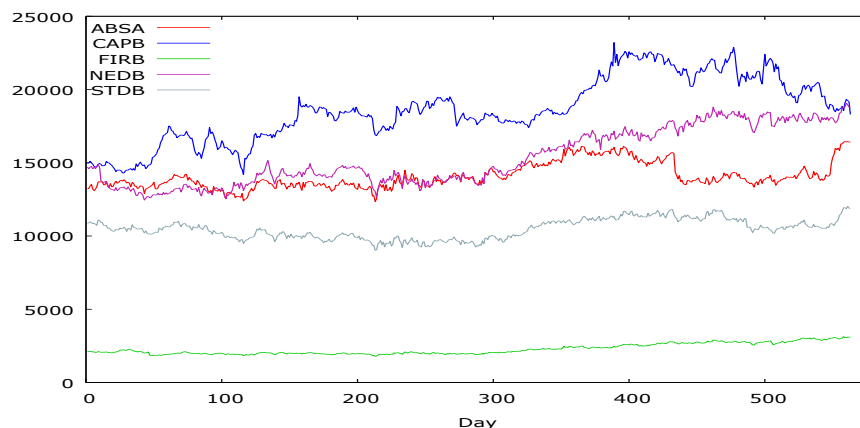


Figure 1: Graphical Representation of the Five Closing Stock Prices

The results reveal that FIRB has the lowest stock prices and is estimated by an upward sloping trend. Stock prices of other banks are explained by irregular increasing patterns with ABSA and NEBD showing convergence at several stages. Given this movements by the stock prices, the data is not stationary at all levels. The series are further checked for nonlinearity by employing the nonlinear test. Since nonlinearity in time series may occur in several ways, there exists no single test that dominates others in detecting nonlinearity. Therefore the study uses the Regression Specification Error Test (RESET) by Ramsey (1969) and Brock-Dechert-Scheinkman (BDS) by Brock *et al.* (1996) tests for this purpose. The null hypothesis of nonlinearity is rejected if the RESET and the BDS tests are greater than the critical values at a conventional level of significance, implying that the true specification is nonlinear. To determine the stability of the models, a Cumulative Sum (CUSUM) test by Brown *et al.* (1975) is used. The null hypothesis is rejected if the CUSUM test exceeds the critical value. The results of the three tests are summarised in Table 1.

**Table 1: Estimated AR Models with Nonlinearity Tests**

		ABSA	CAPB	FIRB	NEBD	STDB
Parameter Estimate	$\alpha_0$	263.614 (1.8731) [0.0616]	197.824 (2.001) [0.0459]	2.9902 (0.3089) [0.7575]	47.3057 (0.6324) [0.5274]	182.023 (2.013) [0.0446]
	$\alpha_1$	0.862676 (20.5228) [0.0000]	0.9897 (187.60) [0.0000]	0.9995 (237.80) [0.0000]	0.8596 (20.51) [0.0000]	0.9828 (114.0) [0.0000]
	$\alpha_2$	0.119039 (2.8155) [0.0050]			0.1379 (3.279) [0.0011]	
RESET Test for Specification	Test Statistic	4.00483 [0.0188]	3.4352 [0.0329]	3.6984 [0.0254]	4.9172 [0.0076]	8.7728 [0.0002]
CUSUM Test for Parameter Stability	Test Statistic (Harvey-Collier)	2.58915 [0.0099]	0.6004 [0.5485]	1.7090 [0.0880]	2.6447 [0.0084]	0.2375 [0.8123]
Test for ARCH Effects	LM	3.1967 [0.07379]	71.2252 [0.0000]	3.0925 [0.0787]	5.9051 [0.0151]	12.1992 [0.0022]
BDS	z-statistics	3.1967 [0.0000]	3.1967 [0.07379]	3.1967 [0.0000]	3.1967 [0.0000]	3.1967 [0.0000]

Figures in (•) are t-statistics while figures in [•] are p-values

Results from the RESET tests of the five variables suggest that the use of a linear regression modelling technique was inappropriate. In addition, the residuals from various autoregressive (AR) models fitted to the data were found to have ARCH structures, further supporting the use of nonlinear modelling methods. There is no evidence of structural change in the data according to the BDS tests. The preliminary results proves that the data is suitable for the application of STR, TAR, MS-AR models.

### Modelling and Forecasting models

This section presents the results of the three nonlinear time series models suggested. Note that estimation of the AR model was based on maximum lag five chosen with the aid of the Swartz Bayesian Criterion (SBC). This estimation was done to fulfil the requirements for the models.

### Threshold Autoregressive Models for Closing Stock Price

Switches between one regime and another depend on a threshold variable and threshold value. This study followed the Hsu *et al.* (2010) structural break concept in selecting the thresholds.

	Variable	Coefficient	Std. Error	t-Statistic	Prob.	R-Square	Adj. R-Square
ABSA(t)	C(1)	2372.968	1048.48	2.263247	0.024	0.94899	0.948254
	C(2)	0.825795	0.077052	10.71732	0.0000		
	C(3)	2386.699	650.5343	3.668829	0.0003		
	C(4)	0.820463	0.048884	16.78401	0.0000		
	C(5)	3733.219	874.7009	4.267995	0.0000		
	C(6)	0.732856	0.062796	11.67038	0.0000		
	C(7)	1840.186	692.9351	2.65564	0.0081		
	C(8)	0.881074	0.045018	19.57142	0.0000		
	C(9)	1.000339	0.001225	816.4269	0.0000		
CAPB(t)	C(1)	1.000446	0.001563	640.0509	0.0000	0.98512	0.984908
	C(2)	1441.481	636.0464	2.266315	0.0238		
	C(3)	0.920211	0.035552	25.88318	0.0000		
	C(4)	1.000821	0.001209	827.7597	0.0000		
	C(5)	2983.457	824.0883	3.620313	0.0003		
	C(6)	0.716250	0.084546	8.471738	0.0000		
	C(7)	0.374637	0.097122	3.857384	0.0001		
	C(8)	-0.228059	0.084111	-2.711422	0.0069		
	C(9)	0.997983	0.001449	688.9222	0.0000		
FIRB(t)	C(1)	0.998951	0.001915	521.575	0.0000		
	C(2)	355.5556	87.66296	4.055939	0.0001		
	C(3)	0.819949	0.044442	18.44991	0.0000		
	C(4)	1.0021588	0.001678	597.3176	0.0000		
	C(5)	1.001118	0.00145	690.4768	0.0000		
	C(6)	1.001137	0.00135	741.3204	0.0000		
NEDB(t)	C(1)	1414.938	1354.395	1.044701	0.2966	0.92195	0.92067
	C(2)	0.892528	0.102173	8.735495	0.0000		
	C(3)	-20560.01	2762.216	-7.443303	0.0000		
	C(4)	-2.562839	0.192509	-13.31283	0.0000		
	C(6)	1726.918	1550.095	1.114073	0.2657		
	C(7)	0.876889	0.110954	7.903171	0.0000		
	C(8)	0.717100	0.191316	3.748249	0.0002		
	C(9)	0.284765	0.191779	1.484858	0.1382		
	C(10)	3017.190	1984.132	1.52066	0.1289		
	C(11)	0.832901	0.110275	7.552972	0.0000		
	STDB(t)	C(1)	1485.843	643.2156	2.310023		
C(2)		0.859032	0.060756	14.13897	0.0000		
C(3)		1557.045	564.578	2.757892	0.0060		
C(4)		0.843761	0.056492	14.93596	0.0000		
C(5)		1771.688	462.4663	3.830956	0.0001		
C(6)		0.816977	0.047865	17.06845	0.0000		
C(7)		1384.418	396.2032	3.494212	0.0005		
C(8)		0.877154	0.035288	24.8571	0.0000		
C(9)		1.000742	0.001336	748.8949	0.0000		

In particular, assuming that the numbers of thresholds are unknown, the Bai-Perron multiple breakpoint method was applied. The final estimated TAR models were obtained and the results are presented as equations below.

### Smooth Transition Regression Analysis

This section provides the results for the STR modelling technique. Also shown are the forecasts of the model for the five variables. Table 2 presents the results from the LSTAR model.

**Table 2:** Estimated LSTR Models

Dep. Var.	Variable	Estimate	t-Stat	p-Value	R-Square	Adj. R-Square
ABSA(t)	----- Linear Part -----				0.94704	0.9471
	CONST	1407.01652	4.4602	0.0000		
	ABSA(t-1)	0.89731	38.7516	0.0000		
	---- Nonlinear Part ----					
	ABSA(t-1)	0.01213	3.8226	0.0001		
	Gamma	14.19718	0.7681	0.4427		
	C1	14598.19084	138.9240	0.0000		
CAPB(t)	----- Linear Part -----				0.98467	0.9847
	CONST	244.32152	2.3887	0.0172		

	CAPB(t-1)	0.98762	177.9250	0.0000		
		---- Nonlinear Part ----				
	CONST	-244.32152	-2.3887	0.0172		
	Gamma	17.97761	1.2362	0.2169		
	C1	22476.78769	94.3540	0.0000		
FIRB(t)		---- Linear Part ----			0.99051	0.9905
	CONST	239.56090	3.8898	0.0001		
	FIRB(t-1)	0.87937	28.3434	0.0000		
		---- Nonlinear Part ----				
	CONST	-206.50774	-3.1367	0.0018		
	FIRB(t-1)	0.10941	3.3907	0.0007		
	Gamma	800.73372	0.1317	0.8953		
	C1	2132.44609	347.5154	0.0000		
NEDB(t)		---- Linear Part ----			0.98693	0.9870
	NEDB(t-1)	1.00023	1121.5372	0.0000		
		---- Nonlinear Part ----				
	CONST	813.8	2.2596	0.0242		
	NEDB(t-1)	-0.0	-2.2466	0.0251		
	Gamma	14.1	0.7563	0.4498		
	C1	15526.9	30.4711	0.0000		
STDB(t)		---- Linear Part ----			0.96003	0.9601
	CONST	268.61160	1.8589	0.0636		
	STDB(t-1)	0.99933	95.3909	0.0000		
		---- Nonlinear Part ----				
	CONST	-266.04	-2.2890	0.0225		
	Gamma	7.29	1.9281	0.0544		
	C1	9284.20	98.9096	0.0000		

As revealed by the results, all five variables have been have autoregressive processes since their lags are significant in both the linear and nonlinear parts. By observation the estimated models seem good judging from the high  $R^2$  and  $R_{adj}^2$  values. Again, the transition values (C1), for ABSA, CAPB, NEDB, and STDB suggest that closing stock price of these banks switch between two regimes. In fact, a closing stock price less than C1 are regarded as low stock yield periods for these banks. Larger closing stock price implies even higher stock prices.

### Markov-Switching AR Models for Stock Prices

Prior to estimating the MS-AR model, the study identifies the number of regime switching models for the variables. This task is fulfilled by applying the linearity likelihood ratio (LR) test. The criterion is to reject the null hypothesis in favour of the alternative if the test is less than the conventional level of significance. Judging from the results presented in Table 3, it is clear that the LR test is in support of a two-state regime for all the five variables. These findings are in accordance with those by S by Ismail and Isa (2006).

**Table 3:** Linearity LR Test of Two-Regime Switch

Variable	Chi-Square Test Statistic	P-value
ABSA	53.794	0.0000
CAPB	100.1	0.0000
FIRB	21.788	0.0006
NEDB	11.296	0.0796
STDB	12.042	0.0610

The results for MS-AR (1) models shown in Table 4, The variances for regime 2 associated with ABSA, CAPB and FIRB the variances of Regime 2,  $\sigma^2(s_t = 2)$ , is greater than the variance of Regime 1,  $\sigma^2(s_t = 1)$ , suggesting that for these three closing stock prices, regime 2 is more volatile than Regime 1. In other words, regime 2 captures the behaviours in ABSA, CAPB and FIRB in an unstable manner and the opposite does not apply to regime 1. Regime 1 is reported to be stable for other banks. The findings also report that for ABSA, FIRB, NEDB and STDB, the estimated regime-dependent intercepts (expected daily increments in closing stock prices) are higher in Regime 1 than in Regime 2 while the opposite holds in the case of CAPB. In other words, changes in ABSA, FIRB, NEDB and STDB closing stock prices increases in a stable state while opposite holds for NEDB.

**Table 4:** Two-Regime MS-AR Modelling Results

	ABSA	CAPB	FIRB	NEDB	STDB
$\mu(s_t = 1)$	13749.6	17853.9	2276.56	15390.1	10507.0
$\mu(s_t = 2)$	13642.6	18761.8	2194.16	14488.6	10457.0
$\phi_1(s_t = 1)$	0.996758	1.00108	0.998810	0.994343	0.973702
$\phi_1(s_t = 2)$	0.531652	0.945820	1.259510	1.000960	1.180030
$\sigma^2(s_t = 1)$	178.457	201.356	34.4960	241.296	137.350
$\sigma^2(s_t = 2)$	241.037	331.188	116.217	190.232	18.7274
$p_{11}$	0.989355	0.98621	0.995928	0.995980	0.94730
$p_{12}$	0.061359	0.041621	0.999979	0.004151	0.69051
$p_{21}$	0.010645	0.013793	0.0040724	0.004019	0.052702
$p_{22}$	0.938640	0.958380	0.0000212	0.995850	0.309490
$E[D(s_t = 1)]$	16.2975	24.0263	1.0000	240.8884	1.4482
$E[D(s_t = 2)]$	93.9408	72.5005	245.5554	248.8181	18.9746

The results further shows that the probabilities of a closing stock price remaining in Regime 1,  $p_{11}$ , are smaller than the probability of a closing stock price staying in Regime 2,  $p_{22}$ , for all the five closing stock prices. In fact, the probabilities of a closing stock price staying in Regime 1 lie in the range of 0.947 to 0.996 with an expected duration,  $E[D(s_t = 1)]$ , of 1 to 241 days. Similarly, the probabilities of a stock price staying in Regime 2 lie in the range 0.000 to 0.958 with an expected duration,  $E[D(s_t = 2)]$ , of 19 to 249 days. This means that closing stock prices can stay slightly longer in Regime 2 than in Regime 1.

### Model performance

This section provides the results of the forecast performance of the three models. Forecasted the future is of great importance for planing, decision-making and policy formulation. The evaluation of nonlinear models is based on the properties of resulting

residuals. Using the residuals, various tests for misspecification, including non-normality, parameter stability and autocorrelation checks were conducted. The diagnostic test statistics for these assumption (not presented here) rendered the models accurate and sufficient. On the basis of reliability, validity and wide use, the performance (error) measuring metrics are recommended for evaluating the efficiency of models in forecasting. The study uses the four error metrics such as RMSE, MAE, MAPE, and RMSPE. The model that generate the least forecast error is chosen and suggested for further analysis. Table 5 provides the results for the four error measures.

**Table 5:** Forecast Comparison among LSTR, TAR and MS-AR Models

Measure	Method	ABSA	Capitec	FRIB	NEDB	STDB
RMSE	LSTR	200.2572	270.9698	35.48659	219.5906	133.0790
	TAR	196.5424	266.1471	35.48629	210.4875	131.0235
	MS-AR	186.7458	217.5940	35.32322	213.6210	129.6859
MAE	LSTR	148.9902	189.5397	27.03180	167.8142	103.7846
	TAR	147.2353	186.6499	26.93976	162.2681	101.3549
	MS-AR	143.0377	160.6033	27.34744	165.5507	97.6963
MAPE	LSTR	0.010624	0.010107	0.011973	0.011023	0.009965
	TAR	0.010502	0.009945	0.011929	0.010653	0.009735
	MS-AR	0.010188	0.008587	0.012121	0.010863	0.009400
RMSPE	LSTR	0.251849	0.239601	0.283848	0.261327	0.236247
	TAR	0.248965	0.235339	0.282786	0.252104	0.230791
	MS-AR	0.241512	0.203568	0.287354	0.257530	0.222846

According to the results, the four error metrics select the MS-AR(1) model for ABSA, CAPB and STDB, and TAR model for NEDB accordingly. MAE, MAPE and RMSPE select the TAR model for FIRB, RMSE selects the MS-AR(1) model for FIRB. The results are in accordance with those by Dacco and Satchell (1999), whose study identified the FIRB as best modelled by the MS-AR(1).

#### 4 Conclusion Remarks

The study explored the performance of the TAR, STAR and the MS-AR models in modelling and forecasting daily stock prices series of five banks of South Africa. The five banks considered are the ABSA, Capitec, First Rand Bank, Nedbank, and Standard Bank for the period from 2010 to 2012. The suggested models perform better when applied to nonlinear series. Appropriate test for this assumption proved that all the series are nonlinear. The estimation of the three models was based on an optimal lag five suggested by the Swartz Bayesian Criterion. The three models were successfully estimated using this lag. To evaluate the performance of the three models, the study used the four forecast error metrics which were in favour of the MS-AR model. Generally, the results proved that the MS-AR performed better in most cases compared to the LSTR and TAR models. From the discussions of the results, the following conclusions can be drawn:

- All five closing stock prices are nonlinear in nature.
- Various estimated predictive models for the five closing stock prices are robust for purposes of forecasting.

The study is recommending the used of MS-AR in modelling and forecasting the economic and financial data. This is motivation by the results of the current study



results and the study of Ismail and Isa (2006), Wasin and Bandi (2011) and Yarmohammadi *et al.* (2012).

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