DOI: 10.20472/IAC.2015.019.116

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BEHAVIOR OF COVARIANCE MATRICES WITH EQUI-CORRELATION APPROACH

Abstract:

Funds and asset managers are increasingly concerned with quantitative and econometric model in order to apply their portfolio models. The main goal of this publication is to study the behavior and the proportions of a stock portfolio from CAC All-Tradable with these kinds of models and compare the results with the historical approach. A GARCH (1,1) process has been used for modelling each asset volatility and Engle dynamic equi-correlation model to forecast covariance matrices. From a small amount of underlying values, the question is raised whether forecasted covariance matrix is more relevant than traditional variance-covariance matrix in a context of minimum variance portfolio model.

Keywords:

Volatility - Correlation – Equi-Correlation - GARCH (1,1) - Portfolio Selection - Asset Allocation-Covariance Matrix – Minimum Variance Portfolio.

JEL Classification: G11, C02, C40

1. Introduction

In asset management theoretical and applied studies [1], mathematical models are developed to measure and to quantify the risk of investments. As such, they play the role of decision-making tools for administrators, investors and regulators. Mathematical and quantitative models are used in almost all sectors of modern finance: portfolio management, evaluation of products, regulation of banks, standards control and risk management [2,5]. Two main lines of approach have been developed in two different modeling trends. Historical line is resting on stock market predictability from direct observation of past data. On the contrary stochastic line is based on possibility of constructing possible stock market future paths from mathematical systems once hypotheses have been set with the expectation to find the most efficient one. Asset managers use these mathematical models to evaluate parameters like volatility or correlation in order to estimate profits or loss even if each model has its limits. Because correlation measures the relationship between two or more financial variables in time, it appeared rapidly as a key element to develop a forecasting method for portfolio strategy [7]. Correlation is a significant notion in portfolio management because it is directly linked to portfolio diversification according to Markowitz theory [4]. Since the first correlation models by Engle [3], many improvements have been found [13]. DECO (Dynamic Equi-correlation) model [6] is the latest one based on correlation and has the advantage of being able to handle many assets and of providing consistent parameter evaluation even if data is noisy. So here DECO model is utilized to forecast covariance matrices, and minimum variance portfolio model has been applied afterward to define portfolio strategy. The method for comparing these matrices to historical one is to study the proportions of each asset in both forecasted and historical portfolios.

2. Equipment and Methods

2.1 Data

The first step has been to select N assets for the portfolio where N≥100. For this step, data have been taken from the website Yahoo Finance which provides a large amount of relevant information concerning stock market prices, financial lexicon and generally recent trends in financial markets. Assets have been taken from the CAC All-Tradable in order to have an unconditional idea of portfolio correlation.

The model of asset allocation has been constructed according to Markowitz theory of diversification and minimum variance. Using Excel and R, the aim is to use solver capacity to calculate and find proportions of each asset with chosen constraints. So, the selection criteria for the portfolio is its minimum variance (note that portfolio variance is the sum of variances of each column of covariance matrix). To test the model, a portfolio constructed with a basket of 8 assets: {Accor, AGF, Air Liquid, Aventis, Axa, BNP-Parisbas, Bouygues, Cap Gemini} has been used. There are 209 periods starting on 01/01/2002 up to 31/12/2005. Weekly data are chosen for better precision.

2.2 Returns and First Calculations:

For each asset its return $r_{i,t}$ is calculated on a precise period of time by using log returns:

$$r_{i,t} = Ln\{S(t)/S(t□1)\} \quad (1)$$

where S(t) is the asset closing price at time t. These returns are stored on one Excel sheet. Expected return and historical standard deviation of each asset can be inferred over the studied period from 01/01/2002 until 31/12/2005. Building the historical variance covariance matrix Q_t, portfolio variance is calculated by changing the proportions and then minimizing the variance with Excel solver. On the other hand, T periods of data are deleted in order to forecast the returns on these periods later.

3. GARCH Model:

Garch model [9] is useful here because it allows generate conditional volatility during both historical and forecasted periods. Model equation is:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \quad ; \quad \varepsilon_{i,t} = \frac{r_{i,t}}{\sigma_{i,t}} \quad (2)$$

where ω, α and β are the constants of the model and are estimated by Quasi-Maximum likelihood method, $r_{i,t}$ is the expected return of asset i and $\varepsilon_{i,t}$ is its residual. The residual highlights the gap between calculated results and results in the real life. An initialization is needed in order to have first σ value (noted σ_{garch}). For that, one uses the unconditional variance, which only depends on historical data. For the first measures of ω, α and β, the following values $\alpha = 8{,}24.\,10^{-2}, \beta = 0{,}8\,, \omega = 3{,}9.\,10^{-6}$ are taken.

So, firstly, $\varepsilon_{i,1} = r_{i,1}/V$ where V is the unconditional variance; and thanks to $\varepsilon_{i,1}$ and V, the first conditional variance can be computed. The first element of log-normal-likelihood function is also calculated when assuming that each return asset follows a Gaussian distribution. Secondly, $\varepsilon_{i,2} = r_{i,2}/\sigma_{2,Garch}$ and from $\varepsilon_{i,2}$ and σ_{2Garch}^2 computed by the model, one gets L and stock it again. These operations are continued up to T, last period of historical data. All likelihood elements are added and the sum is maximized. For the forecasting part, another residuals expression $\varepsilon_{i,t} = \sigma_{i,t} * Z(t)$ is considered where Z(t) is a stochastic term like a shock which follows a Gaussian distribution. The quantities $Z(t < T) = \varepsilon/\sigma$ are calculated and stored in order to use them during forecast. Different paths (for example 5 paths) are computed with these shocks and 5 different forecasted volatilities which follow also a GARCH process.

4. DECO Model

To use the DECO model, equi-correlation matrix is defined by:

$$R_t = (1-\rho_t)\, In + \rho_t J_n \tag{3}$$

where J_n is n-dimensional matrix of ones, I_n n-dimensional identity matrix, □t is equi-correlation obtained from Q matrices in the form

$$\rho_t = \frac{2}{N(N-1)} \sum_{\substack{i,j \\ i=0,j=0}}^{N} \frac{q_{i,j,t}}{\sqrt{q_{i,i,t}\, q_{j,j,t}}} \tag{4}$$

where N is the number of assets and $q_{i,j,t}$ are the elements of Q_t with

$$q_{i,j,t} = \bar{\rho}_{i,j} + \alpha_{deco}\left(\varepsilon_{i,t-1}\varepsilon_{j,t-1} - \bar{\rho}_{i,j}\right) + \beta_{deco}\left(q_{i,j,t-1} - \bar{\rho}_{i,j}\right) \tag{5}$$

$\bar{\rho}_{i,j}$ is the unconditional correlation between the returns r_i and r_j, and □i□t□□□ri,t/□i,t are residuals for each return series. Initial values α_0,β_0 and Q_0 are chosen assuming that $\overline{Q}=Q_0$. From them, correlation parameters α_{deco} and β_{deco} are retrieved by maximizing the following log-likelihood function

$$L = \frac{-1}{T}\sum[N\, log(2\pi) + log(1-\rho_t)^{N-1}[1+(n-1)\rho_t] + \frac{1}{1-\rho_t}[\sum(\varepsilon_{i,t}^2) - \frac{\rho_t}{1+(N-1)\rho_t}\sum(\varepsilon_{i,t}^2)]] \tag{6}$$

where T is the number of historical periods.
So the square matrix Q_1 is computed with Q_0, α, β and the first ε generated with GARCH model, ρ_1 is computed with Q_1, and L_1 is obtained with ρ_1 and ε and so on up to the last historical period see Figure **1**.

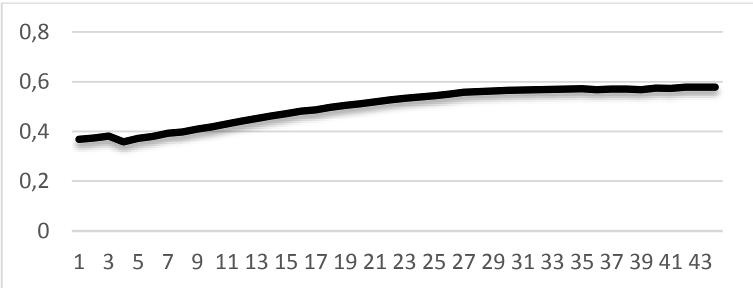

Figure 1. Equi-Correlation vs Historical Periods

During forecasted periods, the different ε given by selected 5 paths are used to store Q matrices and to calculate the equi-correlations, as reported for 9 periods on Figure **2**.

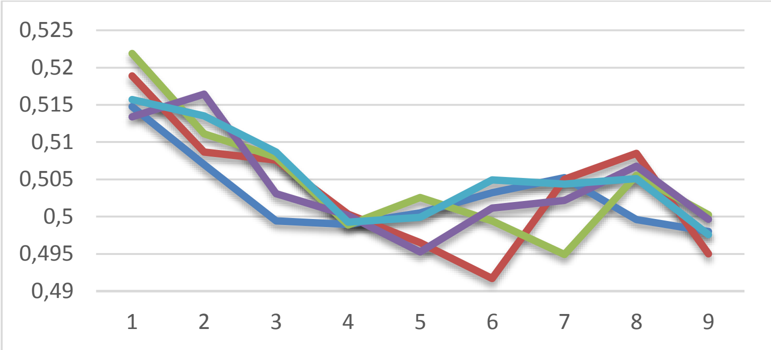

Figure 2. First nine forecasted Equi-correlations with 5 GARCH (1, 1) Processes

5. Discussion and Results

Up to now, the different variance covariance matrices have been calculated with the two empirical and stochastic methods. For the historical data, the unconditional variance has been calculated on the diagonal (corresponding to asset i = asset j); otherwise, the unconditional covariance has been obtained by statistical calculation. At the same time, 5 other variance-covariance matrices (from the 5 different considered paths) given by

$$Cov\left(x_{i,t+x}, x_{j,t+x}\right) = \rho_{DECO,T+x}\, \sigma_{i,t+x}\sigma_{j,T+x} \quad (7)$$

have also been evaluated with here $x\epsilon[1,10]$.

Now the variance of complete portfolio is minimized by varying all assets proportion with Excel solver. Recall that these proportions are obtained at the end of the calculation after having obtained forecasted variance given by GARCH process and equi-correlation given by DECO model. Then one can compare the proportions, see Figure **3,** and conclude that the forecast is remarkably relevant compared to historical results. In present approach highest portfolio proportions are corresponding to less correlated stocks on top of having smallest volatility. This is verified on Figure **3** with selected stocks in opposition to historical method, and in particular with Aventis stock which has highest proportion of the portfolio.

This shows as stressed in present approach that correlation is a relevant parameter in asset allocation in stochastic approach.

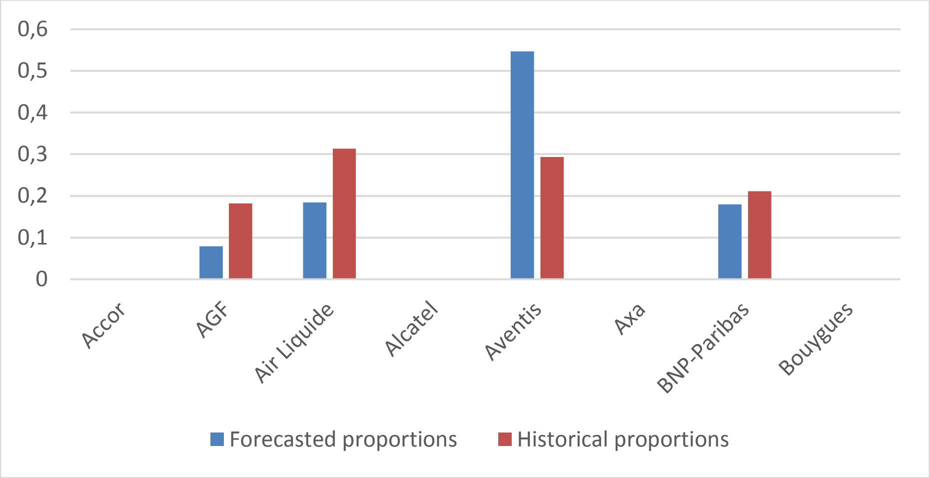

Figure 3. Proportions from Covariance Matrices at Time t+9

6. Conclusion

To improve portfolio management strategy, a new approach has been proposed mainly composed of two steps aimed at getting most efficient assets allocation. These two steps are covering the most important effects acting on portfolio dynamics. First one deals with volatility aspect impairing daily stocks evolution, and is taken here as GARCH model which provides an adapted representation of volatility. Second one is aimed at grasping the interrelation between stocks resulting from market globalism. The used model in present approach is DECO model which is well suited to calculate the effect of this interrelation by supposing that it reduces to a single correlation. Application of present model to simple eight component portfolio has been compared to direct analysis based on statistical calculations with interesting forecasting results.

Aknowledgments

The authors are very much indebted to ECE Paris School of Engineering for having provided necessary environment where the work has been performed, to V. Lebreton and Dr. Y. Rakotodrasimba for advices and guidance during the study and to Pr M. Cotsaftis for help in preparation of the manuscript

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