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A DYNAMIC ANALYSIS OF RELEVANT VARIABLES IN THE SPANISH ECONOMY USING DECOMPOSITION DATA SERIES WITH DAUBECHIES WAVELETS

Abstract:

We illustrate some aspects of the economic situation in Spain following the dynamic of data series of six relevant economic and financial variables: A financial index (IBEX35), a row material (Crude Oil Price in euros, COP), a foreign exchange index (EUR/USD), a bonus (Spain 10-Years, S10YB), the total state debt (Spain State Debt, SSD) and Consumer Price Index (CPI) variables (from es.investing.com, indexmundi.com, www.bde.es and www.ine.es). We analyse the decomposition of non-stationary rates monthly series in the period 2000M1-2014M12 using Daubechies wavelets db8 to visualized high frequency variance, seasonality and trend. We present several figures illustrating the decomposition in both time and frequency domains and tables containing the relevant frequency indices for the different detail series obtained.

Keywords:

Spanish Economy, Dynamic Analysis, Wavelets

JEL Classification: C02, C89, E30

1 Introduction

A topic of great importance for making decisions in a wide range of scientific and social fields is finding a system for processing information and obtaining certain properties in order to ascertain the evolution of relevant variables and, under certain requirements, to predict them.

This study involves two main phases: the knowledge of the variables that represent the data (or, in terms of time series analysis, the *fiiting*), and a sensitivity analysis (or, in terms of time series analysis, *prediction*).

Originally, there was a strong division between methods of time series analysis in the time domain (early twentieth century) and frequency domain (second half of the twentieth century), which persists to this day in most widely used tools (De Gooijer and Hyndman, 2006).

In general, the time domain is useful for describing the dynamics over time of one or more variables by studying the influence of the past and, perhaps, certain expectations regarding their current and future development. The frequency domain is used to describe a sinusoidal series in terms of its amplitude, period or frequency and phase.

In the first case, autocorrelation functions, covariance matrices and parametric models such as ARIMA models are used to describe the dynamic dependency of the series (Box and Jenkins, 1976). The second case includes spectral analysis or power spectra of a process which is calculated on the basis of the Fourier transform of the autocorrelation function (Brockwell and Davis, 1991).

Considerable research work has been undertaken to bring these two approaches together. One such area of work is based on the use of orthogonal wavelet functions.

This approach has yielded an important generalisation of the Fourier transform and provides simultaneous information in both domains. Other proposals have been made involving Kalman filters (Harvey, 1989) or Monte Carlo methods (Hendry, 2007), spurred on by recent computational advances in displaying methods in various applications. Any of them promotes a constant "feedback" between theory and practice.

The literature on wavelets has expanded rapidly over the past 20 years. Many papers have been published using this methodology in a wide range of fields. Applications using wavelets are emerging in astronomy, engineering, medicine, physics and many other fields of study, including, in recent years, finance and economics. For instance, see Ramsey and Lampart, 1998; Gencay, Selcuk and Whitcher, 2002; Ramsey, 2002; Crowley, 2007; Aguiar-Conraria and Soares, 2011a-b; González-Concepción, Gil-Fariña and Pestano-Gabino, 2010, 2012; Hafner, 2012; Mingming and Jinliang, 2012; Rua and Nunes, 2012; Bai et al, 2015; Michis, 2015.

Wavelets are certain families of orthogonal or quasi-orthogonal functions that have many desirable properties, some of which have proven useful in economics and finance, while others have not. For example, Ramsey and Lampart (1998) explore four ways in which wavelets might be used to enhance the empirical toolkit in economics and finance (exploratory analysis, density estimation and local inhomogeneity, time scale decomposition, forecasting, etc.). The drawbacks involve the sample characteristics and potential numerical instabilities. Wavelet-based techniques rely on equally-spaced data, a condition which does not always hold in economic series. Even when it does, the cycles over which the economic activity takes place may not be homogeneous with respect to the type of data. Moreover, some techniques based on wavelets require dyadic samples and a certain number of initial values in order to begin the calculation process. Certain instabilities can also arise when attempting to decompose a signal using very high-order polynomials, due to the requirement to calculate their roots.

The type of data involved (daily, weekly, monthly, quarterly, annual, or other frequency schedules) is essential for drawing conclusions. These can be modelled using wavelet decompositions to reach conclusions regarding frequencies that cannot be observed directly in the original data series. However, when highly disaggregated series are available (data with an hourly or higher frequency), it is also of interest to aggregate data in order to obtain another type of periodicity (weekly, monthly, etc.).

The goal of this paper is to illustrate the application of the wavelet technique, bearing in mind the drawbacks in the multiresolution analysis for economic and financial modelling in Spain. To this end, we have obtained simultaneous time and frequency information for six relevant Spanish economic variables in the period 2000M1-2014M12 by way of the following monthly data series: a foreign exchange index (EUR/USD), a financial index (IBEX35), a bond (Spain 10-Years, S10YB), a raw material (Crude Oil Price in euros, COP), the total state debt (Spain State Debt, SSD) and the Consumer Price

Index (CPI). The examples illustrate the importance of being able to decompose the original signal into a trend component and various detail components, which allow the different frequencies and their time model to be observed separately.

Section 2 offers a brief summary of the methodology we employ in the data to contrast the three type domains: time, frequency and scale. Applying the wavelet technique to this last domain may be viewed as a necessary expansion of the Fourier and short-time Fourier techniques. Given their practical interest, we focus on discrete and continuous wavelet filters, specifically those devised by Daubechies (1992) for their theoretical properties.

Section 3 describes the economic and financial data series considered. In Section 4 we present the Wavelet Multiresolution Decomposition (WMRD) and Fast Fourier Transform (FFT) obtained and the relevant frequency indices detected. The calculations were carried out using MATLAB 7.0. To conclude, we present the more relevant findings and the references cited in the text.

2 Models in Time-Frequency-Scale

The trigonometric functions that serve as the basis of Fourier analysis (sines and cosines) are very helpful when working with time series that are stationary or whose periodicity is highly homogenous over time. This happens in economics when analysing data for an economic activity that exhibits certain cycles in its deterministic structure; for instance, the production of organic agricultural products considering monthly data or more disaggregated data if we consider the influence of seasonal crops or the consumption of basic goods such as water and energy.

Most economic/financial time series, however, exhibit rather complicated patterns over time and the Fourier transform is inefficient at capturing the properties of the series. In fact, if the frequency components are not stationary and are able to change over time, the Windowed Fourier Transform (Gabor) or the transform wavelet appear as alternatives.

The former provides unique information in the frequency domain, while the latter combines information from both the time and frequency domains (Gencay, Selcuk and Whitcher, 2002).

Wavelets are not homogenous over time; instead, they are functions with more irregular waves than those of sines and polynomials. Wavelets are more flexible than the basic trigonometric waves. They are able to adapt to frequency changes over time and yield information simultaneously in both domains through the so-called scale domain. Their use has important similarities and differences with the Fourier analysis

There are numerous families of wavelet functions, but not all are equally appealing from a financial application standpoint. We will focus here on the discrete case and on the Daubechies (denoted dbN) wavelets (Daubechies, 1992).

This family includes the Haar wavelet, written as db1, the simplest wavelet imaginable and certainly the earliest. dbN are not symmetrical and for some, the asymmetry is large. The regularity increases with the order and the functions become more regular at some points than at others. The analysis is orthogonal.

Using these wavelets we built a model for each variable based on the WMRD (*Wavelet Multi-resolution Decomposition*).

In general, the WMRD of the variable or signal s_t at level N is given by $\{a_N, d_N, d_{N-1}, ..., d_1\}$, such that $s_t = a_N + d_N + d_{N-1} + ... + d_1$.

$$s_t = \underbrace{\underbrace{\underbrace{\underbrace{a_{N} + d_{N}}_{a_{N-1}} + d_{N-1}}_{\vdots}}_{a_2} + d_2}_{a_1} + d_1$$

a_N is called the trend signal at level N and d_n is the detail signal at level n ($1 \le n \le N$), constructed using the *father* (or scaling function, representing the smooth, trend or low-frequency) and *mother* (representing the detailed or high-frequency) wavelets, respectively (Gencay, Selcuk and Whitcher (2002), among others. Our contribution focuses on the practical interpretation of the WMRD of a discrete signal for the financial data selected.

In economics, $\{s_t\}_{t=1...T}$ is normally a discrete real-time signal. Then, we can directly use a *discrete wavelet transform* (DWT) or the *continuous wavelet transform* (CWT), if we assume a continuous signal $x(t)$ approximating s_t. In this paper, we use the discrete case directly.

The aim of the WMRD is to obtain multiple series from the original data series, each of which highlights a level that is often not easy to see in the original data series. In practice, some aspects should be taken into account:

- The number of observations, T, restricts the choice of the wavelet and the number N of scale, dj, that can be produced. It must hold that $T \geq 2^N$.
- The use of Daubechies wavelets requires dyadic samples, although some approaches have recently been proposed that avoid this unrealistic requirement in a large number of applications (Gencay, Selcuk and Whitcher, 2002).
- The calculation of a large decomposition may result in the presence of correlations in the results.
- The calculation of a decomposition using high-degree wavelets can yield numerical instabilities, though a more regular asymptotic behaviour is achieved if these are avoided. Moreover, a certain number of initial values are needed to begin calculations.
- WMRD analysis assumes that data are sampled at equally-spaced intervals.
- Results are sensitive to the data type used in the study. That is, for a study of economic data, and given the nature of economic cycles in particular, a decomposition beyond level 3 for annual data, level 6 for monthly data, etc., would be of no interest.
- The interpretation of the scales when using more highly disaggregated series, such as those involving weekly, daily or hourly data, for example, is more delicate since the number of hours/day, hours/week, weeks/month, days/month, hours/month, weeks/year, days/year, hours/year, etc., does not have a set whole value.
- Depending on the type of activity, each sample may contain data associated with only some days or hours (e.g. daytime or night-time activity, activity on certain days of the week).

3 Data description

Six economic and financial monthly data series from 2000M3 to 2014M12 have been selected to illustrate the Spanish economy. Figures 1 and 2 contain the original and monthly rates series, respectively.

a) A foreign exchange index (EUR/USD).

http://es.investing.com/currencies/eur-usd-historical-data

Price and change rate monthly series for the EUR/USD were considered for the period in question.

It should be noted that the most important economic indicator that influences the EUR/USD exchange rate is interest rates. In addition, employment and other economic factors, such as Gross Domestic Product, Trade Balance imports, Retail Sales, Consumer Price, Producer Price Indexes and international news, exert a considerable influence on the EUR/USD rate.

When the Euro was first issued in January 1999, the popularity of the consolidated Euro currency led EUR/USD to gradually increase, crossing back above the 1.00 parity level in November, 2002. The currency pair continued to trend higher, eventually reaching a peak value of 1.56 in July 2008 in the wake of the U.S. sub-prime mortgage crisis that led to the collapse of Lehman Brothers in September 2008.

Since reaching its peak, EUR/USD has traded in a wide range, bounded on the downside by a low in June 2010 as the Greek sovereign debt crisis increased worries about a possible Eurozone breakup. Although the pair has since recovered considerably, concerns about the debt burden of less affluent European nations have continued to weigh on the rate.

b) A financial index (IBEX35).

http://es.investing.com/indices/spain-35-historical-data

The IBEX 35 is the official index of the Spanish Continuous Market. The index tracks the performance of the 35 most liquid stocks traded on the continuous market on the Bolsa of Madrid. It is a free-float, capitalisation-weighted index.

From 2000 to mid-2002, due to the bursting technology bubble, corporate scandals and the Latin American crisis, the IBEX fell by 59.39%. In particular, the Latin American crisis in 2002 shrank the IBEX by 28.11% and the bursting of the technology market bubble in 2000 devalued the index by 21.75%, the second and third highest falls recorded.

The index recovered through mid-2007 when, as a consequence of the global financial crisis, the bankruptcy of Lehman Brothers and the bursting of the housing bubble, the stock market index collapsed.

In 2008 the IBEX fell by 39.4%, the worst figure in its history. This was partially offset by the 30% rebound experienced by the index in 2009.

The largest increase in the history of the IBEX 35 was in May 2010 following the great financial operation to clamp down on speculation against the sovereign debt of some member States, which stemmed the decline of the euro. The European Union agreed to an unprecedented loan package to cover the requirements of member countries in financial need (Greece) and to defend the single currency. The IBEX 35 rose by 14.43%, though overall the index ended 2010 with a 17% drawdown.

It was in 2010 when the viability of Spanish public finances began to be questioned, as Spain's rating was cut below the highest triple A level. The IBEX strung together three consecutive losing years (a decline of 17% in 2010, 13% in 2011 and 4.6% in 2012), setting a new low in 2012, its lowest level since 2002. In 2012, the IBEX again featured the high volatility inherited from the previous two years as the risk premium reached its highest level (642 points)

In 2013, and for the first time since 2009, the Spanish stock market ended the year with gains (21.41%). Renewed confidence in the stability of the euro area, the relaxation of constraints in the debt market, the expansive monetary policies of the main central banks, the return of international investors and the incipient signals of an economic recovery all boosted the Spanish stock market.

In 2014, the increase was about 3.66%, the first time since 2009 the IBEX again closed above 10,000 points. By the end of the year the risk premium stood at 110 points, signalling a return to the risk premium levels from 2008, when it began to exceed 100 points.

c) A bond (The Spanish 10-year Government bond, S10YB).

http://es.investing.com/rates-bonds/spain-10-year-bond-yield-historical-data

Monthly prices and change were considered for the period in question.

The price of Spain's 10-year bond is the interest rate or yield offered to investors. It is used to calculate the risk premium over Germany.

Of particular interest in the period under study is the summer of 2012, when the returns on 10-year bonds peaked above 7.6% as euro sovereign bonds were given a BBB rating. From then on the returns started to decline to eventually settle below 3% in May 2014, following the improved economic outlook.

d) A raw material (Crude Oil (petroleum) Monthly Price in euros, COP).

http://www.indexmundi.com/commodities/?commodity=crude-oil&currency=eur

The Crude Oil (petroleum) data selected consists of the simple average of three spot prices: Dated Brent, West Texas Intermediate, and the Dubai Fateh, in Euros per Barrel.

The price trend since 2000 has been marked by political instability in several producing countries, a growing demand and the war in Iraq and Lebanon, all of which led to an all-time high in early 2008.

The bankruptcy of Lehman Brothers caused the price to drop by almost 30 euros in late 2008. From that level, in 2011, the presence of turmoil in countries like Egypt, Yemen, Libya and the increased demand in China and India brought about a price increase. Starting in June 2014, the prices registered significant falls.

e) Total state debt (Spain State Debt, SSD).

http://www.bde.es/webbde/es/estadis/infoest/indeco.html

Data series provide information on the state debt under the Excessive Deficit Procedure (EDP). It is the debt of government agencies and the instruments through which this materialises: currency and deposits, short- and medium-term debt securities, short- and long-term loans, government liabilities and the debt of public enterprises.

The economic and financial crisis has caused a substantial increase in the government debt of the European Union (EU) to far exceed that prior to 2008. This fact, together with the difficulties of tackling the expansive dynamic, has put the sustainability of public finances at the centre of the economic policy debate in Europe. There was an increase in the government debt ratios in developed countries to maximum levels in many cases.

Since 2008 Spain's debt has been increasing in a context of high public deficits. The ratio of public debt to GDP rose over this period to 84% of GDP in 2012. The persistent and high primary deficits accumulated during the period are the main factor that explains this decline, also influenced by loan interest.

The rapid increase in public debt in some euro area countries was one of the main factors that led to the sovereign debt crisis in early 2010.

In the Spanish case, the ratio of public debt to GDP at the end of 2013 was 93.9%, a figure that for the first time since the establishment of the Economic and Monetary Union was above the aggregate euro area (92.6% in 2013). By the close of 2014, the ratio stood at 97.67% of GDP.

f) Consumer Price Index (CPI).

http://www.ine.es/jaxiT3/Tabla.htm?t=10030 (Monthly data)

http://ine.es/jaxiT3/Tabla.htm?t=10013 (index series from 2002M1).

Previous data series are built by using previous rates.
The trend in the Consumer Price Index (CPI) data, which aims to measure the evolution of prices for consumer goods and services acquired by resident households in Spain, was collected.

The economic crisis and especially the sharp contraction in private consumption resulted in a substantial moderation of inflation in Spain, proxied by the growth rate of the CPI.

Throughout the period, Spain was characterised by moderate inflation, though the economy experienced a deflationary period in 2009 (-0.3% change in prices in 2009)

However, the CPI increased by 3% in December 2010, 2.2 percentage points above the rate observed in the same month the previous year. This trend was mainly due to a reduced number of factors such as rising commodities prices in international markets, tax increases that were part of the fiscal consolidation package, and an increase in certain regulated prices.

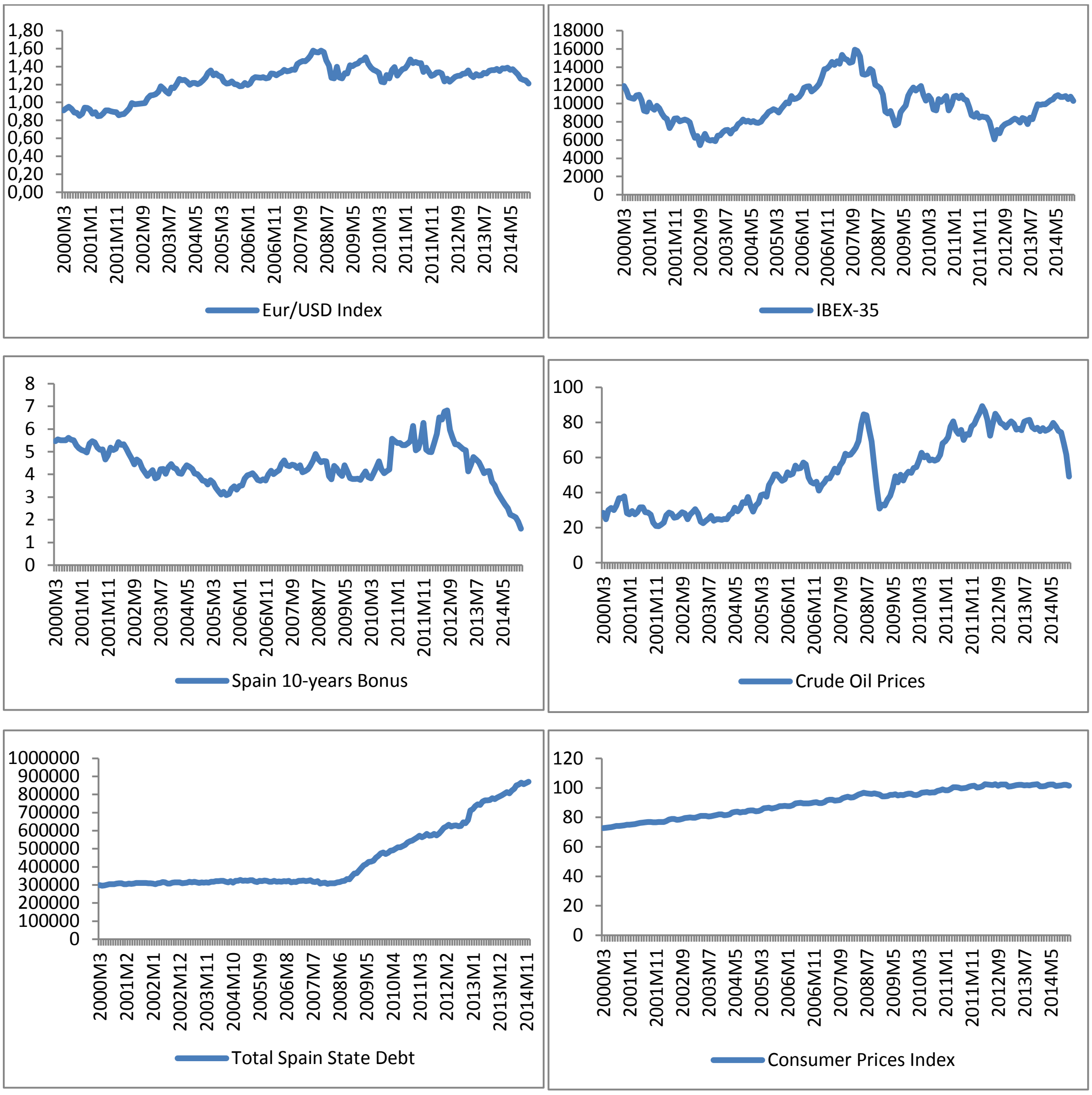

Figure 1. The original data series

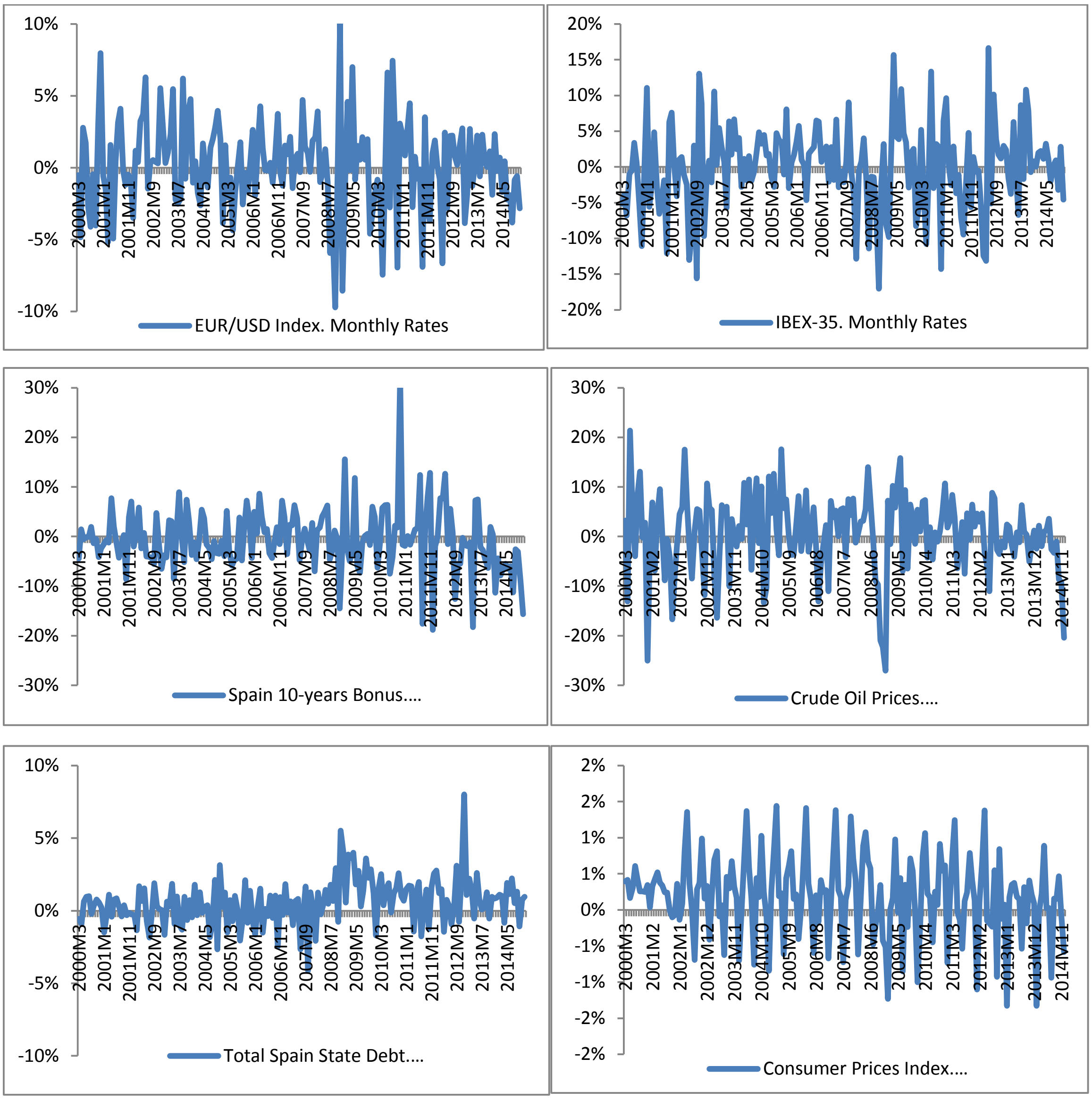

Figure 2. The Monthly Rates data series

4 WMRD, FFT and Frequency Indices of Original Series and their Monthly Rates

Figure 3-8 (a) illustrates the analysis of the original variables (non-stationary data series) using WMRD obtained from db8, level 5 (there are T=180 observations, $180>2^5$). In addition, for each variable we applied FFT to their five detail series (Figures 3-8 (b)) to obtain the Relevant Frequency Indices given in Table 1. The comparison shows that the WMRD captures information on these indices that displays possible changes over time.

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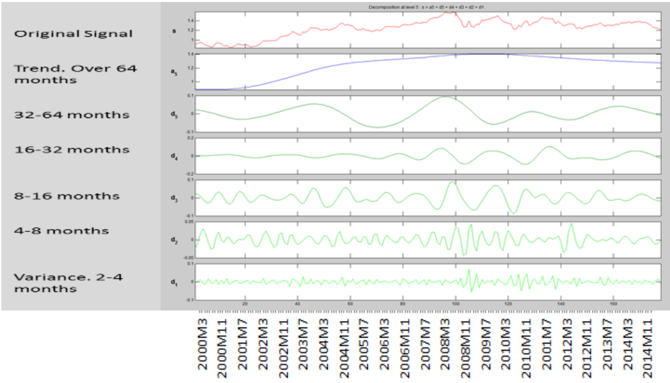


Figure 3(a). WMRD db8, level 5, to EUR/USD

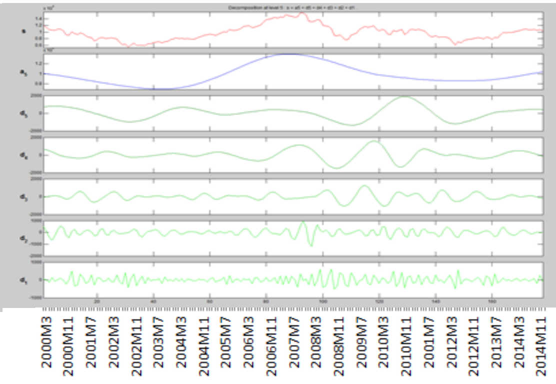


Figure 4(a). WMRD db8, level 5, to IBEX35

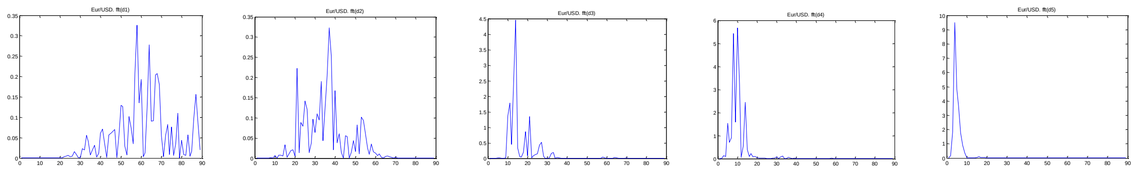


Figure 3(b). FFT to detail series EUR/USD

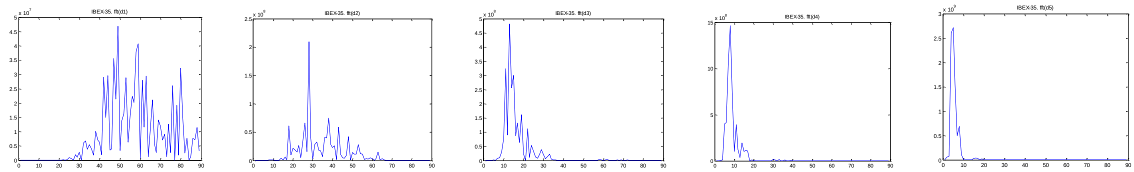


Figure 4(b). FFT to detail series IBEX35

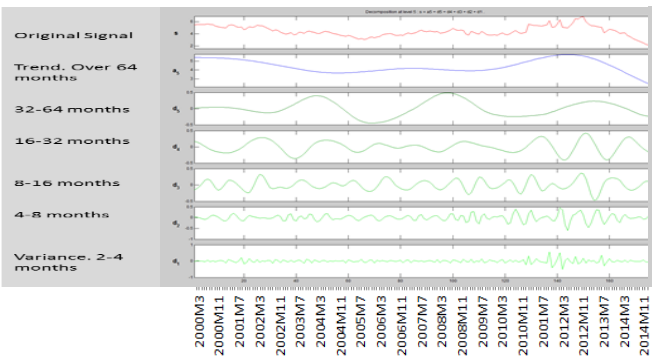


Figure 5(a). WMRD db8, level 5, to S10YB

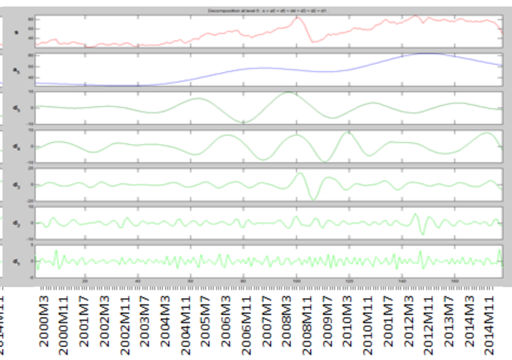


Figure 6(a). WMRD db8, level 5, to COP

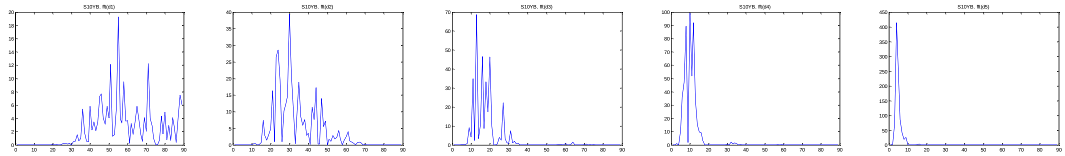


Figure 5(b). FFT to detail series S10YB

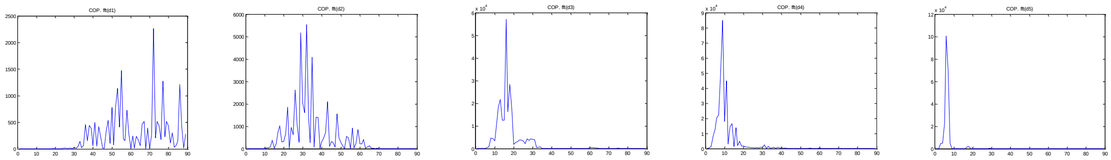


Figure 6(b). FFT to detail series COP

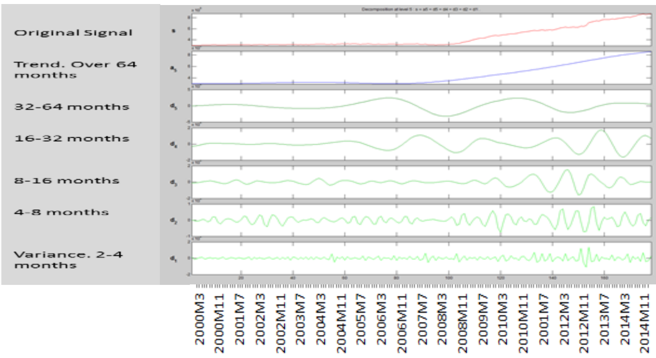


Figure 7(a). WMRD db8, level 5, to SSD

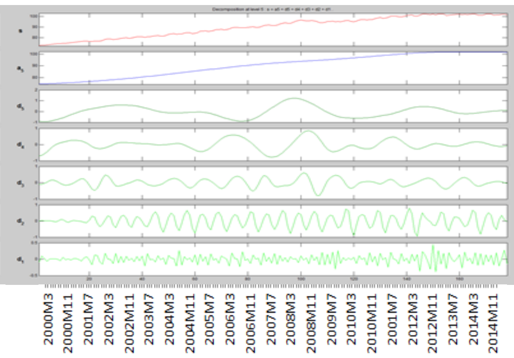


Figure 8(a). WMRD db8, level 5, to CPI

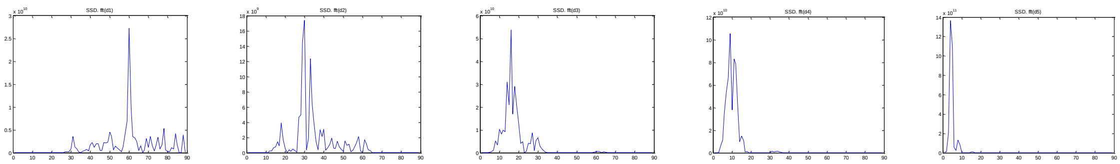


Figure 7(b). FFT to detail series SSD

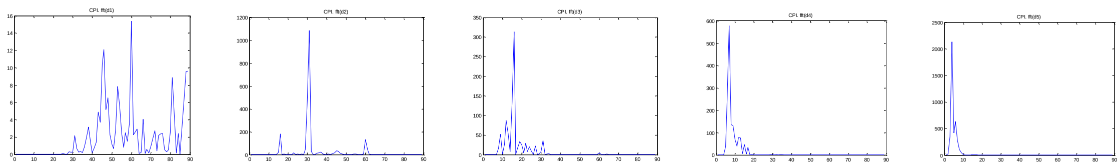
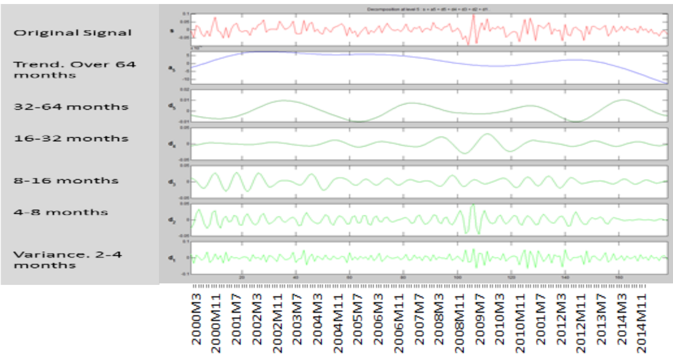


Figure 8(b). FFT to detail series CPI

	EUR/USD					IBEX-35					S10YB				
db8	d_1	d_2	d_3	d_4	d_5	d_1	d_2	d_3	d_4	d_5	d_1	d_2	d_3	d_4	d_5
Most relevant frequency index	58	37	14	10	4	49	28	13	8	5	55	30	13	10	4
Monthly period	3.1	4.8	12.7	17.8	44.5	3.6	6.4	13.7	22.2	35.6	3.2	5.9	13.7	17.8	44.5
	COP					**SSD**					**CPI**				
db8	d_1	d_2	d_3	d_4	d_5	d_1	d_2	d_3	d_4	d_5	d_1	d_2	d_3	d_4	d_5
Most relevant frequency index	72	32	16	9	6	60	30	16	9	4	60	31	16	7	4
Monthly period	2.5	5.6	11.1	19.7	29.7	3.0	5.9	11.1	19.7	44.5	3.0	5.7	11.1	25.4	44.5

Table 1. Relevant Frequency Indices of detail series

Next, in Figure 9-14 (a) we illustrate a similar analysis for the Monthly Rates of the selected variables, which are also non-stationary. We also present the FFT of their five detail series (Figure 9-14 (b)) and the Relevant Frequency Indices (Table 2).



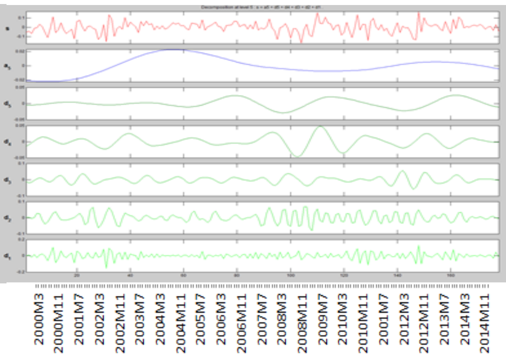


Figure 9(a). WMRD db8, level 5, to EUR/USD MR Figure 10(a). WMRD db8, level 5, to IBEX35 MR

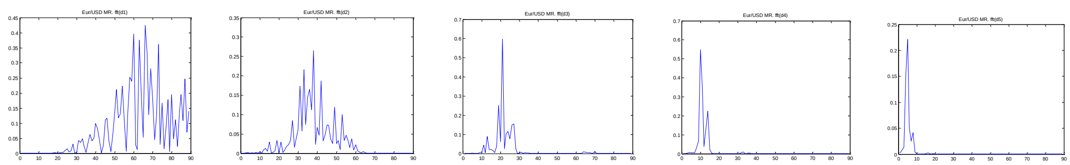


Figure 9(b). FFT to detail series EUR/USD MR

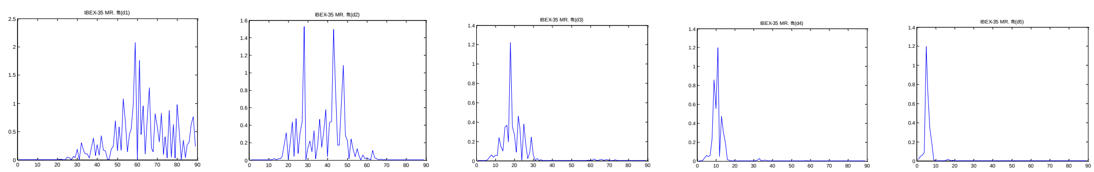
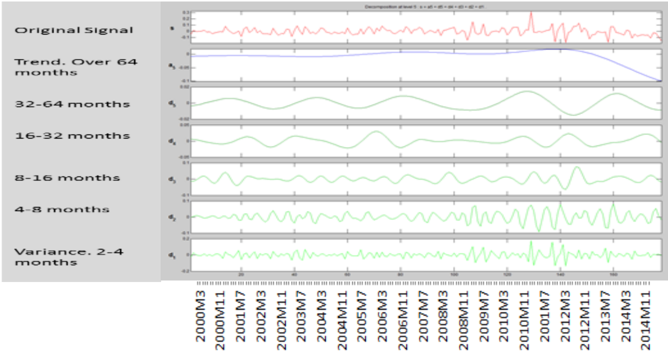


Figure 10(b). FFT to detail series IBEX35 MR



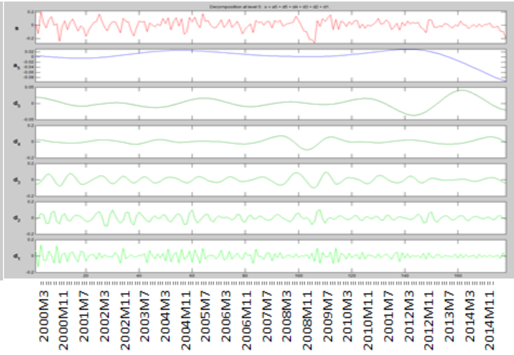


Figure 11(a). WMRD db8, level 5, to S10YB MR

Figure 12(a). WMRD db8, level 5, to COP MR

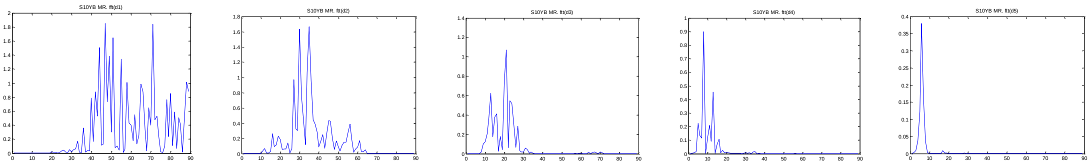


Figure 11(b). FFT to detail series S10YB MR

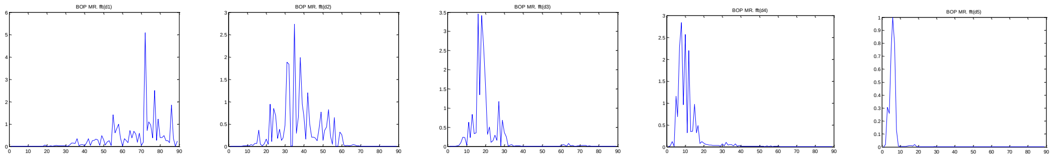
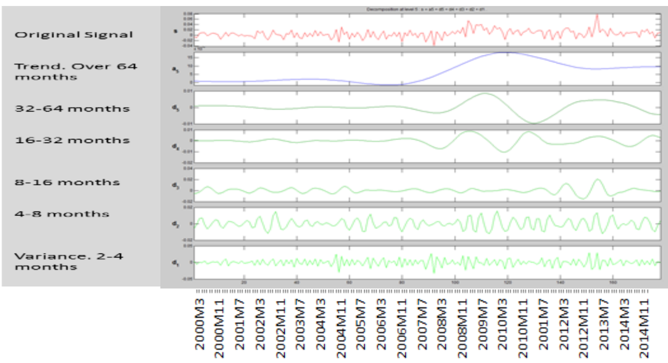


Figure 12(b). FFT to detail series COP MR



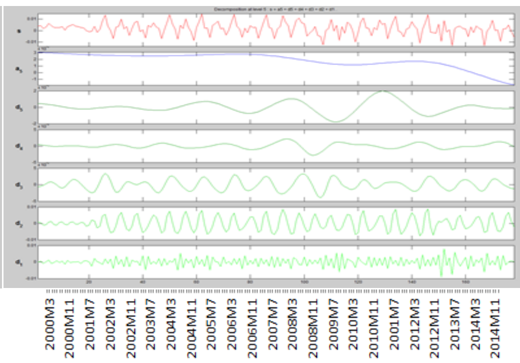


Figure 13(a). WMRD db8, level 5, to SSD MR

Figure 14(a). WMRD db8, level 5, to CPI MR

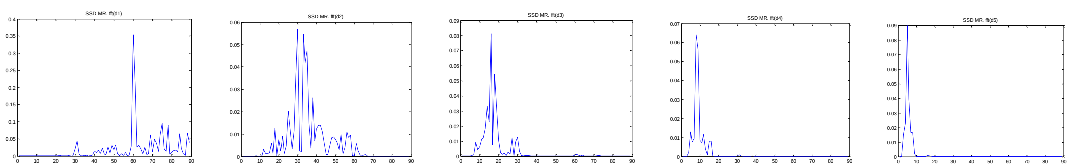


Figure 13(b). FFT to detail series SSD MR

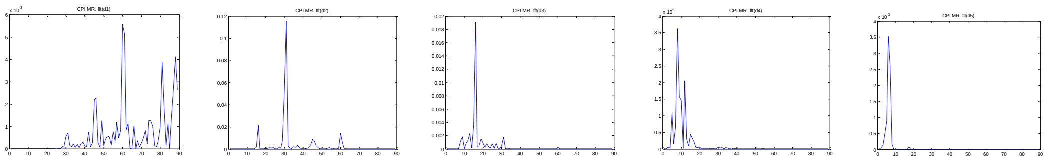


Figure 14(b). FFT to detail series CPI MR

	Eur/USD MR					**IBEX-35 MR**					**S10YB MR**				
db8	**d_1**	**d_2**	**d_3**	**d_4**	**d_5**	**d_1**	**d_2**	**d_3**	**d_4**	**d_5**	**d_1**	**d_2**	**d_3**	**d_4**	**d_5**
Most relevant frequency index	66	38	21	10	5	59	28	18	11	5	47	35	21	8	6
Monthly period	2.7	4.7	8.5	17.8	35.6	3.0	6.4	9.9	16.2	35.6	3.8	5.1	8.5	22.2	29.7
	COP MR					**SSD MR**					**CPI MR**				
db8	**d_1**	**d_2**	**d_3**	**d_4**	**d_5**	**d_1**	**d_2**	**d_3**	**d_4**	**d_5**	**d_1**	**d_2**	**d_3**	**d_4**	**d_5**
Most relevant frequency index	72	35	16	8	6	60	30	16	8	5	60	31	16	8	6
Monthly period	2.5	5.1	11.1	22.2	29.7	3.0	5.9	11.1	22.2	35.6	3.0	5.7	11.1	22.2	29.7

Table 2. Relevant Frequency Indices of Monthly Rates Series

5 Conclusions

We have studied the dynamics of six relevant economic variables in Spain by decomposing the original data series and their monthly rates over the period 2000M3-2014M12, using Daubechies wavelets of order 8 (db8) and level 5. This is a way of showing the importance of the variance or short term (detail level d_1), medium term (detail level d_2-d_5) and trend or long term (a_5). In general, higher variances are observed in the short term and significant differences between the interval before 2008 and the other after it, in the medium and long terms. The series are not stationary and the most relevant index indicates approximate multiples of quarterly periods (3 months).

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