DOI: 10.20472/IAC.2016.023.103

GLORIA GHENO

Ca'Foscari University of Venice and ECLT, Italy

MEDIATION IN LOG-LINEAR MODEL

Abstract:

The analysis of the causality is important in many fields of research. I propose a causal theory to obtain the causal effects in a causal loglinear model. It calculates them using the odds ratio and the concepts proposed by Pearl's causal theory where it is possible. My analysis can be divided into 2 parts. In the first part the effects are calculated distinguishing between a simple mediation model with 1 mediator (model without the multiplicative interaction effect between exogenous variable and mediator) and a mediation model with 1 mediator and the multiplicative interaction effect between exogenous variable and mediator. In both models it is possible also to analyze the cell effect, which is a new interaction effect. Then in a causal loglinear model there are three interaction effects: multiplicative interaction effect, additive interaction effect and cell effect. In the second part the effects are calculated distinguishing between a mediation model with 2 parallel uncorrelated mediators and a mediation model with 2 parallel correlated mediators. In parallel mediation model with correlated influencing variables, Pearl’s theory cannot be used (Pearl, 2014) and and no alternative theory has been proposed. For this reason I propose a new causal concept with relatively formulas to calculate the causal effects in a mediation model with 2 parallel correlated mediators. These types of models are many important in marketing field: for example in customer satisfaction it is important to analyze a model where quality influences the positive and negative emotions and these 3 variables influence the future behavior. Then I show some applications of my causal theory to understand marketing problems.

Keywords:

customer satisfaction, direct effect, indirect effect, interaction, loglinear model, mediation model, parallel mediators, total effect

JEL Classification: C30, C39, M31

Introduction

The analysis of the causality is important in many fields of research, for example in economics and in social sciences, because the analyst seeks to understand the mechanisms of the analyzed phenomena using the relations among the variables (i.e. the relations cause-effect, where some variables are the causes, other variables the effects). The variables can influence in causal way directly, indirectly or in both ways other variables. The set of all causal effects which influence a variable is called "Total effect". The direct effect is, instead, the causal effect of a variable on another variable without any intervening variables, while the indirect effect is the causal effect of a variable on another variable considering only the effect through the intervention of other variables, called mediators. Wright (1921) defines a diagram, called path diagram, to represent the relations among the variables. In the path diagram, the causal direct relation between 2 variables is represented by an arrow which goes from the influencing variable to the influenced variable while the correlation between 2 variables is represented by a double arrow. If 2 variables are not connected, then there is not direct relation between them. To explain better the direct, indirect and total effects then I use the first path diagram represented in Figure 1: the arrow, which goes from X to Y, represents the direct effect of X on Y; the set of the two arrows, which go from X and Z to Z and Y, represents the indirect effect of X on Y through Z and the arrow, which goes from Z to Y, represents the direct effect of Z on Y. Then the indirect effect is the causal effect of X on Y mediated by Z. An analyst, then, who is interested in the variable Y, will be interested to understand what affects Y and then he will study the direct, indirect and total effects.

It is possible to complicate these effects by introducing the concept of interaction. The interaction occurs when the effect of one cause-variable may depend in some way on the presence or absence of another cause-variable and vice versa. In literature the interaction effect can be measured on additive or multiplicative scale and in many case induces that the effect of one variable on another varies by levels of a third and vice versa. The second path diagram of Figure 1 shows a model with interaction, where X and Z influence directly Y but also their joint effect XZ influences Y. Another possible complication is linked to the number of the mediators in a mediation model. The path diagrams of models with 2 mediators Z and W are shown in Figure 2. Hayes (2013) calls ”parallel multiple mediators model” the first model of Figure 2 while he calls ”serial multiple mediators model” the second model of Figure 2.

If the parallel mediators Z and W are correlated (double arrow in Figure 2), according to Pearl (2014) it is possible to calculate the indirect effect only if the model is linear. For this reason in this article I propose a new method to calculate the causal effects in a nonlinear-in-parameters model as that loglinear using the odds ratio and a modified version of Pearl's causal theory. This new method overcomes the problem of the correlated mediators and that of analyzing the causal effects in a loglinear model. A problem of using the loglinear models is, indeed, the inability to calculate all causal effects and this can be considered its limitation.

Figure 1: Simple mediation model with 1 mediator and simple interaction model

Source: Own path diagrams

The remaining part of this article is organized as follows. Section 2 introduces the loglinear model and its causal version. Section 3 describes my causal theory in a simple mediation model as that of Figure 1 and in its version with the addition of the multiplicative interaction term. Section 4 describes my causal theory in a model with uncorrelated and correlated parallel mediators. Section 5 illustrates some applications of my causal theory in marketing field.

Loglinear model and causal loglinear model

Before introducing the method to calculate the effects, I explain the transition from a loglinear model to a causal loglinear model which represents a loglinear model where the variables have a causal role, i.e. for example X becomes the cause and Y the effect. Vermunt (1996), indeed, distinguishes between loglinear models and causal loglinear models. The loglinear model describes the observed frequencies, it does not distinguish between dependent and independent variables and it measures the strength of the association among variables. The causal loglinear model, introduced by Goodman (1973) and also called "modified path analysis approach", is a loglinear model which considers a causal order of the variables a priori. This model, as written by Vermunt (2005), consists of specifying a "recursive" system of logit models where the variable, which appears as dependent in a particular logit equation, may appear as one of the independent variables in one of the next equations. To show the difference between the 2 models, I consider a model with 3 categorical variables X, Z and Y. In the loglinear model the joint probability of these 3 variables is written so:

$$P(X=x, Z=z, Y=y) = \eta \mu^{X=x} \mu^{Z=z} \mu^{Y=y} \mu^{X=x,Y=y} \mu^{X=x,Z=z} \mu^{Z=z,Y=y} \mu^{X=x,Z=z,Y=y} \quad (1)$$

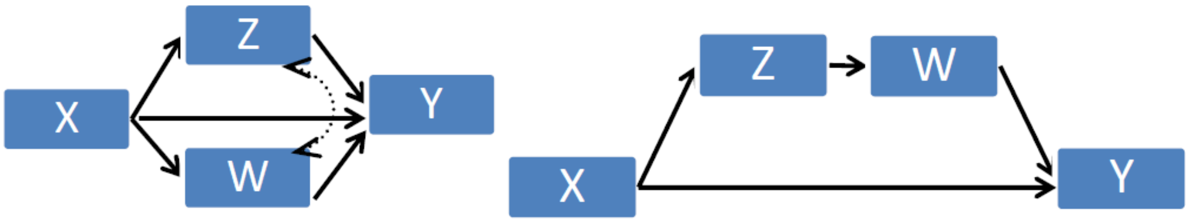
Now I suppose that X, Z and Y are binary (0 or 1) and I constraint the parameters using the dummy code criterion so that the parameters are identified, that is:

 ISBN 978-80-87927-23-6, IISES

$$\mu^{Y=0} = \mu^{X=0} = \mu^{Z=0} = 1$$
$$\mu^{Z=0,Y=0} = \mu^{X=0,Y=0} = \mu^{Z=0,Y=1} = \mu^{X=0,Y=1} = \mu^{Z=1,Y=0} = \mu^{X=1,Y=0} = 1$$
$$\mu^{X=0,Z=0} = \mu^{X=0,Z=1} = \mu^{X=1,Z=0} = 1$$
$$\mu^{X=0,Z=0,Y=i} = \mu^{X=0,Z=1,Y=i} = \mu^{X=1,Z=0,Y=i} = \mu^{X=1,Z=1,Y=0} = 1 \quad i = 0,1 \tag{2}$$

To transform a loglinear model in a casual loglinear model, considering the causal

Figure 2: parallel multiple mediators model and serial multiple mediators model



Source:Own path diagrams

order of the first model of Figure 1, I must suppose that the three-interaction term $\mu^{X=1,Z=1,Y=1}$ is equal to 1 because, if it is present, it introduces the causal multiplicative interaction term of X and Z on Y. The presence or absence of this parameter, indeed, brings about the presence or absence of the multiplicative interaction. Following the probability structure proposed by Goodman (1973), the causal model of a simple mediation model can be written as a decomposition of the joint probability into conditional probabilities, i.e. P(X,Z,Y)=P(Y|Z,X) P(Z|X)P(X). Then if I write the simple mediation model of Figure 1 in causal loglinear terms, it becomes:

$$P(X = x) = \frac{\mu_c^{X=x}}{1 + \mu_c^{X=1}} = \eta_c^X \mu_c^{X=x} \tag{3}$$
$$P(Z = z|X = x) = \frac{\mu_c^{Z=z}\mu_c^{X=x,Z=z}}{1 + \mu_c^{Z=1}\mu_c^{X=x,Z=1}} = \eta_c^{Z|X=x}\mu_c^{Z=z}\mu_c^{X=x,Z=z}$$
$$P(Y = y|X = x, Z = z) = \frac{\mu^{Y=y}\mu^{X=x,Y=y}\mu^{Z=z,Y=y}}{1 + \mu^{Y=1}\mu^{X=x,Y=1}\mu^{Z=z,Y=1}} = \eta^{Y|X=x,Z=z}\mu^{Y=y}\mu^{X=x,Y=y}\mu^{Z=z,Y=y}$$

where c denotes the causal loglinear parameters and η the normalization factor. The causal loglinear parameters are estimated by the conditional probabilities while the loglinear parameters by the joint probability. The parameters of P(Y=y|X=x,Z=z) are the only which remain equal in both models.

Causal theory in a mediation model with 1 mediator

In the loglinear literature the causal effects are considered in partial way and for this reason a true causal analysis is not made. If the simple mediation model of Figure 1 is considered, the loglinear literature (Bergsma et al., 2009) calculates, using the odds ratio, the total effect and the direct effect but it does not consider the indirect effect. The odds ratio describes the relationship between 2 binary variables; if the variables are categorical, it is necessary a transformation in binary variables to use it. For example if I want to analyze the relation between X and Y, which are categorical variables with 5 categories, I transform them in binary variables: the transformed X and Y are equal to 1 if their original value is 5, 0 otherwise. The relationships considered by the odds ratios can be associative or causal (Zhang, 2008): in the first type the relation is measured using the actual response variable, while in the second using the potential response. If the two types of odds ratio are different, this is due to the influence of a third variable called confounding variable (Zhang, 2008; Szumilas, 2010). This confounding variable is causally linked to the response variable but it is not related causally to other cause or it is linked causally but it is not a mediator variable (Szumilas, 2010). To explain better, I make an example. If the 2 correlated variables X and Z influence causally Y, Z is a confoundering variable of the relation between X and Y (recalling that the correlation is not a causal link). In a simple mediation model without confounders, the total effect (TE) and the direct effect used in the loglinear literature (LDE) are given by the following formulas:

$$Or^{TE}_{x^0,x^1} = \frac{P(Y=1|X=x^1)}{1-P(Y=1|X=x^1)}\frac{1-P(Y=1|X=x^0)}{P(Y=1|X=x^0)} \tag{4}$$

$$Or^{LDE}_{x^0,x^1}(Z) = \frac{P(Y=1|X=x^1,Z=z)}{1-P(Y=1|X=x^1,Z=z)}\frac{1-P(Y=1|X=x^0,Z=z)}{P(Y=1|X=x^0,Z=z)} \tag{5}$$

where the subscript (x^0,x^1) indicates that the causal effect measures the effect of the variation of X from x^0 to x^1. In the next part of my analysis I consider $x^0=0$ and $x^1=1$. I note that these causal effects coincide with the definitions of total effect and of controlled direct effect proposed by Pearl (2009, 2012, 2014). He, however, never uses the odds ratio to calculate the causal effects, but he prefers to calculate them using the conditional moments. I propose, for this reason, a causal analysis for the loglinear models, applying, when it is possible, Pearl's causal theory and the odds ratio. I note that the direct effect is always equal to the causal two-effects parameter $\mu^{X=1,Y=1}$, i.e. it is independent of the value of Z. If in a linear-in-parameters model without interaction the variable X and the variable Z influence Y but X does not influence Z, the total effect of X on Y is equal to the direct effect of X on Y. This is not true in a loglinear model without interaction: I find, indeed, that, when $\mu^{X=1,Z=1,Y=1}$ is equal to 1, the total effect is not equal to the direct effect, but there is another effect, which I call cell effect. The cell effect is present only if more variables influence directly the same variable, as in this case where X and Z influence Y. If there is not the direct effect between X and Y ($\mu^{X=1,Y=1}$=1) or between Z and Y ($\mu^{Z=1,Y=1}$= 1), the cell effect becomes equal to 1 and the total effect is equal to the direct effect of Z on Y or of X on Y. The cell effect formula is equal to:

$$Cell_{x^0,x^1}^{effect}(Z) = \left[\frac{\sum_Z P(Y=1|X=x^1,Z=z)P(Z=z|X=x^0)}{1-\sum_Z P(Y=1|X=x^1,Z=z)P(Z=z|X=x^0)}\frac{1-P(Y=1|X=x^0)}{P(Y=1|X=x^0)}\right] \left[\frac{P(Y=1|X=x^1,Z=z)}{1-P(Y=1|X=x^1,Z=z)}\frac{1-P(Y=1|X=x^0,Z=z)}{P(Y=1|X=x^0,Z=z)}\right]^{-1} \quad (6)$$

In this simple mediation model the cell effect does not depend on Z and then I can write $Cell^{effect}(Z) = Cell^{effect}$, i.e. the cell effect can be interpreted as a constant interaction effect (this is not true in a loglinear model with multiplicative interaction). The total effect and the direct effect used in the loglinear literature are the odds ratio versions of the total effect and the controlled direct effect proposed by Pearl (2009, 2012, 2014), for this reason I propose the odds ratio version of his indirect effect:

$$Or_{x^0,x^1}^{IE} = \left[\frac{\sum_Z P(Y=1|X=x^0,Z=z)P(Z=z|X=x^1)}{1-\sum_Z P(Y=1|X=x^0,Z=z)P(Z=z|X=x^1)}\right]\left[\frac{1-P(Y=1|X=x^0)}{P(Y=1|X=x^0)}\right] \quad (7)$$

and of his decomposition of the total effect

$$Or_{x^0,x^1}^{TE} = \underbrace{Or_{x^0,x^1}^{LDE}(Z)Cell_{x^0,x^1}^{effect}(Z)}_{Or_{x^0,x^1}^{NDE}}\frac{1}{Or_{x^1,x^0}^{IE}} \quad (8)$$

where I add my decomposition of Pearl's natural direct effect in cell effect and LD effect. Pearl, indeed, proposes 2 direct effects: the natural direct effect and the controlled direct effect. The first is the change of Y when X changes and Z is constant at whatever value obtained by the start value of X, while the second is the change of Y when X changes and all other factors are held fixed.

Now I consider a mediation model with 1 mediator and with multiplicative interaction. Then X influences directly Z and Y is influenced directly by the variable X, by the variable Z and by their joint effect due to the three-interaction term $\mu^{X=1,Z=1,Y=1}$. The formulas (4), (5), (6), (7) and (8) remain valid to calculate the causal effects. The direct effect of X on Y used in the loglinear literature becomes a function of Z, i.e. $\mu^{X=1,Y=1}$ $\mu^{X=1,Z=z,Y=1}$. For the same reason also the cell effect becomes a function of Z. The natural direct, indirect and total effects, instead, do not become function of Z. The indirect effect of a model with multiplicative interaction remains equal to that of a model without multiplicative interaction.

Causal theory in a mediation model with 2 parallel mediators

I complicate the simple mediation model adding another mediator in parallel way so I obtain the first model of Figure 2, where the causal effect of the variable X is mediated by 2 mediators Z and W and where there are not interactions, i.e. $\mu^{X=1,Z=1,Y=1}$ = $\mu^{X=1,W=1,Y=1}$ = $\mu^{Z=1,W=1,Y=1}$ = $\mu^{X=1,Z=1,W=1,Y=1}$ =1. Following Goodman (1973), the joint probability becomes equal to P(Y|Z,W,X)P(Z,W|X)P(X) and the loglinear representation of this model is

$$P(X=x)=\eta_c^{X}\mu_c^{X=x}$$
$$P(Z=z,W=w|X=x)=\eta_c^{Z|X=x}\mu_c^{Z=z}\mu_c^{W=w}\mu_c^{X=x,Z=z}\mu_c^{X=x,W=w}\mu_c^{Z=z,W=w}$$
$$P(Y=y|X=x,Z=z,W=w)=\eta^{Y|X=x,Z=z,W=w}\mu^{Y=y}\mu^{X=x,Y=y}\mu^{Z=z,Y=y}\mu^{Z=z,Y=y} \quad (9)$$

where $\mu_c^{Z=z,W=w}$ measures the correlation of the 2 mediators. If the 2 mediators are uncorrelated ($\mu_c^{Z=1,W=1}$=1), the causal effects can be still calculated using the odds ratio version of Pearl's causal theory. Unlike Pearl's causal theory (2014), I prefer to consider the indirect effect of the 2 mediators together, also if the indirect effect for any mediator can be calculated using the formula (7). The formulas of the total effect remain equal to those of the model with 1 mediator (formulas (4) and (8)). The direct effect proposed in literature, the cell effect and the indirect effect become respectively:

$$Or_{x^0,x^1}^{LDE}(Z)=\frac{P(Y=1|X=x^1,Z=z,W=w)}{1-P(Y=1|X=x^1,Z=z,W=w)}\frac{1-P(Y=1|X=x^0,Z=z,W=w)}{P(Y=1|X=x^0,Z=z,W=w)} \quad (10)$$

$$Cell^{effect}_{x^0,x^1}(Z) = \tag{11}$$

$$\left[\frac{\sum_{Z,W} P(Y=1|X=x^1, Z=z, W=w) P(Z=z, W=w|X=x^0)}{1-\sum_{Z,W} P(Y=1|X=x^1, Z=z, W=w) P(Z=z, W=w|X=x^0)} \frac{1-P(Y=1|X=x^0)}{P(Y=1|X=x^0)}\right]$$

$$\left[\frac{P(Y=1|X=x^1, Z=z, W=w)}{1-P(Y=1|X=x^1, Z=z, W=w)} \frac{1-P(Y=1|X=x^0, Z=z, W=w)}{P(Y=1|X=x^0, Z=z, W=w)}\right]^{-1}$$

$$Or^{IE}_{x^0,x^1} = \left[\frac{\sum_{Z,W} P(\mathrm{Y}=1|\mathrm{X}=\mathrm{x}^0, \mathrm{Z}=\mathrm{z}, \mathrm{W}=\mathrm{w}) P(Z=z, W=w|X=x^1)}{1-\sum_{Z,W} P(Y=1|X=x^0, Z=z, W=w) P(Z=z, W=w|X=x^1)}\right] \tag{12}$$

$$\left[\frac{1-P(Y=1|X=x^0)}{P(Y=1|X=x^0)}\right]$$

I note that the direct effect proposed in literature remains equal to $\mu^{X=1,Y=1}$ as in the model with 1 mediator and without multiplicative interaction. If I calculate the causal effects with these formulas in a model with correlated mediators, I would give a causal importance also to the correlation and this is wrong. For this reason I propose a modification to eliminate the role of the correlation. I start recalling that the conditional probability of the variables Z and W given X is equal to

$$P(Z=z, W=w|X=x) = \eta_c^{Z,W|X=x} \mu_c^{Z=z} \mu_c^{W=w} \mu_c^{X=x,Z=z} \mu_c^{X=x,W=w} \mu_c^{Z=z,W=w} \tag{13}$$

Then I propose the elimination of the parameter $\mu_c^{Z=z,W=w}$, which measures the correlation, so that I consider only the causal relations and P(Z,W|X) becomes

$$\tilde{P}(Z=z, W=w|X=x) = \tilde{\eta}_c^{Z,W|X=x} \mu_c^{Z=z} \mu_c^{W=w} \mu_c^{X=x,Z=z} \mu_c^{X=x,W=w} \tag{14}$$

where the mu parameters are the same of the formula (13). Substituting in the previous formulas to P(Z,W|X) its new version, called uncorrelated conditional probability, I obtain the formulas to calculate the causal effects when the mediators are correlated. These new formulas are:

$$\widetilde{\mathrm{O\,r}}^{TE}_{x^0,x^1} = \frac{\tilde{P}(Y=1|X=x^1)}{1-\tilde{P}(Y=1|X=x^1)}\frac{1-\tilde{P}(Y=1|X=x^0)}{\tilde{P}(Y=1|X=x^0)} \qquad (15)$$

$$\widetilde{\mathrm{O\,r}}^{TE}_{x^0,x^1} = \underbrace{Or^{LDE}_{x^0,x^1}(Z)\widetilde{Cell}^{effect}_{x^0,x^1}(Z)}_{\widetilde{\mathrm{O\,r}}^{NDE}_{x^0,x^1}}\frac{1}{\widetilde{\mathrm{O\,r}}^{IE}_{x^1,x^0}} \qquad (16)$$

$$\widetilde{\mathrm{O\,r}}^{IE}_{x^0,x^1} = \left[\frac{\sum_{Z,W} P(Y=1|X=x^0,Z=z,W=w)\tilde{P}(Z=z,W=w|X=x^1)}{1-\sum_{Z,W} P(Y=1|X=x^0,Z=z,W=w)\tilde{P}(Z=z,W=w|X=x^1)}\right] \qquad (17)$$

$$\left[\frac{1-\tilde{P}(Y=1|X=x^0)}{\tilde{P}(Y=1|X=x^0)}\right]$$

$$\widetilde{Cell}^{effect}_{x^0,x^1}(Z) = \qquad (18)$$

$$\left[\frac{\sum_{Z,W} P(Y=1|X=x^1,Z=z,W=w)\tilde{P}(Z=z,W=w|X=x^0)}{1-\sum_{Z,W} P(Y=1|X=x^1,Z=z,W=w)\tilde{P}(Z=z,W=w|X=x^0)}\frac{1-\tilde{P}(Y=1|X=x^0)}{\tilde{P}(Y=1|X=x^0)}\right]$$

$$\left[\frac{P(Y=1|X=x^1,Z=z,W=w)}{1-P(Y=1|X=x^1,Z=z,W=w)}\frac{1-P(Y=1|X=x^0,Z=z,W=w)}{P(Y=1|X=x^0,Z=z,W=w)}\right]^{-1}$$

where $\tilde{P}(Y|X) = \sum_{Z,W} P(Y|X,Z,W)\tilde{P}(Z,W|X)$. I note that only the formula of the direct effect proposed in literature remains unchanged, while all other formulas of the causal effects change if the mediators are correlated.

Numerical studies

In this section I apply my causal theory to empirical datasets where the variables are binary (0=low value, 1=high value). The first 2 examples consider the relations among a typical product (in this case Sauris' ham), the satisfaction about its festival and the customer future behavior. The third example considers the relation among the quality of a fast food (in this case Mc Donald), the positive and negative emotions of a customer and his future behavior. This analysis is developed in marketing but it can be applied in many other economic fields or in social sciences.

The first dataset is composed of 3 dichotomous variables (X measures the interest about Sauris' ham considering the possibility of buying Sauris' ham, Z measures the satisfaction about Sauris' festival considering the happiness which an individual has if he thinks about Sauris' festival and Y measures the future behavior considering if an individual will buy Sauris's ham more often). The estimated parameters of the causal loglinear model are shown in Table 1. The two-effects parameters are all significant (i.e. all are different from 1). According to the traditional loglinear literature, the causal two-effects parameters are the causal direct effects. In this case, because all causal two-effects parameters are greater than 1, an increase of the variable X produces an increase of the variable Z and the same result occurs for the relation between X and Y and for that between Z and Y. Now I calculate the effects using the formulas (5), (6), (7) and (8). The total effect is equal to 2.4008, then an increase of X produces an increase of Y while the indirect effect is equal to 1.2845: an increase of X produces, indirectly, an increase of Y. The cell effect is equal to 0.9741 and for this reason it mitigates the controlled direct effect: the presence of 2 variables, which influence Y, decreases the direct effect of X on Y, which becomes equal to 1.8741.

Table 1: causal loglinear parameters

	First dataset	Second dataset	Third dataset
$\mu^{X=1,Z=1,Y=1}$		2.8826*	
$\mu^{X=1,Y=1}$	1.9240**	1.4042	3.9711***
$\mu^{Z=1,Y=1}$	2.4038***	3.5385**	2.9848**
$\mu^{W=1,Y=1}$			9.3261***
$\mu^{Y=1}$	0.4881***	0.2826***	0.0408***
$\mu_c^{X=1,Z=1}$	3.3059***	3.5534***	2.5099***
$\mu_c^{X=1,W=1}$			2.8914**
$\mu_c^{Z=1}$	0.4659***	0.3390***	0.2198***
$\mu_c^{W=1}$			0.0362***
$\mu_c^{X=1}$	1.7132***	1.2278°	0.1832***
Signif. Codes: 0”***” 0.001”**” 0.01”*” 0.05”°” 0.1”” 1			

Source: Data used as examples in Gheno (2011) and in Gheno (2015)

Then also for the natural direct effect an increase of X produces an increase of Y. From this analysis, I conclude that if a customer becomes interested in Sauris' ham, he will buy Sauris' ham more often also thanks to the happiness due to Sauris' festival. In marketing research, this means that an event linked to the product can increase its sell. The role of the event is minus important than the role of the interest about the product (indirect effect/ total effect < direct effect/total effect) and their joint effect decreases the direct effect used in the loglinear literature (cell effect < 1).

Now I consider a second dataset. This dataset is composed of 3 dichotomous variables (X measures the interest about Sauris’ ham considering the possibility of testing Sauris’ ham, Z measures the satisfaction about Sauris’ festival considering the quality of products presented during Sauris’ festival and Y measures the future behavior considering if an individual will suggest to go to Sauris’ festival). The values of parameters are shown in Table 1. It is necessary to consider $\mu^{X=1,Y=1}$,also if it is not significant, because $\mu^{X=1,Z=1,Y=1}$ is significant. The total effect is equal to 3.1886, then an increase of X produces an increase of Y. The indirect effect is equal to 1.4493: an increase of X produces, indirectly, an increase of Y. Now I consider the effect of the multiplicative interaction, whose parameter is bigger than 1 and which influences the LD effect and the cell effect. The cell effect is 0.42705 with Z=1, i.e. it mitigates the LD effect, while it is 1.2310 with Z=0, i.e. it increases the LD effect. When Z is high (Z=1), the LD effect is equal to 4.0477 (i.e. $\mu^{X=1,Y=1}$ $\mu^{X=1,Z=1,Y=1}$) while when Z is low (Z=0), it is equal to 1.4042 (i.e. $\mu^{X=1,Y=1}$). For any value of satisfaction, then, the overall interaction effect (cell effect (Z) x $\mu^{X=1,Z=z,Y=1}$) is always equal to 1.2310 and for this reason the natural direct effect is always equal to 1.7286. From this analysis, I conclude that if a customer becomes interested in Sauris' ham, then he will suggest to go to Sauris' festival more often thanks also to the quality of the presented products and to the overall joint effect of the interest and of the satisfaction.

The third dataset is composed of 4 dichotomous variables (X measures McDonald atmosphere considering the music in the fast food, Z measures the positive emotions, W measures the absence of negative emotions and Y measures the will of returning of the customer). I do not consider the interactions. The parameter, which determines the presence of the correlation between Z and W, is significant ($\mu_c^{Z=1,W=1}$= 4.1166, p-value=0) and for this reason I apply the formulas (15), (16), (17) and (18). The remained parameters are significant and they are shown in Table 1. In this case, because all causal two-effects parameters are greater than 1, an increase of the influencing variables produces an increase of the influenced variables according to the traditional loglinear literature. The total effect is equal to 5.1348, the indirect effect is equal to 1.5664, the cell effect is equal to 0.9035 and then the natural direct effect is smaller than the direct effect present in literature (3.5879<3.9711). From this analysis I conclude that a good atmosphere of the fast food (quality of the local) influences positively both directly and indirectly the return of the clients.

Conclusions

When a researcher analyzes the data, he is interested in understanding the mechanisms which govern the changes of the variables. To know these mechanisms he uses the causal effects. When the researcher uses the loglinear models to study the data, unfortunately he has not available a causal theory, but only few comments on various papers where the odds ratios are used. For this reason, using the causal concepts provided by Pearl (2009, 2012, 2014), I provide a causal theory to calculate the effects in the loglinear models using the odds ratio so that the parameters have the same interpretation given by the loglinear literature. Making so I find a new effect which I call cell effect. It can be interpreted as an interaction effect which occurs whenever I consider two variables affecting directly a third. Another problem which is present in the loglinear model is linked to the calculus of the indirect effect when the parallel mediators are correlated. To solve this problem I propose a new concept which I call uncorrelated conditional probability. Substituting this in the formulas for the model with uncorrelated mediators, I can calculate the causal effects eliminating the problem of the correlation.

References

BERGSMA, W. P., CROON, M. A., & HAGENAARS, J. A. (2009). *Marginal models for dependent, clustered, and longitudinal categorical data*. New York: Springer.

GHENO, G. (2011). *Modelli strutturali con variabili categoriali: un approccio a classi latenti*. Master Thesis, University of Padua.

GHENO, G. (2015). S*tructural equation models with interacting mediators: theory and empirical results*. Phd Thesis, University of Padua.

GOODMAN, L. A. (1973). The analysis of multidimensional contingency tables when some variables are posterior to others: a modified path analysis approach. *Biometrika* 60(1), pp.179–192.

HAYES, A. F. (2013). *Introduction to mediation, moderation and conditional process analysis:a regression-based approach*, New York: The Guilford press.

PEARL, J. (2009). Causal inference in statistics: An overview. *Statistics Surveys* 3, pp. 96–146.

PEARL, J. (2012). The mediation formula: a guide to the assessment of causal pathways in nonlinear models. In C. Berzuini, P. Dawid, and L. Bernardinelli, ed. *Causality: Statistical Perspectives and Applications*, Chirchester: John Wiley & Sons, pp. 151–179.

PEARL, J. (2014). Interpretation and identification of causal mediation. *Psychological Methods* 19, pp. 459-481

SZUMILAS, M. (2010). Explaining odds ratios. *Journal of the Canadian Academy of Child and Adolescent Psychiatry* 19(3), pp. 227–229.

VERMUNT, J. K. (1996). Causal log-linear modeling with latent variables and missing data. In U. Engel and J. Reinecke ed. *Analysis of Change: Advanced Techniques in Panel Data Analysis*, Berlin/New York: Walter de Gruyter, pp. 35–60.

VERMUNT, J. K. (2005). Log-linear models. In B. Everitt and D. Howell ed. *Encyclopedia of Statistics in Behavioral Science*, Chichester: Wiley, pp. 1082–1093

WRIGHT, S. (1921). Correlation and causation. *Journal of agricultural research* 20(7), pp. 557–585

ZHANG, Z. (2008). Estimating a marginal causal odds ratio subject to confounding. *Communications in Statistics-Theory and Methods* 38(3), pp. 309–321.