PAWEŁ KLIBER

Poznan University of Economics, Poland

PORTFOLIO ANALYSIS IN JUMP-DIFFUSION MODEL WITH POWER-LAW TAILS

Abstract:

The classic portfolio analysis given by Markowitz theory and Capital Asset Pricing Model is based on the assumption that the assets' returns are normally distributed. In this situation one can use only two criteria: expected return and variance of return as the measures of possible gains and risk, respectively. However there is a growing evidence that the assets' returns and in particular returns of shares in the stock markets fail to obey Gaussian distribution. Therefore different measures of risk should be considered.

In the paper we analyze the portfolio problem in the situation when stock prices follows jump-diffusion model with the tails of jumps obeying power-law. We consider a portfolio problem with two risk criteria: risk in the situation of normal market circumstances and the risk of jumps. We propose a method for numerical computing the former risk using Fast Fourier Transform (FFT). Finally we present the examples of portfolio analysis with the new method for the shares from Warsaw Stock Market Exchange.

Keywords:

portfolio analysis, jump-diffusion models, power-law, risk of extremes, Fast Fourier Transform

JEL Classification: G11, C61, C58

1   Introduction

The classic portfolio analysis, proposed in (Markowitz, 1952) and developed by Sharpe (1963) into Capital Asset Pricing Model, is based on the assumption that assets returns are normally distributed. This assumption is sometimes made only implicitly. For example, in the Markowitz approach it is assumed that investor take into account two criteria: potential profitability, measured by expected value, and risk, measured by variance. These two characteristics would provide a full description of distribution of returns, if this distribution was Gaussian. In other cases usually more characteristics are needed.

It is also well-known from many empirical studies, that the true distribution of assets returns is not Gaussian. Moreover, prices usually do not follow continuous changes. As it was indicated for example in (Cont, Tankov, 2006), (Jondeau et.al., 2007) or (Malevergne, Sornette, 2006), it is a stylized fact in the finance, that asset returns are not normally distributed (usually their distributions have excessive kurtosis and skewness) and that there are discontinuities ("jumps") in processes of assets prices. A sound theory of portfolio analysis should take these facts into account.

In the paper we analyze the portfolio problem in the situation when stocks prices follow jump-diffusion model with the tails of jumps obeying power-law. We consider a portfolio problem with two risk criteria: risk in the situation of normal market circumstances and the risk of jumps. We propose a method for numerical computing the former risk using Fast Fourier Transform (FFT).

The article is organized as follows. After this short introduction, the brief introduction of jump-diffusion models of asset prices is presented in the section 2. This section contains main mathematical notions connected with this class of models, such as Levy processes, jump measure and Lévy-Itô decomposition. In the third section we present the proposition for augmenting classical portfolio analysis for a criterion connected with the risk of sudden jumps in asset prices. In the section fourth we propose a specific example of jump-diffusion model with the distribution of jumps that obeys power law (two-sided Pareto distribution). This section contains also details concerning numerical computations of the risk criterion. The section five contains empirical example for the portfolios of stocks from Polish market quoted on the Warsaw Stock Exchange. Section six concludes.

2   Jump-diffusion models of asset prices

Lévy process L is a stochastic *cadlag*[1] process which starts at zero and fulfils the following conditions.

- Its increments are independent and stationary, i.e. for any $t_1 < t_2 < \cdots < t_n$ the variables $L_{t_2} - L_{t_1}, L_{t_3} - L_{t_2} \ldots, L_{t_n} - L_{t_{n-1}}$ are independent and the distribution of $L_{t+h} - L_t$ depends only on h and not on t.

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[1] That is its trajectories are right-continuous and have left limits (fr. - *continue à droite, limite à gauche*, see for example [Schryaev, 1999]).

- The process is stochastically continuous, that is $\forall \varepsilon > 0 \; \underset{h \to 0}{lim}(|L_{t+h} - L_t| \geq \varepsilon) = 0$, which means that the jumps of the process are random – the probability that the process jumps at any given moment t equals 0.

The Lévy processes are closely connected with infinitely divisible distributions, i.e. with distributions that can be represented as a sum of n identically distributed random variables (for any n). The infinitely divisible distributions are the broadest class of distributions that can appear in limit theorems concerning sums of independent variables[1]. It is true that the distribution of Lévy process at any moment of time is infinitely divisible[2]. On the other hand – for any infinitely divisible distribution f there exists a Lévy process L such that $L_1 \sim f$. Thus the Lévy processes are the widest class of processes which can be interpreted as a result of many small and independent random increments.

2.1　Lévy-Khinchin representation

According to Lévy-Khnitchin theorem (see [Appelbaum, 2004], [Cont, Tankov, 2004] or [Kyprianou, 2006]) any Lévy process L is completely described by its characteristic exponent, that is by the logarithm of the characteristic function of L_1. We have

$$E[e^{iuL_t}] = e^{t\psi(u)}, \tag{1}$$

where the function ψ (characteristic exponent) is given by

$$\psi(u) = -\frac{1}{2}\sigma^2 u^2 + i\mu u + \int_R \left(e^{iux} - 1 - iux1_{|x|\leq 1}\right)dv(x), \tag{2}$$

where $\sigma^2 \in R_+$, $\mu \in R$, and v is a measure on R (so called Lévy measure) which fulfils

$$\int_{|x|\leq 1} x^2 dv(x) < \infty \text{ and } v(\{x \in R : |x| > 1\}) < \infty. \tag{3}$$

The measure v describes jumps of a process – the value $v(R)$ is the number of jumps in the unit of time. The value $v([c,d])$ denotes relative frequency of jumps which have a size between c and d. If the measure v fulfils

$$\int_{|x|\leq 1} |x|\, dv(x) < \infty, \tag{4}$$

then (2) can be reformulated as

$$\psi(u) = -\frac{1}{2}\sigma^2 u^2 + i\mu u + \int_R \left(e^{iux} - 1\right)dv(x) \tag{5}$$

and μ denotes the drift of the process.

2.2　Lévy-Itô decomposition

According to Lévy-Itô theorem any Lévy process can be decomposed into a sum of a linear trend, a Wiener process (continuous, with Gaussian distribution), a Poisson process of large jumps and a completely discontinuous martingale:

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[1] See (Feller, 1967), chapter XVII.
[2] See for example (Sato, 1999).

$$L_t = \mu t + \sigma W_t + P_t + M_t^d \,, \tag{6}$$

where W is a standard Wiener process, P is a compound Poisson process with jumps in $(-\infty, 1] \cup [1, +\infty)$ and M^d is a completely discontinuous martingale with jumps in $(-1, 1)$. If the Lévy measure fulfils (4), then we can rewrite (6) as

$$L_t = \mu t + \sigma W_t + \sum_{s \leq t} \Delta L_t, \tag{7}$$

where $\Delta L_t = L_t - \lim_{s \to t^-} L_s$. The equations (6) and (7) describe the structure of Lévy processes. According to (6) the process can be decomposed into the sum of continuous Gaussian process and discontinuous jumps. The parameters μ and σ describe trend and volatility of the continuous part of the process.

2.3   Jump-diffusion models

We assume that prices of all considered assets are driven by Lévy processes. The price of the asset i at the moment t is given by the following stochastic differential equation[1]:

$$dS_t^i = S_{t-}^i dL_t^i. \tag{8}$$

The solution is a stochastic exponent of the process L^i and logarithmic returns of the assets have infinitely-divisible distribution. Each process driving the prices is described by its characteristic triple (μ_i, σ_i, v_i), where μ_i and σ_i are, respectively, trend and volatility parameters of the continuous part of the process driving prices of the i-th asset and v_i is its Lévy measure. The continuous parts of the driving processes can be correlated. Let σ_{ij} denote the covariance between Gaussian parts of the processes L^i and L^j. Of course $\sigma_{ii} = \sigma_i^2$.

Models like (8) are usually called jump-diffusion models. Although in the literature this term denotes most often models with finite measure of jumps ($v(R) < \infty$), here we understand this term more broadly. As examples of such models we can indicate Merton model (see [Merton, 1976]) or Kou model (see [Kou, 2002]). In jump-diffusion models the returns of asset are described by three parameters: mean μ, variance of Gaussian part σ^2 and Lévy measure v. This kind of models apply also if distributions of returns are α-stable, have Student distribution or belong to generalized hyperbolic family of distributions (these assumptions are frequent in financial literature and models based on them fit quite well to data, see for example [Mandelbrot, 1997] or [Mandelbrot, Hudson, 2005]).

To fully specify the model one has also to describe the interdependences between jumps of various assets. The most general way to do this is to specify a Lévy measure of jump vector. The Lévy measure v is then a measure on R^n (where n is the number of assets), which can be interpreted as follows: for any $A \subset R^n$ such that $0 \notin A$ the value $v(A)$ is the expected number of jumps (in the unit of time) of all assets, such that $(\Delta L_t^1, \Delta L_t^2, \dots, \Delta L_t^n) \in A$. However this method of describing common jumps is not effective, because one has to specify the measure on n-dimensional space. Kallsen and Tankov (2006) proposed a method based on Lévy copula functions, which describes the dependences in tails of jumps distributions of various assets. There

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[1] Alternatively, one can assume that the assets' prices are given as ordinary exponents of Lévy processes $S_t^i = S_0^i e^{L_t^i}$. As it was shown in (Kallsen, 2000), both approaches are equivalent.

is also another method which assumes that one can divide jumps in returns into idiosyncratic jumps (characteristic for given asset) and market jumps, which affect the whole market. The process L^i, governing the prices of asset i, is then defined as

$$L_t^i = \mu_i t + \sigma_i W_t + U_t^i + \gamma_i U_t^0, \tag{9}$$

where U^i is the process of jumps of the asset i, U^0 is the process of market jumps, and the parameter γ_i is describes how the market jumps affect the prices of the given asset.

The model (9) can be easily augmented allowing for the existence of jumps characteristic for specific sector or market segments. The driving process is then given by the following equation:

$$L_t^i = \mu_i t + \sigma_i W_t + U_t^i + \gamma_i U_t^0 + \sum_{k=1}^{K} \delta_{ik} U_S^k 1_{i \in A_k}, \tag{10}$$

where U_S^k is the process describing the jumps in the sector k, δ_{ik} is the parameter that describes how jumps in the sector k affect the prices of the asset i and A_k is the set of indexes of assets in the sector k. Alternatively, one can assume that there are not any jumps characteristic for individual assets and all discontinuities are due to market jumps. The model is then given by the equation

$$L_t^i = \mu_i t + \sigma_i W_t + \gamma_i U_t^0 = \mu_i t + \sigma_i W_t + \sum_{j=1}^{N_t} \gamma_i C_t, \tag{11}$$

where N is a Poisson process and C_t are independent and identically distributed random variables that describe the size of market jumps. If we assume that C_t are normally distributed, we obtain multidimensional version of classical Merton model.

3   Risk of jump as a criterion in portfolio analysis

We consider a portfolio of n assets. By α_1, α_2, ..., α_n we denote the proportions of the wealth invested in consecutive assets. Of course, $\sum_{i=1}^{n} \alpha_i = 1$. The structure of the portfolio is thus given by the vector $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)^T$. We assume that there are no transaction costs and the investor is able to manage the portfolio, by buying and selling assets, to maintain the assumed structure. Let us denote the value of the portfolio (or the wealth of the investor) at the moment t by V_t. It follows from the eqn. (8) that the value process is given by the stochastic differential equation

$$dV_t = V_{t-} dL_t^\alpha, \tag{12}$$

where $L_t^\alpha = L_t^1 + \cdots + L_t^n$. The process L^α is also a Lévy process with the variation parameter of the Gaussian part of the process equal to

$$\sigma_\alpha^2 = \sum_{i,j=1}^{n} \alpha_i \alpha_j \sigma_{ij}. \tag{13}$$

The Lévy measure of the value process is given by

$$\nu_\alpha(A) = \nu(\{(x_1, ..., x_n) \in R^n : \alpha_1 x_1 + \cdots + \alpha_n x_n \in A\}). \tag{14}$$

If the measure ν is absolutely continuous and has a density function f, then the measure ν_α also has a density function f_α, which can be calculated by integration:

$$f_\alpha(x) = \frac{1}{\alpha_n} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} ... \int_{-\infty}^{+\infty} f\left(x_1, ..., x_{n-1}, \frac{1}{\alpha_n} x - \frac{\alpha_1}{\alpha_n} x_1 - \cdots - \frac{\alpha_{n-1}}{\alpha_n} x_1\right) dx_1 \, dx_2 \, ... \, dx_{n-1}. \tag{15}$$

Let us define some function $U: R \to R_+$ which serves as a measure of "disutility" of jumps. We can define the criterion of jumps' risk as follows:

$$K_3(\alpha) = \int_{-\infty}^{+\infty} U(x) d\nu_\alpha(x). \tag{16}$$

Accounting for classical criteria in the portfolio analysis, we obtain a three-criterial optimization problem with the following objectives:

1. Maximize expected return of a portfolio

$$\max_{\alpha} K_1(\alpha) = \alpha \cdot \mu , \tag{17}$$

2. minimize variance of the return of a portfolio

$$\min_{\alpha} K_2(\alpha) = \sigma_{\alpha}^2, \tag{18}$$

   where σ_{α}^2 is given by (13), and

3. minimize the risk of jumps, where the jumps' risk measure is given by the eqn. (16).

As in other multicriterial problems, we should focus on efficient solutions, i.e. the portfolios for which it is impossible to improve any criterion without worsening others. Such solutions can be obtained for example by solving the following optimization problem:

$$\max_{\alpha} K_1(\alpha) - \lambda_2 K_2(\alpha) - \lambda_3 K_3(\alpha), \tag{19}$$

where $\lambda_2, \lambda_3 > 0$ are coefficients that describe investor's risk aversion concerning, respectively, "normal" market circumstances and jumps. However solving the problem (19) can be computationally difficult, because in order to derive the measure v_{α}, one has to perform multidimensional integration, as it is given in the eqn. (15). The problem is much simpler in the setup with market jumps.

In the model with market jumps, given by the eqn. (9), the process that drives the value of the portfolio can be expressed as follows:

$$L_t^{\alpha} = \sum_{i=1}^{n} \alpha_i \mu_i\, t + \sum_{i=1}^{n} \alpha_i W_t^i + U_t^{\alpha}, \tag{20}$$

where the jump process of the portfolio value is defined as

$$U_t^{\alpha} = (\sum_{i=1}^{n} \alpha_i \gamma_i) U_t^0 + \sum_{i=1}^{n} \alpha_i U_t^i = \sum_{i=0}^{n} \alpha_i U_t^i , \tag{21}$$

where

$$\alpha_0 = \sum_{i=1}^{n} \alpha_i \gamma_i . \tag{22}$$

It can be shown, that the Lévy measure of the portfolio jumps process is given by

$$dv_{\alpha}(x) = \sum_{i=0}^{n} \frac{1}{\alpha_i} dv_i \left(\frac{x}{\alpha_i}\right) 1_{\alpha_i > 0}. \tag{23}$$

4   Tempered stable tails and computations using FFT

We assume that the jump process of each asset (as well as the process of market jumps) has finite intensity. For each asset the jumps occur independently – i.e. the moments of jumps for different assets form Poisson process with intensity $v_i(R)$. This number is also the expected number of jumps of asset i in the unit of time. We assume that jumps sizes follow the two-sided Pareto distribution, i.e. the densities of positive and negative jumps are given respectively by

$$f_i^1(x) = C_i^1 \left(\frac{h_i^1}{x}\right)^{\beta_i^1} \text{ for } x > h_i^1 > 0 \tag{24}$$

and

$$f_i^2(x) = C_i^2 \left(-\frac{h_i^2}{x}\right)^{\beta_i^2} \text{ for } x < -h_i^2 < 0, \tag{25}$$

where C_i^1, C_i^2 are normalizing constants and β_i^1, β_i^2 are indexes characterizing the decay in tails of the distributions. We assume that positive and negative jumps are greater than some critical values: h_i^1 and h_i^2, respectively. The size of jumps obeys power law, which proved to be valid in many circumstances concerning financial markets and extreme events (see for example [Malevergne, Sornette, 2006] or [Rachev et.al., 2011]).

We can define the jump risk of the portfolio as in the previous sections. However there are some problems with the criterion $K_3(\alpha)$. The expected number of jumps of portfolio in the unit of time is equal to

$$\nu_\alpha(R) = \sum_{i=0}^n \nu_i(R) 1_{\alpha_i>0} \tag{26}$$

and $K_3(\alpha)$ as a function of α is discontinuous when $\alpha_i \to 0$ for some i. To avoid this problem we modify the third criterion. Let

$$\widetilde{\nu_i}(dx) = \nu_i(dx)/\nu_i(R) \tag{27}$$

be the distribution of jumps sizes for the asset i. The distribution of jumps of the process $\alpha_i U^i$ is given by the measure $\widetilde{\nu_i}\left(\frac{dx}{\alpha_i}\right)/\alpha_i$. Define $\widetilde{\nu_\alpha}$ as the convolution of these measures:

$$\widetilde{\nu_\alpha}(dx) = \frac{1}{\alpha_0 \cdots \alpha_n} \widetilde{\nu_0}(dx/\alpha_0) * \cdots * \widetilde{\nu_n}(dx/\alpha_n). \tag{28}$$

The convulsion describes the distribution of the sum of variables, thus $\widetilde{\nu_\alpha}$ can be interpreted as the average size of a jump in the value process. We modify this measure to take into account intensities of jumps in various components of the portfolio to obtain the following measure:

$$\widetilde{\eta_\alpha}(dx) = \left(\sum_{i=0}^n \alpha_i \nu_i(R)\right) \widetilde{\nu_\alpha}(dx). \tag{29}$$

The risk of jumps criterion is now defined as

$$\widetilde{K_3}(\alpha) = \int_{-\infty}^{+\infty} U(x) \widetilde{\eta_\alpha}(dx). \tag{30}$$

The advantage of the new criterion is that the measure $\widetilde{\nu_\alpha}(dx)$ can be relatively easy computed numerically. If $\phi_j(u)$ is the Fourier transform of the measure $\widetilde{\nu_j}$:

$$\phi_j(u) = \int_{-\infty}^{+\infty} e^{iux} \widetilde{\nu_j}(dx), \tag{31}$$

then the Fourier transform of the measure $\widetilde{\nu_\alpha}(dx)$ is given by

$$\phi_\alpha(u) = \int_{-\infty}^{+\infty} e^{iux} \widetilde{\nu_\alpha}(dx) = \phi_0(\alpha_0 u)\phi_1(\alpha_1 u)\cdots\phi_n(\alpha_n u). \tag{32}$$

The measure $\widetilde{\nu_\alpha}$ can be computed numerically as an inverse Fourier transform of the function ϕ_α. The calculation can be performed effectively with the usage of Fast Fourier Transform (FFT) algorithm.

If the jumps sizes are given by two-sided Pareto distribution, as it was assumed by eqn. (24) and (25), the Fourier transform of each measure is given by

$$\phi_j(u) = C_j(\beta_j^1 + 1)C_j^1(-iuh_j^1)^{\beta_j^1+1}\Gamma(-\beta_j^1 - 1, -iuh_j^1) +$$

$$+ (\beta_j^2 + 1)C_j^2(-iuh_j^2)^{\beta_j^2+1}\Gamma(-\beta_j^2 - 1, iuh_j^2), \qquad (33)$$

where C_j is a normalizing constant and $\Gamma(a, x)$ is an incomplete gamma functions, defined as[1]

$$\Gamma(a, x) = \int_x^{+\infty} t^{a-1}e^{-t}dt. \qquad (34)$$

5 Empirical example – portfolios of shares from Polish stock exchange

We have performed an empirical analysis concerning computations of jump risk criterion for efficient portfolios build with the stocks quoted in the Polish Stock Market in Warsaw. We analyzed portfolios build with thirty biggest companies in the Warsaw Stock Exchange, which form the index WIG30 (the index of biggest and most liquid shares in the market). The calculations were based on intraday observations of the stocks in the period from 12th May till 14th October 2016. In the estimations 5th minutes returns were used.

In order to estimate parameters of distributions we had to identify jumps in the sample. We used a procedure proposed in (Andersen et.al. 2010) and (Ané, Métais, 2010). For each trading day we have performed test for the presence of jumps, based on the difference between realized volatility and bi-power variation. The test statistic is

$$BNS_m = \frac{BV_m\sqrt{N}}{\sqrt{\Omega_m}}(\ln RV_m - \ln BV_m), \qquad (35)$$

where RV_m is the realized volatility in the day m (i.e. $RV_m = \sum_{k=1}^{N} r_{k,m}^2$, where $r_{1,m}, \ldots, r_{N,m}$ are intraday returns in the day m), BV_m is bi-power variation in this day ($BV_m = \sum_{k=1}^{N-1}|r_{k,m}r_{k+1,m}|$) and Ω_m is the quaricity in the day m ($\Omega_m = (\pi^2/4)\sum_{k=4}^{N}|r_{k-3,m}r_{k-2,m}r_{k-1,m}r_{k,m}|$). As it was shown in (Barndorf-Nielsen, Shephard, 2006) the test statistic has asymptotically standard normal distribution. For each trading day we performed the test and if it had shown the existence of jumps, then the return with the highest absolute value was identified as a jump and removed from the sample. Then the procedure was repeated. In this way we obtained a sample of jumps, which allowed us to estimate the parameters of jump distribution. The results are presented in the Table 1.

Table 1. Parameters of the two-sided Pareto jumps distribution for stock from Polish index WIG30

Symbol	C_j^1	h_j^1	β_j^1	C_j^2	h_j^2	β_j^2
ACP	0.0650	0.00018	1.37	0.0610	0.00018	1.39
ALR	0.0350	0.00016	1.34	0.0470	0.00016	1.32
ATT	0.4070	0.00013	1.39	0.5010	0.00013	1.36
BHW	0.4190	0.00013	1.34	0.5290	0.00013	1.33
BRS	0.0000	0.00152	2.48	0.0000	0.00152	2.52
BZW	0.0580	0.00016	1.33	0.0510	0.00016	1.34

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[1] See for example (Abramovitz, Stegun, 1972).

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CCC	0.0580	0.00027	1.45	0.0630	0.00027	1.45
CPS	0.0320	0.00041	1.48	0.0270	0.00041	1.49
ENA	0.0140	0.00091	1.66	0.0170	0.00091	1.61
EUR	0.0320	0.00022	1.34	0.0220	0.00044	1.45
GTC	0.0300	0.00130	1.72	0.0210	0.00134	1.81
ING	0.0820	0.00034	1.44	0.0800	0.00034	1.43
JSW	0.0310	0.00066	1.42	0.0270	0.00067	1.42
KER	0.0730	0.00016	1.35	0.0700	0.00016	1.35
KGH	0.0040	0.00215	1.79	0.0080	0.00202	1.76
LPP	0.1650	0.00001	1.18	0.1700	0.00001	1.18
LTS	0.0290	0.00035	1.43	0.0240	0.00033	1.43
LWB	0.1270	0.00026	1.37	0.1270	0.00026	1.37
MBK	0.0590	0.00014	1.30	0.0640	0.00014	1.30
MIL	0.0010	0.00173	2.33	0.0010	0.00173	2.45
NET	0.0180	0.00193	2.07	0.0100	0.00192	2.24
OPL	0.0000	0.00169	3.00	0.0010	0.00166	2.58
PEO	0.0030	0.00242	2.14	0.0200	0.00242	1.89
PGE	0.0060	0.00077	1.71	0.0060	0.00077	1.67
PGN	0.0050	0.00177	2.09	0.0160	0.00176	1.89
PKN	0.0020	0.00161	1.91	0.0210	0.00189	1.60
PKO	0.0170	0.00040	1.44	0.0110	0.00040	1.47
PZU	0.0140	0.00035	1.43	0.0120	0.00035	1.45
SNS	0.0001	0.00245	3.42	0.0011	0.00244	3.52
TPE	0.0012	0.00318	3.83	0.0009	0.00318	4.12

Source: Own computations

Having eliminated jumps one can estimate the parameters of the continuous part of the process – namely, expected returns, as well as variances and covariances of returns. Base on this one can consider generalized portfolio problem. Here we consider a set of effective portfolio in the classical sense – i.e. the portfolios with the lowest variance, given the assumed expected return. We consider the solutions to the following problem:

$$\min_{\alpha \geq 0} K_2(\alpha), \text{ subject to } K_1(\alpha) \geq r ,$$

with respect to different assumed expected return r, where the critera K_1 and K_2 are defined by the eqn. (17) and (18). The results are presented in Figure 1, which depicts the minimal variance of a portfolio return given the assumed mean return. Figure 2 contains the values of jumps risk criterion $K_3(\alpha)$ for the effective portfolios in mean-variance sense, assuming that the disutility of jumps function in the eqn. (30) is quadratic: $U(x) = x^2$. As can be seen on the graphs, there exists a tradeoff between the two types of risk. The effective portfolios with lower risk of the continuous part (measured by variance of returns) usually have higher risk of jumps. This suggest that the interdependences between the three considered criteria can be nontrivial and are worth further considering.

Figure 1. The minimal variance with respect to mean return of a portfolio
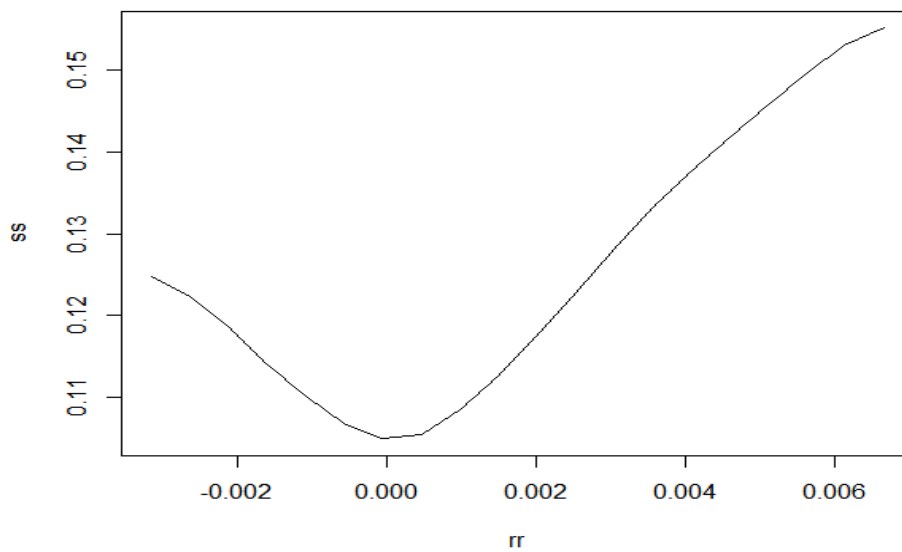
Source: Own computations

Figure 2. Jump risk measure with respect to mean return of a portfolio
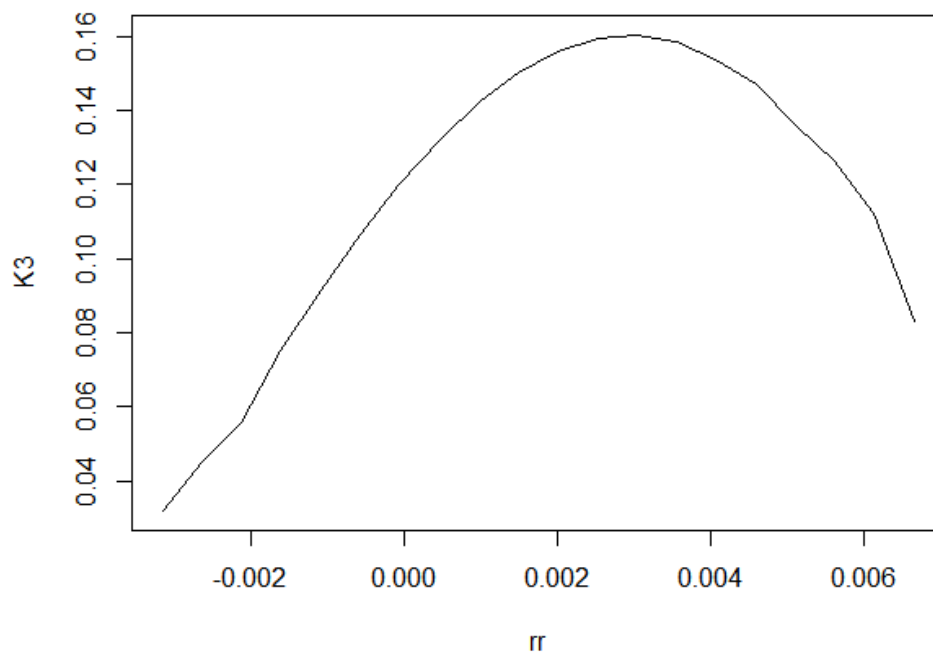
Source: Own computations

6   Conclusions

In the paper we consider the extension of the classical portfolio analysis for one more criterion, measuring the risk of sudden changes in asset prices and in the value of portfolio ("jumps"). We

have shown that there are various of ways in which one can account for this additional source of risk. We propose to measure this risk using expected value of disutility of jumps, calculated according to the measure defined as the convulsion of measures describing jumps of individual assets in the portfolio. Such criterion is both intuitive and relatively easy to handle numerically.

In some cases, as it was shown in (Kliber, 2008) and (Kliber, 2013), it is possible to obtain analytical formulae for the value of the jumps risk criterion. In this paper we consider the case in which jumps have two-sided Pareto distribution, i.e. jumps sizes obey power law. Under such assumption the analytical solution is unknown. One can make computations numerically using Fourier transform of measures describing jumps sizes and adopting the FFT (Fast Fourier Transform) algorithm. However, it should be noted that even such computations can be costly, as the calculations of Fourier transform of Pareto distributions involve the computation of incomplete gamma functions, what can be time-consuming.

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