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# GLOBAL PORTFOLIO MANAGEMENT UNDER STATE DEPENDENT MULTIPLE RISK PREMIA

## Abstract:

In this paper, we assess the benefits from international factor diversification under a regime based portfolio construction framework that takes into account the dynamic changes in stock markets. We show that there are significant costs to investors who fail to (a) pursue an international diversification strategy using sources of return other than the market premium and (b) take into account the existence of regimes in portfolio construction and asset allocation. Short sale and tracking error constraints reduce but do not eliminate the gains from a dynamic global factor portfolio. Implementation through commercially available, investable factor indices to provide efficient and low cost building blocks to construct a dynamic diversified factor portfolio in practice preserves most of the benefits from state dependent portfolio construction.

## Keywords:

Diversification benefits, Factor returns, Regime Switching Models.

**JEL Classification:** G11

## 1. Introduction

Research since the 1980's has challenged the view that market risk is the only priced factor in asset returns and that the equity premium is the key source of excess returns. Value, size, and momentum risk premia are regarded as separate, independent sources of excess returns. In the presence of multiple risk premia, an investor in the world market equity portfolio, which is no longer efficient, should also consider exposure to non-market risk premia. If factor returns are independent across countries, investing in a global portfolio of 'style' funds should provide considerable efficiency gains to a global equity portfolio. Our evidence suggests that there are significant costs to equity investors who fail to pursue an international diversification strategy using sources of return other than the market premium.

Investors have long recognized that returns, risk and correlations are different across bull and bear markets. Regime models represent an attractive alternative to the extreme of either no change or continuous changes in the asset return distribution. The evidence reported in Hamilton and Susmel (1994), Guidolin and Timmermann (2008), Tu (2010) and Ang and Timmermann (2012) suggest that market returns and Sharpe ratios are low or negative in the high volatility regime and significantly positive in the low risk regime. The practice of using the same risk and return models if returns are driven by different regimes could lead to inefficient portfolios. In the presence of different risk states, investors should scale down the volatility of their portfolios when volatility is high and increase the risk of their portfolios in the low risk state. We provide evidence suggesting that augmenting the global market portfolio with globally diversified portfolios of size, value and momentum factors using a regime based portfolio construction framework enhance significantly the efficiency of the world market portfolio.

Strategies for capturing systematic equity premia by the investors require the construction of portfolios that mimic the behavior of the risk factors. For example to capture the market risk premium investors use the capitalization weighted market portfolio. Building portfolios to capture other risk premia is more demanding. Even ignoring transaction costs and liquidity considerations, creating a portfolio that tracks factor returns requires a dynamic strategy as the portfolio of stocks held changes as risk attributes change. In fact, over the last five years index providers like S&P Dow Jones, MSCI, Russell Investments and FTSE using different construction methodologies have created indices designed to track risk factor exposure. These indices provide practical tools for tracking performance subject to turnover constraints and non-target factor neutrality. For investors who prefer to invest in long-only portfolios value, growth, large cap, small cap and high momentum benchmark indices exist for some time. In the final part of the paper, we use investable factor and style indexes and demonstrate that the proposed asset allocation strategies can be implemented profitably in practice.

Our paper combines multiple equity factors in a regime-based framework to shed light on the following research issues:

1. Examine the diversification benefits from investing in a portfolio of global market premia. We build global portfolios combining the world market portfolio as proxied by the capitalization weighted market portfolio and study the improvement in the return to risk ratio from the addition of a global portfolio of country value, size and momentum premia. This is consistent with a core-satellite strategy where the core is the world market index and satellites the global factor funds. In this first part of the research, we assume stable distributions and therefore static allocations to risk premia.
2. Identify statistically a joint 2-state regime-switching model for global market and risk premia and build state regime dependent globally diversified optimal risk premia portfolios. We test whether the regime-switching model adds value compared to the static risk premia allocations studied in (1). We link the probabilities of the regime-switching model to observed financial and economic variables. Linking the estimated state probabilities to observable financial variables aids our understanding about the economic determinants of the predicted regimes and increase our confidence to the estimated model since the asset allocation decisions are driven by the underlying economic conditions and not only by statistical relations.
3. Examine the cost of short selling and tracking error constraints. For many investors (pension funds, mutual funds, and individual investors) short selling is either too expensive or impossible to implement. For these investors we construct long-only factor funds. Reflecting the current institutional practice of managing portfolios against benchmarks we also construct portfolios under tracking error constraints. The difference in performance between unconstrained and constrained portfolios returns reflects the cost of constraints in portfolio performance.
4. Investigate the cost of implementing factor-based investment strategies in practice. We use investable factor and style commercially available indices as efficient and low cost building blocks to construct a diversified factor portfolio and implement the tactical asset allocation switches indicated by the regime model.

This paper extends the work of Ang and Bekaert (2004), Eun, Huang and Lai (2008), and Eun, Lai, de Roon, and Zhang (2010). We follow the methodology of Ang and Bekaert (2004) to construct the state dependent portfolio. However, unlike Ang and Bekaert (2004) we incorporate into our analysis practical considerations such as portfolio constraints and transactions costs. The benefits form diversifying across factor funds has been explored in recent papers by Eun, Huang and Lai (2008) and Eun, Lai, de Roon, and Zhang (2010). We extend their work by considering the effects of regimes on global factor premia portfolio construction. We also study in more detail the implementation issues arising when an investor implements theoretical factor portfolios in practice.

Our evidence suggests that a globally diversified portfolio of capitalization weighted factor premia increases the Sharpe ratio of the market portfolio from 0.40 to 1.60. The regime-based global factor premia portfolio increases the Sharpe ratio to 1.84. The benefits are reduced but remain significant under tracking error constraints. For low active risk portfolios (tracking error 2%), the Sharpe ratio of the static strategy is 0.59 while regime based asset allocation increases the Sharpe ratio further to 0.82.

The superiority of the global factor portfolio vis a vis the market portfolio remains intact when we examine out-of-sample evidence. The return to risk ratio of the global factor portfolio is almost double of the return to risk ratio of the world equity market portfolio. Imposing short-selling constraints reduces considerably the benefits from factor diversification but the portfolios remain superior in terms of return to risk compared with the market portfolio.

The benefits of an internationally diversified portfolio of factor premia remain significant when we use investable long-short indices to replicate the international factor portfolios. Using the Dow Jones US Thematic market neutral size, value and momentum indices to implement portfolio construction in practice produces performance similar to the theoretical factor funds. Finally, for investors with short sale constraints the use of the investable MSCI Global style indices to replicate the theoretical factor portfolios produces a small but economically significant improvement to the return to risk tradeoff offered by the world market portfolio.

The rest of this paper is organized as follows. Section 2 provides an overview of the literature on the size, value and momentum premia. Section 3 presents the data, summary statistics and the two-state regime model describing the joint distribution of risk premia and market returns. Section 4 presents the in sample empirical evidence under both single and two state regimes and various constraints. Section 5 contains the out of sample evidence while in Section 6 we discuss implementation issues and show evidence on the ability of investable indices to replicate the results we get with theoretical portfolios. Section 7 concludes the paper.

## 2. The Size, Value and Momentum Premia

It is almost three decades since Banz's (1981) discovery of the small capitalization effect. Since Banz's (1981) finding that a portfolio of small capitalization stocks has on average a higher return compared to a portfolio of large capitalization stocks there has been a growing number of papers studying the size effect[1]. Two recent papers (Fama and French, 2012 and Cakisi, Fabozzi and Tan, 2012) examine international data for 23 developed and 18 emerging markets, respectively. The empirical evidence presented in these papers, based on data since 1989, casts serious doubt as to whether the size premium is still present in capital markets.

The value effect, the observation that stocks with low valuation ratios tend to outperform stocks with high valuation ratios is one of the most robust finding in the finance empirical literature. The value premium has a long history, discussed first by Graham and Dodd (1934), and has since attracted a lot of interest from both practitioners and academics. The most popular ratios used to measure value are the Price-to-Book (PB) and the Price-to-Earnings (PE) ratios. Results are similar if instead of PB or PE one uses dividend yield or sales yield or cash flow yield. The

[1]See Reinganum (1983), Brown, Kleidon and Marsh (1983), Lamourex and Sanger (1989), Fama and French (1992, 1993, 1998, 2012), Berk (1995), Horowitz, Loughran, and (2000), Dimson, Marsh and Staunton (2002) among others. For a recent survey of the size effect see Mathijs van Dijk (2011).

literature related to the value premium is so vast, it is impossible to provide a comprehensive review here. A partial list of the most important papers for the US market includes Basu (1977), Rosenberg, Reid and Lanstein (1985), Fama and French (1992) and Lakonishok, Shleifer and Visney (1994). Recent papers looking at the international evidence on the value effect are Fama and French (1998, 2012) and Cakisi, Fabozzi and Tan (2012). Fama and French (2012) study 23 developed markets grouped in four regions (America, Europe, Japan and Asia Pacific) and find robust evidence of a value premium. The monthly average value premium ranges between 0.33% for North America and 0.62% for Asia Pacific. The value premium is statistically significant for all regions except North America. A global portfolio of all regions has a monthly value premium of 0.45% (standard deviation 2.46%). The world market premium over the same period was 0.44% per month with standard deviation of 4.37%. In terms of return to risk, the value premium offers almost twice the return to risk reward compared with the market portfolio. Cakisi, Fabozzi and Tan (2012) report similar results for 18 emerging markets. The all-emerging markets portfolio achieved a monthly average return of 1.15% with a standard deviation of 4.87%. If anything, the value premium is consistent across countries and over time.

Many capital market observers regard the momentum premium, the difference in returns between past winners and past losers as the premier anomaly[2]. The evidence in Fama and French (2012) suggests that a global portfolio of high momentum stocks outperforms low momentum stocks on average by 0.62% per month (standard deviation 4.2%). The return to risk ratio is slightly lower than the value premium but better than the market premium. The momentum premium is positive in North America, Europe and Asia Pacific and insignificant only in Japan. Cakisi, Fabozzi and Tan (2012) report similar results for emerging markets. An all-emerging markets portfolio long in past winners and short in past losers has an average monthly return of 0.86% (standard deviation 5.55%). With the exception of the Eastern European region, the momentum premium is statistically significant in the other regions.

We now have a wealth of empirical evidence on the existence of size, value and momentum premia. The evidence suggests that, with the exception of the size effect, the market rewards consistently value and high momentum investors. Rational explanations of these premia argue that they represent compensation to exposure to systematic risks (Fama and French 1992, 1993, 1996). However, non-market risk premia might be the result of market inefficiency due to investor irrationality (Lakonishok, Shleifer and Vishny, 1994). The academic debate on the subject is not yet settled. Does it matter from a practical portfolio perspective? From a practical point of view whether risk premia are the result of rational or irrational behavior matters to the extent that if premia are the result of market irrationality they may disappear as investors arbitrage them away. The persistence of the value premium

[2]Evidence for a momentum premium in the US market was first presented in Jegadeesh and Titman (1993). Papers by Rouwenhorst (1998), Chui, Titman and Wei (2010), Griffin, Ji and Martin (2003), Grundy and Martin (2001) extent the empirical evidence to both developed and emerging markets.

since its discovery 30 years ago and the momentum premium 20 years ago suggests that either the risks are real and will continue in the future or if there are the result of irrational behavior there must be significant limits to arbitrage. The popularity of small capitalization and value/growth funds among investors including establishment of ETFs in most developed markets and the adoption of a value or growth investment philosophy by many institutional investors is evidence of investors' recognition of the value of non-market premia in their portfolios. Recently introduced momentum ETFs is evidence of investors' interest in capturing the momentum premium.

## 3. Data

We obtain data from Thomson Datastream and cover all stocks (dead or alive) from July 1981 to December 2012 (378 monthly observations) in the G7 markets: Canada, France, Germany, Japan, Italy, U.K and the U.S. Equity data from Thomson Datastream are cleaned using the filters described in the works of Ince and Porter (2006), Hou, Karolyi and Kho (2011), Guo and Savickas (2008), and Busse, Goyal and Wahal (2013) to minimize the risk of data errors and to account for potential peculiarities of the dataset (see Appendix 1 for details).

We follow closely the Fama and French (1992) methodology to construct the style portfolios. At the end of June we sort all stocks in a country based on their market capitalization and the book value per share to form the SMB and HML portfolios. We set as missing negatives or zero values of book value per share while the fiscal year ending in year $t-1$ is matched with the returns and the market capitalization of year $t$ and hence there is no looking ahead bias in our dataset.

At the end of June of each year, we form the six Fama and French (1993) portfolios and calculate the value-weighted monthly returns over the next 12 months. To create the SMB portfolio we use the median of the market value, while for the book to market portfolios we set the breakpoints at the 30$^{th}$ and 70$^{th}$ percentiles of the book to market ratio. We calculate the momentum for month $t$ as the cumulative monthly returns for $t-1$ to $t-12$. Combined with the market capitalization we construct every month six value weighted portfolios to form the momentum factor by using the median of the market value and the 30$^{th}$ and 70$^{th}$ percentiles of the momentum. Finally, we construct the global HML, SMB, and MOM factors as country capitalization weighted averages. The return of the world market portfolio is the capitalization weighted average of the seven countries market portfolios.

Table 1 presents descriptive statistics for country monthly market returns and value, size and momentum premiums. It also shows descriptive statistics for the capitalization weighted world market return and global factor premiums. Country market returns are positive and statistically significant for all countries and for the world market portfolio. The monthly average world market return is equal to 0.90% per month with a standard deviation of 4.48% per month.

The world value premium is 0.60% and it ranges between 0.76% (Germany and Japan) and 0.46% (USA). The value premium is statistically significant for all countries and the global value portfolio. Monthly average value premiums estimates are similar to those reported by Fama and French (2012) and Busse, Goyal and

Wahal (2013). Fama and French (2012), using data for the November 1991- March 2011 for the 23 developed countries, estimate the average global value premium as 0.46% per month.

The momentum premium is present in all markets except Japan[3]. The average monthly momentum premium is positive and statistically significant in all markets (except Japan) and ranges from 0.09% (Japan) to 1.08% (Canada). The momentum premium is the most volatile among the risk factors with monthly country momentum standard deviations ranging between 3.09% (UK) and 4.54% (Germany). The average world momentum premium is equal to 0.46% with a standard deviation of 3.09% and it is statistically significantly different from zero. Fama and French (2012) report an average global momentum premium of 0.62% per month (t-statistic of 2.30).

The world size premium is close to zero (0.06%) with a standard deviation of 2.33% and it does not differ statistically from zero in line with the evidence presented in Fama and French (2012).

Table 1 also reports monthly Sharpe ratios for the market and factor premiums. The capitalization weighted world market portfolio has a monthly Sharpe ratio of 0.12 (0.40 annualized). The annualized Sharpe ratios of the value and momentum premiums are 0.97 and 0.52 respectively, significantly better than the market portfolio. With the exception of Japan, for all countries the value and momentum factors have a better return tradeoffs compared with the market portfolios. Combining value and momentum with the market portfolio should improve the Sharpe ratio of the market portfolio irrespective of the correlation between factor premiums and the market portfolio.

Creating a global multi-factor portfolio should lead to further diversification benefits if market and factor returns are not perfectly correlated. Table 2 shows the correlation between world and country market returns and factor premiums. The correlation between the market premium and factor premia is close to zero and in many cases slightly negative. The average correlation between country market and the size premium is -0.24, the market and the value premium -0.01 and the market and the momentum premium -0.18. The correlation between factor premia is also close to zero and in many cases slightly negative. The average correlation between the country value and size premia is -0.23, between the value and momentum premia -0.25 and between the size and momentum premia 0.08. The low correlation between market returns and factor premia and factor premia across countries implies diversification benefits for country based multi-factor portfolios. For the global portfolios, we also observe low and in most of the cases negative correlations between market and factor portfolios and between the factor portfolios. On average the correlation between factor and market returns is -0.18 and between the factor portfolios -0.13. Adding factor funds to market portfolios promises significant diversification benefits to domestic portfolios.

*3.1. A Multivariate Regime Model for Risk Premiums*

[3] Fama and French (2012) report that the monthly momentum premium for Japan is 0.08% and statistically insignificant.

Investors have long recognized that return, risk and correlations are different across bull and bear markets. Modeling time variations of the distribution of equity returns has been the subject of many academic papers. However, most of this research assumes that the distribution of returns remain constant, at least over the period used to estimate its parameters. Eun, Lai, de Roon, and Zhang (2010) use the "static" (or single state) mean-variance framework to find the portfolio with the maximum Sharpe ratio by combining the global market portfolio and factor funds. At the other extreme, there are models that assume that there is a continuous change in the structure of asset returns. We use a two-state multivariate regime model as an alternative to the extremes of no or continuous change in the return distribution.

A 2-state regime-switching model with a multivariate normal distribution (MVN) in each regime is described as:

$$y_t = \mu_{S_t} + \varepsilon_t, \varepsilon_t \sim \mathrm{MVN}(0, \Sigma_{S_t}), \qquad (1)$$

where $y_t = [\mathrm{MR}_t, \mathrm{SMB}_t, \mathrm{HML}_t, \mathrm{MOM}_t]$, $\mu_{S_t}$is a 4x1 mean matrix and $\Sigma_{S_t}$is a 4x4 variance-covariance matrix. Both $\mu_{S_t}$and $\Sigma_{S_t}$are state dependent at time $t$. Following Hamilton (1989), we hypothesize that the process is a first-order Markov and is described by a latent variable $S_t = 1,2$ while its transition matrix $\Pi$ is characterized by constant probabilities $(P, Q)$. Table 3 presents the estimation results of equation 1.

To evaluate the quality of regime classification, we follow Ang and Bekaert (2002) and calculate the regime classification measure: $RCM = 400 * \frac{1}{T}\sum_{t=1}^{T} p_t(1 - pt,$ where $pt=St|\mathcal{T}T$. The $RCM$ is equal to 28.17 providing strong indications that the 2-state regime switching model classifies correctly the periods of high and/or low risk.

The Akaike, Schwarz, and Hannan-Quinn information criterion show that the state dependent specification describes better the joint distribution of risk premia compared to the single state process. A likelihood ratio test shows that the test statistic is greater than the corresponding value of the chi-square distribution[4].

Figure 1 plots the smoothed probability of state 1 (high-risk environment) conditioned on all information in the sample (Kim, 1994). In the same figure we also plot the market and factors returns.

In all countries, market and factors returns switch to the high volatility environment in 1997-2002 and in 2008-2009, but there are also periods of high risk that are country specific. On average the market stays in the low (high) risk environment 24.03 (8.21) months. The probability that the world equity market moves into regime 1 (the high variance state) generally coincides with an increase in market volatility and the beginning of recession, a finding that suggests that equity risk might be related to the economic activity.

[4]The underlying distribution of the likelihood ratio test is unknown and hence this test must be used with caution.

The characteristics of the state dependent means and volatilities of the market portfolio are similar to those reported by Ang and Bekaert (2002, 2004). The average market return during the high volatility periods is statistically insignificant, and lower than the corresponding average return during the low volatility periods that differs from zero. The monthly standard deviation of market returns during high-risk periods is almost 1.8 greater than that during the less turmoil periods. Value stocks perform better than the growth stocks in both states. The average value premium is 0.75% and marginally statistically significant in the high variance regime and 0.57% and statistically significant in the low variance regime. The momentum premium is significant in the low variance regime with an average return of 0.68% but negative (-0.15%) and insignificant in the high variance regime. The size premium is close to zero and insignificant in both regimes.

The 2-state regime model partitions the sample in high and low variance periods where risk premia have low returns and high volatility and periods where risk premia are positive with low volatility. The model suggests that in general the return to risk (Sharpe) ratios are higher during quite periods. These patterns in returns and volatilities suggest that mean-variance investors should hold different portfolios depending on the regime. We explore the implications of our findings for portfolio construction in the next section.

Panel B of Table 3 presents state dependent correlations of the global risk premiums. Correlations between risk factors are in general low across both regimes. Specifically, the average correlation between market and factor portfolios in the high (low) risk environment is equal to -0.26 (-0.05), while between the factors -0.14 (-0.13).

The regime model identified above uses purely statistical analysis of risk premium data. To explore the relation between regimes and economic conditions we estimate the following probit regression:

$$\begin{aligned} \mathrm{P}(\mathrm{D_t}=1) = \mathrm{F}\big(\alpha + \beta_1 Default_t + {+\beta_2} Term_t + \beta_3 DY_t + \beta_4 Illiquidity_t + \beta_5 ADS_t \\ + \beta_6 Wvol_t + \beta_7 Wdisp_t\big), \end{aligned} \tag{2}$$

where $\mathrm{D_t} = 1$ when the state probability is greater than 50% (regime 1) and $\mathrm{D_t} = 0$ otherwise (regime 2). The probability of being in regime 1 is modeled as a function of the following financial and business cycle variables: (a) the default premium (Default) defined as the difference between the return of US BBB and AAA corporate bonds (b) the term spread (Term) defined as the difference between the ten-year USA treasury constant maturity yield and the three month T-Bill rate (c) the world market dividend yield (DY) (d) world stock market liquidity (Liquidity) using the liquidity measure of Pástor and Stambaugh (2003) (e) the business conditions index (ADS) which is designed to track real business conditions[5] (f) world stock market volatility (Wvol) calculated using daily world stock market returns and (g) world stock return

[5] For more information on the ADS business index, the reader is referred to the work of Aruoba, Diebold and Scotti (2009).

dispersion[6] (Wdisp) defined as the cross-sectional standard deviation at time t using all G7 markets stocks covered by DataStream. In particular we calculate monthly return dispersion as $\sqrt{\sum_{i=1}^{N} w_{i,t}\,(r_i - r_m)^2}$ where $r_i$ is the stock return, $r_m$ is the return of the capitalization weighted market portfolio, $N$ is the number of stocks and $w_{i,t}$ is the market capitalization weight of stock $i$ in month $t-1$.

Columns 1-7 of table 4 show estimated coefficients, z-statistics and McFadden R-squares for each variable using equation 2. The relation between each variable and the state probability is as expected from theory and statistically significant. An increase in the default premium, an inverted term structure slope, a low dividend yield, a fall in stock market liquidity and deterioration in economic conditions is associated with an increase in the probability of the high-risk state. Decreases in market volatility and dispersion are associated with a low-risk state. World dispersion and world market volatility have the greatest explanatory power followed by default ADS. When we include all the variables in the regression all variables, except ADS and world dispersion, lose their statistical significance. Return dispersion seems to be a good proxy of the other financial and economic variables including world market volatility. Jointly the determinants explain 55.2% of state probability. The coefficient estimate of return dispersion is positive and statistically significant and suggests that an increase of risk coincides with an increase in the probability the market is in the high-risk environment. The evidence suggests that the estimated state probabilities are linked to observable financial and economic variables and in particular return dispersion.

## 4. Portfolio Management in Multi-Factor World: Single vs State Dependent Environment

The advice of modern portfolio theory to an investor in a world where the CAPM holds is very simple: split your portfolio between a bank account and a broadly based passively managed index fund that approximates the market portfolio. Is this advice valid in a multi-factor world? Cochrane (1999), based on the model developed by Merton (1973), argues that in a multifactor world the investor might not invest only in the market portfolio. Instead, the investor will hold three funds: a risk free fund, the market portfolio held by the average investor and one additional multifactor efficient portfolio. In a multi-factor world where investors are rewarded systematically for bearing market, value, size, and momentum risk in addition to market risk, the market portfolio will not be on the efficient frontier (it will not be an efficient portfolio). The multifactor portfolio will include positions in small, value and momentum stocks in excess to the exposure given by investing in the market portfolio.

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[6] Stock market return dispersion provides a timely, easy to calculate at any time frequency, model free measure of volatility. It measures the extent to which stocks move together or are diverging and has been used by both finance academics and practitioners to measure trends in aggregate idiosyncratic volatility, investors' herding behavior, micro-economic uncertainty, trends in global stock market correlations and as an indicator of potential alpha and a proxy for active risk. Academic research suggests that return dispersion is an effective proxy of the investment opportunity with predictive power for risk premia and the business cycle (Garcia, Mantilla-Garcia and Martellini (2013), Stivers and Sun (2010)).

We construct mean-variance global optimal portfolios by maximizing the Sharpe ratio under the assumption of single and two-state regimes. We consider a U.S.A. investor who holds the market index and invests in the three factor funds. Specifically, in the single state the optimal portfolio maximizes the Sharpe ratio: $\frac{W\mu - r_f}{\sqrt{W\Sigma W^T}}$, where $W$ is $1\text{x}4$ matrix of the weights,μ is an $4\text{x}1$ mean matrix, and Σ is an $4\text{x}4$ variance-covariance matrix. This portfolio is named single-state optimal (SS-optimal).

In the state dependent environment, we form the portfolio by maximizing the Sharpe ratio:

$$\frac{\pi_1 W_1 \mu_1 + \pi_2 W_2 \mu_2 - r_f}{\sqrt{\pi_1 \sigma_1^2 + \pi_2 \sigma_2^2 + (1-\pi_1)\pi_1(\mu_2 - \mu_1)^2}} \quad (3)$$

where $\pi_1(\pi_2)$ is the steady-state probability of state 1(2), $\mu_1$ $(\mu_2)$ is a $4\text{x}1$ mean matrix of state 1 (2), $\sigma_1^2 = W_1 \Sigma_1 {W_1}^T$ is the variance of state 1, and $\sigma_2^2 = W_2 \Sigma_2 {W_2}^T$ is the variance of state 2. If the smoothed probability in month $t$ is greater (lower) than 50%, we classify the month as a high (low) risk and we use the corresponding weights to calculate the returns of the portfolio. This portfolio is named regime-optimal (R-optimal).

Following current institutional investment practices we also construct portfolios designed to have 2% and 5% tracking error against the benchmark. For the optimal portfolio the benchmark is the world market portfolio and for the regime-optimal portfolio the benchmark is the optimal portfolio.

We evaluate the performance of portfolios using the following criteria: Sharpe ratio, return loss (RL) and information ratio (IR).

i. The Sharpe ratio of portfolio i ($SR_i$) defined as $\mathrm{SR_i} = \frac{\overline{\mu_\imath - r_f}}{\sigma_i}$ where $\overline{\mu_\imath - r_f}$ is the average portfolio excess return and $\sigma_i$ is the standard deviation of portfolio excess returns.

ii. The return loss of portfolio i ($RL_i$) defined as $RL_i = (SR_b - SR_i)\ \sigma_i$ where $SR_b\ and\ SR_i$ are the Sharpe ratios of the benchmark and portfolio i. The return loss is the difference in expected returns between the global factor portfolio and the world market portfolio with the same standard deviation. In other words, the return loss that an investor will experience if she had invested in the world market portfolio levered up or down, to have the same volatility as the global factor portfolio.

iii. The information ratio ($IR_i$) of portfolio i defined as $\mathrm{IR_i} = \frac{\overline{\mu_\imath - \mu_B}}{\sigma_{r_i - r_B}}$ where $\overline{\mu_\imath - \mu_b}$ and $\sigma_{r_i - r_B}$ are the average and standard deviation of portfolio's i excess return against the benchmark.

We calculate the tracking error of each portfolio against the world market portfolio. For the regime-optimal portfolio we also calculate tracking error against the single state optimal portfolio. We measure portfolio turnover as the average sum of the absolute value of trades across the market and the factor portfolios. $\mathrm{Turnover} = 12 * \frac{1}{\mathrm{T}-1}\sum_{\mathrm{t}=1}^{\mathrm{T}-1}\sum_{\mathrm{i}=1}^{\mathrm{Assets}}\left(\left|\mathrm{w}_{\mathrm{i,t+1}} - \mathrm{w}_{i,\mathrm{t}}\right|\right)$ where $w_i$ is the weight to portfolio i. We also calculate the break-even transactions costs for the global factor portfolios, defined as the fixed transaction cost that makes the excess return of the optimal and the regime-optimal against the world market portfolio equal to zero.

### *4.1. No short sales constraints*

Table 5, panel A, shows statistics for the market, single state optimal, and regime-optimal portfolios constructed under the assumption of no constraints and 2% and 5% tracking error constraints. To create the optimal portfolio we maximize the Sharpe ratio of a portfolio consisting of the market portfolio and the global size, value and momentum long-short portfolios. For an investor who holds the market portfolio, the optimal portfolio represents the optimal combination of global non-market risk premia with the market portfolio. The unconstrained optimal portfolio achieves an annual return of 41.86% and a standard deviation of 23.27%. The Sharpe ratio of the optimal portfolio, 1.60, is significantly higher than the Sharpe ratio of the world market portfolio (0.40). The return loss to an investor without access to the global factor portfolio is 28.03%. The optimal portfolio outperforms the world market portfolio with significant tracking error and has an information ratio of 1.27.

Imposing tracking error constraints result in more realistic portfolios. The Sharpe ratio of the optimal portfolio with 2% tracking error against the world market portfolio (0.59) is, as expected, less than the return to risk of the unconstrained case but still significantly higher than the market portfolio. The optimal global factor portfolio outperforms the world market portfolio by 2.58%. An investor who chooses to invest in the world market portfolio would lose 2.94% compared to an investment to an equally risky global factor portfolio. Allowing for a higher tracking error (5%) increases both the return and risk and improves the optimal portfolio's Sharpe ratio (0.88). The optimal portfolio outperforms the world market portfolio by 6.33%, consistent with an information ratio of 1.27.

Exploring multiple regimes in portfolio construction using the multivariate regime model estimated in section 3.1 improves further a globally diversified multifactor portfolio. In the unconstrained case the regime-optimal portfolio achieves a Sharpe ratio of 1.84. An investor who invests in the single state global factor optimal portfolio but ignores regimes suffers a return loss of 10.86%. The information ratio of the regime-optimal portfolio relative to the optimal portfolio equals 1.53. We note however, the high turnover of the unconstrained case (1117.8%) and the high active risk of the unconstrained (tracking error 45.51%) case make these portfolios unrealistic for most investors.

The regime based optimal portfolio with 2% tracking error has a Sharpe ratio of 0.82, a significant improvement compared with the Sharpe ratio of the single state optimal portfolio. The regime optimal portfolio outperforms the single state optimal portfolio by 3.35% p.a. and achieves an information ratio of 1.67. There are significant return losses (3.37%) for investors who choose to ignore regimes and invest in the single state global factor optimal portfolio. The annual turnover of the strategy is 44.55% and the break-even transaction costs that will equalize the world market and regime-optimal global factor portfolio are more than 13%. Imposing a 5% tracking error constraint improves significantly all performance measures compared with the 2% tracking error case.

The success of the regime-optimal portfolio reflects both the ability of the joint regime-model to identify periods of high/low risk and the portfolio construction methodology. For portfolios with 2% (5%) tracking error constraint[7], the investor invests 40% (100%), 14% (36%) and 15% (37%) in value, size and momentum funds in the high-risk state and 60% (149%), 14% (37%) and 35% (89%) in the low risk state. The allocation to factor funds for the optimal single state portfolio is 26% (64%) to value, 7% (19%) to size and 11% (29%) to momentum. The investor increases exposures to value and momentum funds when future risk is expected to be low and scales back exposures when risk is expected to be high. In other words, the regime model suggests a more risky portfolio when risk is low and a more conservative portfolio when risk is high. This is consistent with the higher premia observed during low risk periods. The detailed results are presented in panels B and C of table 5.

*4.2. Short sales constraints*

Short sale constraints are the norm in institutional investing. In this section, we assume that short sales are not allowed. For example, investors can get exposure to the value premium by investing in a long-only portfolio of value stocks. If the short positions in a long-short value portfolio make a significant contribution to the value premium, the return of the long-only investor will be lower. The evidence presented in Israel and Moskowitz (2012) suggest that about half of the value and momentum premiums come from long positions. We provide evidence on the importance of shorting within a portfolio context.

As panel A of table 6 shows, imposing a short sale constraint reduces but does not eliminate the benefits from global factor investing[8]. For the 2% tracking error-constrained investor, the single state optimal factor portfolio increases the Sharpe ratio of the world market portfolio from 0.40 to 0.52. Considering regimes improves the Sharpe ratio to 0.65, an improvement of 0.25 compared with the world market portfolio. The return loss of investing in the world market portfolio rather than the single state global factor portfolio is 1.93%. Ignoring regimes in portfolio construction results in a further return loss of 1.80%. The information ratio of the single state optimal portfolio is 0.78 and for the regime optimal portfolio 0.83. The turnover of the regime driven strategy is only 7.81% and as a result the strategy is clearly profitable after transaction costs (the breakeven transaction cost is 41.10%).

[7] For the unconstrained case the optimal portfolio exposure to the value, size and momentum factors are 314%, 92% and 141% respectively. Regime based optimal portfolios have 295%, 125% and 58% in value, size and momentum in the high risk regime and 779%, 158% and 576% in the low risk state. The high risk associated with these extreme exposures make these portfolios unsuitable for most institutional investors.

[8] We use equation 1 to estimate the joint regime model for the market, value, small and high momentum long-only portfolios. The detailed results are presents in table 1A in the Appendix.

In the last two columns of panel A of table 6 shows results for portfolios that allow for 5% maximum tracking error constraint. For a more active investor the Sharpe ratio of the single state global factor portfolio increase to 0.71 and the return loss 4.40%. The Sharpe ratio of the regime optimal portfolio is 0.78 and the total return loss if the investor ignores the global factor funds and regimes in portfolio construction and invests in the world market portfolio is 5.90%. We note that the total risk of the regime optimal portfolio is very close to the single state portfolio and similar to the regime-optimal portfolio in the unconstrained case. For the regime optimal portfolio the maximum tracking error against the single state optimal global factor achievable during the period was only 2.32%.

Under short sale constraints an investor with 2% (5%) maximum tracking error will allocate 66% (15%) in the world market portfolio, 22% (53%) in the value fund and 12% (38%) in the momentum fund. In the low risk state the investor invests 23% (0%) in the market portfolio, 42% (76%) in the value fund and 35% (24%) in the momentum fund. In the high risk state the weights of the market, value and momentum portfolios are 40% (0%), 42% (100%) and 18% (0%), respectively. The detailed results are presented in panels B and C of table 6.

Comparing the Sharpe ratios with and without short sale constraints provides an estimate of the costs of constraining short sales. For the 2% tracking error portfolios not allowing short sales reduces the Sharpe ratio of the single state optimal portfolio by 12% and the regime-optimal portfolio by 20%. The corresponding numbers for the 5% tracking error portfolios are 20% for the optimal portfolio in the single state case and 42% in the two-state regime case. The reduction in portfolio efficiency as measured by the decrease in portfolio Sharpe ratios depends on active risk portfolio constraints. For low active risk portfolios, typical of pension fund portfolios, the short sale constraint has a small cost especially if the higher implementation costs of long-short portfolio management are taken into account.

**5. Out-of-sample Evidence**

The in sample evidence presented suggests that both the single state optimal and the regime-optimal portfolio enhances relative the performance. In this section we explore the out-of-sample performance of these two strategies.

To assess the economic benefits of global factor portfolios out of sample we create optimal portfolios using return, risk and state probability forecasts using data available at the time of the portfolio construction decision. We use an expanding window approach. More specifically, we use monthly data over the period from July 1981 to December 2003 to estimate the parameters of the multivariate regime-switching model and calculate the optimal weights of the assets. To minimize turnover, the portfolio weights calculated at the end of the year[9] are kept constant for the next twelve months and as forecast for next month's state of the market we use the next twelve month forecasts of state probability. We then add twelve more months in the dataset and repeat the described procedure. This methodology ensures that

[9] We examine also a monthly rebalancing scheme. The results are qualitatively similar to the reported in Table 7.

there is no look-ahead bias as it utilizes only information that was available at month t. In table 7 we present portfolio statistics for the more realistic and therefore more practically relevant low and high active risk portfolios (maximum tracking error 2% and 5% respectively).

*5.1 Out-of-sample evidence for the factor portfolios*

In the out of sample period (January 2004-December 2012) the world market portfolio achieved a return of 6.50% p.a., volatility of 15.93% p.a. and a Sharpe ratio of 0.30. The low active risk single state optimal portfolio improves the Sharpe ratio of the market portfolio from 0.30 to 0.41 with an information ratio of 1.00. Investing in the market portfolio rather than the global factor portfolio results in return loss of 1.75%. A regime based portfolio construction strategy improves the Sharpe ratio to 0.62. Ignoring regimes in portfolio construction costs the investor 3.41%. In general the Sharpe and information ratios in the sample results are very similar with the out-sample evidence. Portfolio turnover is low by market practice standards at 43% and similar to the in sample evidence (45%). The break-even transaction costs estimates suggest that impossibly high transactions costs are necessary to make excess portfolio returns negative.

Allowing for a maximum tracking error of 5% produces qualitatively very similar results in and out of sample. Although the Sharpe ratios are smaller in the out of sample period, the relative improvement as the investor moves from the world market portfolio to the global factor portfolio and then a dynamic regime based portfolio construction is very similar to the in sample evidence. Annual turnover at 88% is within the portfolio turnover rate range observed in institutional investment management practices. Breakeven costs are significantly higher than even the most conservative costs estimates suggesting that the proposed strategies are robust to transaction costs adjustments.

Short sale constraints reduce considerably the benefits from investing in global factor funds (see panel B of table 7). Imposing a short sale constraint reduces the Sharpe ratio of the low active risk single state global factor portfolio from 0.41 to 0.36. Short sale constraints reduce the Sharpe ratio of the dynamic regime driven global factor portfolio proportionately more (from 0.62 to 0.43). The benefits however remain economically significant and similar albeit lower than the in-sample evidence. Investing in a static global factor portfolio improves the market Sharpe ratio from 0.30 to 0.36. Taking into account regimes improves the Sharpe ratio further to 0.43. The single state optimal portfolio outperforms the market portfolio by 0.74% p.a. The regime-optimal portfolio outperforms the single state optimal portfolio by 1.09% and the world market portfolio by 1.83%.

Under short sale constraints the maximum tracking error achievable in this period for the single state global factor portfolio against the market is 3.49% and for the dynamic regime based global factor portfolio against the single state optimal portfolio 2.37% (see panel B of table 7, last column). Similar to the in-sample evidence, short sale constraints limit the investors' ability to create high-risk dynamic regime based global portfolios when the benchmark is the single state optimal portfolio.

## 6. Implementation Issues

Most academic research on factor based portfolio construction pays little attention at the complex issues involved in implementing the theoretical portfolios in practice. Building portfolios to capture risk premia is more demanding than creating and managing the capitalization weighted market portfolio. Creating a factor portfolio requires a dynamic strategy as the portfolio of stocks held changes as risk attributes change. The turnover generated raises issues such as transaction costs and liquidity. Our database in December 2012 would have included 14844 stocks. To create the global momentum factor portfolio would have required roughly 4000 short and an equal number of long positions. Full replication of this monthly-rebalanced portfolio is almost impossible. Index providers recognizing the challenges involved in creating factor portfolios that are investable and with controlled turnover have developed passive indices that replicate factor performance. Some of these indices are being used by the mutual fund and ETF industry as the basis for passive investment products.

*6.1 Replicating the theoretical global factor portfolios using the DJ Thematic Market Neutral indexes*

In this section, we use commercially available indices as proxies for the theoretical portfolios studied in the previous sections of the paper. For investors with no short-sale constraints we use the Dow Jones US thematic market neutral size, value and momentum indices. Index history covers the period January 2002 to December 2012. The indices are designed to be both market and sector neutral, are based on the largest 1000 US stocks screened by liquidity and consist of 200 short and 200 long positions[10].

The DJ Thematic indices are US based and therefore unlikely to replicate perfectly the world size, value and momentum portfolios used in this study. Table 8 shows descriptive statistics of the DJ Thematic indices and compares their performance with the world factor portfolios constructed in this study. The average return for the DJ thematic market neutral value index is 0.63% close to that of the global value factor portfolio and its correlation with the global value factor is 0.57. The DJ momentum thematic factor is also highly correlated with the theoretical index (0.89) but during the period the index underperformed the theoretical global momentum portfolio by 0.21%. The size factor portfolios are the least correlated (0.19) and during the period the DJ Thematic size index did much better than the theoretical global factor portfolio (0.50% versus 0.04%). The DJ Thematic indices have higher volatility than the theoretical global factor portfolios.

Portfolio performance results for the 2% and 5% tracking error cases, when our estimates of global premia are replaced with the DJ thematic market neutral indices, are presented in table 9. For the out-of-sample period, using the DJ indices rather

[10] For information on the DJ thematic indices see www.djindexex.co/thematicmarketneutral/.

than the theoretical global portfolios makes very marginal differences in portfolio returns and risks. The Sharpe ratios of the single state optimal portfolios when short sales are allowed are reduced from 0.41 to 0.39 in the low active risk case (tracking error 2%) and from 0.56 to 0.51 in the high active risk case (tracking error 5%). The Sharpe ratios of the regime-optimal portfolios using the DJ indices are slightly lower but still significantly better than the return to risk of the world market portfolio. Marginal differences are also observed for the information ratios and the return loss metric. The DJ indices track the world market portfolio as well as the theoretical global factor portfolios.

*6.2 Replicating the theoretical long-only global factor portfolios using the MSCI Indices*

For investors facing short sale constraints and invest in long-only portfolios, value, growth, large cap, and small cap benchmark indices exist for some time (FTSE, MSCI, S&P and Russell Style Indices). Even more important from a practical investment management point of view, for these risks premia there are ETFs for some countries and regions. In this section of the paper, we replicate the long-only portfolio performance results using the MSCI world size (ACWI small cap), value (ACWI value standard) and momentum (ACWI momentum standard) indices instead of the theoretical global factor portfolios.

In table 10, we present descriptive statistics of the MSCI indices and the corresponding global style portfolios for the period January 1997 to December 2012. The average returns for the size and momentum MSCI indices are similar to their theoretical counterparts. However, the average return of the MSCI global value index is almost half the theoretical value factor portfolio used in this study. The volatility of the MSCI style indices is higher than the volatility of the theoretical portfolios reflecting the fact that the MSCI indices are less diversified (contain fewer stocks) than the theoretical factor portfolios. The Sharpe ratio of the MSCI indices is lower for all factors. The MSCI indices and the theoretical portfolios are highly correlated (0.9 and above).

Table 11 presents the results from replacing for the out-of-sample study the returns of the global portfolios with that of the MSCI global indices. The average returns using the MSCI indices are marginally lower than the average returns of the theoretical factor portfolios (except for the regime-optimal portfolio with 5% tracking error) while the portfolio volatilities are higher. As a result, the Sharpe ratios of portfolios that use the MSCI indices are lower than those using the theoretical factors. However, the excess return of the single state and regime optimal portfolios using the MSCI indices rather than the theoretical factor portfolios remains economically significant. Ignoring global factor funds and regimes and investing in the world market portfolio cost a low active risk investor 2.07% when factor returns are measured using the theoretical portfolios and 1.13% when the MSCI indices are used instead. Higher active risk investors' using a dynamic regime switching strategy implemented through long-only MSCI style indices achieve the same Sharpe ratio (0.47) as a strategy implemented with long-only theoretical portfolios.

## 7. Conclusions

Long-term evidence from the US market and evidence from both developed and emerging markets in the last 30 years suggest that investment strategies that emphasize small cap, value and high momentum stocks generate positive excess returns and generally higher Sharpe ratios than the market portfolio.

While there is research on the benefits of diversifying across factor premia within the major capital markets there is less work on the benefits of building an internationally diversified portfolio of market and factor premia. Since factor premia tend to be un-correlated across markets, a global style fund should enhance significantly the efficiency of the world market portfolio. We assess the benefits from international factor diversification assuming (a) a single state and a regime-switching model of expected returns, variances and correlations (b) constraints on short sales, cash and portfolio tracking error.

We propose a new investment strategy, beyond the traditional globally diversified equity portfolio, based on investments in a global portfolio of style risk premia in a risk-on, risk-off framework. We show that there are significant costs to investors who fail to (a) pursue an international diversification strategy using sources of return other than the market premium and (b) take into account the existence of regimes in portfolio construction and asset allocation.

Our in and out of sample empirical evidence suggests that short sale and tracking error risk constraints reduce but do not eliminate the benefits from a dynamic global factor based portfolio strategy. Using commercially available indices as proxies for factor funds with low implementation costs, preserves most of the benefits of the proposed investment strategy.

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# Appendix 1. Database construction

1. We include in the database only stocks characterized by Datastream as "equities" (Stock type: EQ).
2. We exclude all companies that are not listed on the primary stock exchange.
3. We use Level 2, Level 3, and Level 4 sector names and the names of the companies to identify and exclude closed end funds, REITs, ADRs and preferred stocks.
4. We set returns as missing if the return index[11] is below 3.
5. We set the returns of two consecutive months as missing if we observe an increase over 300% at month t and a decrease more than 50% at month t+1.
6. We set returns as missing if they are higher (lower) than the 97.5$^{th}$ (2.5$^{th}$) percentiles of the daily or monthly return distribution to mitigate the effect of extreme stock returns.
7. We remove 5% of the stocks with the smallest capitalization.
8. All stocks with less than 10 observations during a month are excluded from the analysis.

[11] A stock is given a total return index value of 100 when entering the database. A return index less than 3 means that the security lost 97% of its value.

# Appendix 2. The Joint Regime Model for the Global Market, Size, Value and Momentum Portfolios – Long Only

**Table A1.** The Joint Regime Model for the Global Market, Size, Value and Momentum Portfolios

Panel A presents the estimations of equation: $y_t = \mu_{S_t} + \varepsilon_t, \varepsilon_t \sim MVN(0, \Sigma_{S_t})$, where $y_t = [MR_t, Small_t, Value_t, Momentum_t]$, $\mu_{S_t}$ is an $4x1$ mean matrix and $\Sigma_{S_t}$ is an $4x4$ variance-covariance matrix. Both $\mu_{S_t}$ and $\Sigma_{S_t}$ are state dependent at time $t$. The transition matrix $\Pi$ is characterized by constant probabilities $(P, Q)$. P values are in parenthesis. It also shows the regime classification measure: $RCM = 400 * \frac{1}{T}\sum_{t=1}^{T} p_t(1-p_t)$, where $p_t = (S_t|\mathcal{I}_T)$ three information criterion (Akaike, Schwarz, and Hannan-Quinn), and the log likelihood values for the 2-state and single state specifications. Panel B presents the implied correlation coefficients of the two states. The sample period is from 07/30/1981 to 12/31/2012

**Panel A.** Estimation Results

| | Market | Value | Small | Momentum |
|---|---|---|---|---|
| Mean, state 1 | 0.0018 (0.83) | 0.0093(0.24) | 0.006 (0.50) | 0.0054 (0.48) |
| Mean, state 2 | 0.0117 (0.00) | 0.0147 (0.00) | 0.0120 (0.00) | 0.0149 (0.00) |
| $\sigma^2$, state 1 | 0.0040 (0.00) | 0.0032 (0.00) | 0.0041 (0.00) | 0.0029 (0.00) |
| $\sigma^2$, state 2 | 0.0012 (0.00) | 0.0012 (0.00) | 0.0013 (0.00) | 0.0013 (0.00) |
| **Transition Probabilities** | | | | |
| P | 0.91 (0.00) | | | |
| Q | 0.97 (0.00) | | | |
| Duration, state 1 | 10.73 | | | |
| Duration, state 2 | 29.04 | | | |
| RCM | 28.170 | | | |
| Log likelihood | 2-state: 4614.06 | Single State: 3762.53 | | |
| Akaike info criterion | 2-state: -24.32 | Single State: -19.89 | | |
| Schwartz criterion | 2-state: -24.01 | Single State: -19.85 | | |
| Hannan-Quin criterion | 2-state: -24.20 | Single State: -19.87 | | |

Panel B. Correlation Analysis

| | | | | |
|---|---|---|---|---|
| **State 1** | | | | |
| Market | 1 | | | |
| HML | 0.89 | 1 | | |
| SMB | 0.85 | 0.91 | 1 | |
| Momentum | 0.89 | 0.87 | 0.94 | 1 |
| **State 2** | | | | |
| Market | 1 | | | |
| HML | 0.92 | 1 | | |
| SMB | 0.85 | 0.95 | 1 | |
| Momentum | 0.93 | 0.93 | 0.94 | 1 |

## Tables and Figures

**Table 1.** Monthly Descriptive Statistics

This table presents descriptive statistics for country monthly market returns and value, size and momentum premiums. It also shows descriptive statistics for the capitalization weighted world market return and global factor premiums At the end of June of each year, we form the six Fama and French (1993) portfolios and calculate the value-weighted monthly returns over the next 12 months. To create the SMB portfolio we use the median of the market value, while for the book to market portfolios we set the breakpoints at the 30th and 70th percentiles of the book to market ratio. We calculate the momentum for month $t$ as the cumulative monthly returns for $t-1$ to $t-12$. Combined with the market capitalization we construct every month six value weighted portfolios to form the momentum factor by using the median of the market value and the 30th and 70th percentiles of the momentum. Finally, we construct the global HML, SMB, and MOM factors as country capitalization weighted averages. The return of the world market portfolio is the capitalization weighted average of the seven countries market portfolios. The table also reports the monthly Sharpe ratios for the market and factor premiums. The data are obtained from Thomson Datastream and cover all stocks (dead or alive) from July 1981 to December 2012 (378 monthly observations) in the G7 markets: Canada, France, Germany, Japan, Italy, U.K and the U.S.

| | mean | Stdev | t-stat | max | min | Sharpe | mean | stdev | t-stat | max | min | Sharpe |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **US** | | | | | | **UK** | | | | | |
| Market | 1.01% | 4.49% | 4.36 | 13.38% | -20.75% | 0.14 | 1.06% | 5.37% | 3.83 | 16.50% | -21.23% | 0.13 |
| HML | 0.46% | 2.98% | 3.03 | 12.17% | -11.82% | 0.16 | 0.49% | 2.40% | 4.01 | 8.76% | -9.42% | 0.21 |
| SMB | 0.18% | 3.11% | 1.10 | 19.32% | -14.01% | 0.06 | -0.23% | 3.00% | -1.46 | 12.50% | -9.19% | -0.08 |
| MOM | 0.48% | 3.70% | 2.53 | 14.52% | -23.65% | 0.13 | 0.93% | 3.09% | 5.82 | 12.44% | -21.24% | 0.30 |
| | **Canada** | | | | | | **Japan** | | | | | |
| Market | 1.01% | 5.52% | 3.55 | 20.33% | -26.50% | 0.11 | 0.69% | 6.36% | 2.11 | 27.00% | -17.47% | 0.05 |
| HML | 0.72% | 3.11% | 4.52 | 11.41% | -19.12% | 0.23 | 0.76% | 2.62% | 5.65 | 9.04% | -10.23% | 0.29 |
| SMB | 0.21% | 2.60% | 1.60 | 17.20% | -11.45% | 0.08 | -0.15% | 3.32% | -0.87 | 14.19% | -13.35% | -0.04 |
| MOM | 1.08% | 4.04% | 5.20 | 12.88% | -15.53% | 0.27 | 0.09% | 4.43% | 0.40 | 13.22% | -30.61% | 0.02 |
| | **Italy** | | | | | | **France** | | | | | |
| Market | 0.84% | 7.25% | 2.25 | 27.09% | -23.11% | 0.06 | 1.24% | 6.16% | 3.91 | 19.85% | -21.53% | 0.14 |
| HML | 0.56% | 3.32% | 3.31 | 17.21% | -15.63% | 0.17 | 0.51% | 3.74% | 2.66 | 16.23% | -28.83% | 0.14 |
| SMB | -0.43% | 3.19% | -2.60 | 9.56% | -18.32% | -0.13 | -0.06% | 3.16% | -0.34 | 20.28% | -9.47% | -0.02 |
| MOM | 0.90% | 4.51% | 3.88 | 16.36% | -22.42% | 0.20 | 1.04% | 3.96% | 5.11 | 15.32% | -19.23% | 0.26 |
| | **Germany** | | | | | | **World Factor Fund – Capitalization Weighted** | | | | | |
| Market | 1.06% | 6.16% | 3.33 | 19.33% | -20.65% | 0.11 | 0.90% | 4.48% | 3.89 | 12.95% | -18.22% | 0.12 |
| HML | 0.76% | 3.05% | 4.83 | 15.81% | -14.61% | 0.25 | 0.60% | 2.17% | 5.40 | 9.45% | -11.02% | 0.28 |
| SMB | -0.37% | 2.89% | -2.49 | 8.66% | -10.60% | -0.13 | 0.06% | 2.33% | 0.47 | 13.36% | -9.52% | 0.02 |
| MOM | 0.82% | 4.54% | 3.50 | 21.86% | -25.61% | 0.18 | 0.46% | 3.09% | 2.89 | 11.93% | -20.54% | 0.15 |

**Table 2.** Correlation Analysis.

This table shows the correlation between world and country market returns and factor premiums. At the end of June of each year, we form the six Fama and French (1993) portfolios and calculate the value-weighted monthly returns over the next 12 months. To create the SMB portfolio we use the median of the market value, while for the book to market portfolios we set the breakpoints at the 30th and 70th percentiles of the book to market ratio. We calculate the momentum for month $t$ as the cumulative monthly returns for $t-1$ to $t-12$. Combined with the market capitalization we construct every month six value weighted portfolios to form the momentum factor by using the median of the market value and the 30th and 70th percentiles of the momentum. Finally, we construct the global HML, SMB, and MOM factors as country capitalization weighted averages. The return of the world market portfolio is the capitalization weighted average of the seven countries market portfolios. The data are obtained from Thomson Datastream and cover all stocks (dead or alive) from July 1981 to December 2012 (378 monthly observations) in the G7 markets: Canada, France, Germany, Japan, Italy, U.K and the U.S.

| | **US** | | | | **UK** | | | |
|---|---|---|---|---|---|---|---|---|
| | Market | HML | SMB | MOM | Market | HML | SMB | MOM |
| Market | 100.00% | | | | 100.00% | | | |
| HML | -31.38% | 100.00% | | | 13.50% | 100.00% | | |
| SMB | 3.40% | -36.90% | 100.00% | | -31.19% | -27.15% | 100.00% | |
| MOM | -14.43% | -21.57% | 12.15% | 100.00% | -15.38% | -32.32% | 16.87% | 100.00% |

| | **Canada** | | | | **Japan** | | | |
|---|---|---|---|---|---|---|---|---|
| | Market | HML | SMB | MOM | Market | HML | SMB | MOM |
| Market | 100.00% | | | | 100.00% | | | |
| HML | -19.55% | 100.00% | | | -31.85% | 100.00% | | |
| SMB | 2.77% | -32.43% | 100.00% | | -9.60% | 9.63% | 100.00% | |
| MOM | -15.30% | -23.42% | 12.88% | 100.00% | -9.77% | -20.70% | -17.94% | 100.00% |

| | **Italy** | | | | **France** | | | |
|---|---|---|---|---|---|---|---|---|
| | Market | HML | SMB | MOM | Market | HML | SMB | MOM |
| Market | 100.00% | | | | 100.00% | | | |
| HML | 26.24% | 100.00% | | | 18.15% | 100.00% | | |
| SMB | -49.17% | -13.75% | 100.00% | | -32.91% | -39.20% | 100.00% | |
| MOM | -22.67% | -20.66% | 1.20% | 100.00% | -21.57% | -34.40% | 25.83% | 100.00% |

| | **Germany** | | | | **World Factor Fund – Capitalization Weighted** | | | |
|---|---|---|---|---|---|---|---|---|
| | Market | HML | SMB | MOM | | Market | HML | SMB |
| Market | 100.00% | | | | Market | 100.00% | | |
| HML | 17.16% | 100.00% | | | HML | -25.91% | 100.00% | |
| SMB | -51.40% | -20.92% | 100.00% | | SMB | -9.33% | -28.32% | 100.00% |
| MOM | -25.57% | -23.91% | 5.72% | 100.00% | MOM | -19.24% | -23.30% | 12.52% |

**Table 3.** The Joint Regime Model for the Global Market, Size, Value and Momentum Factors

Panel A presents the estimations of equation: $y_t = \mu_{S_t} + \varepsilon_t, \varepsilon_t \sim MVN(0, \Sigma_{S_t})$, where $y_t = [MR_t, SMB_t, HML_t, UMD_t]$, $\mu_{S_t}$ is an $4x1$ mean matrix and $\Sigma_{S_t}$ is an $4x4$ variance-covariance matrix. Both $\mu_{S_t}$ and $\Sigma_{S_t}$ are state dependent at time $t$. The transition matrix $\Pi$ is characterized by constant probabilities $(P, Q)$. P values are in parentheses. It also shows the regime classification measure: $RCM = 400 * \frac{1}{T}\sum_{t=1}^{T} p_t(1 - p_t)$, where $p_t = (S_t|\mathcal{I}_T)$ three information criterion (Akaike, Schwarz, and Hannan-Quinn), and the log likelihood values for the 2-state and single state specifications. Panel B presents the implied correlation coefficients of the two states. The sample period is from 07/30/1981 to 12/31/2012

**Panel A.** Estimation Results

| | Market | HML | SMB | Momentum |
|---|---|---|---|---|
| Mean, state 1 | 0.0020 (0.82) | 0.0073 (0.09) | 0.0004 (0.92) | -0.0015 (0.81) |
| Mean, state 2 | 0.0115 (0.00) | 0.0056 (0.00) | 0.0007 (0.55) | 0.0068 (0.00) |
| $\sigma^2$, state 1 | 0.0042 (0.00) | 0.0012 (0.00) | 0.0012 (0.00) | 0.0027 (0.00) |
| $\sigma^2$, state 2 | 0.0010 (0.00) | 0.0002 (0.00) | 0.0003 (0.00) | 0.0003 (0.00) |
| Transition Probabilities | | | | |
| P | 0.878 (0.00) | | | |
| Q | 0.958 (0.00) | | | |
| Duration, state 1 | 8.207 | | | |
| Duration, state 2 | 24.030 | | | |
| RCM | 28.170 | | | |
| Log likelihood | 2-state: 4139.41 | | Single State: 3273.736 | |
| Akaike info criterion | 2-state: -21.80 | | Single State: -18.38 | |
| Schwartz criterion | 2-state: -21.49 | | Single State: -18.34 | |
| Hannan-Quin criterion | 2-state: -21.68 | | Single State: -18.36 | |

Panel B. Correlation Analysis

| | | | | |
|---|---|---|---|---|
| State 1 | | | | |
| Market | 1 | | | |
| HML | -0.335 | 1 | | |
| SMB | -0.058 | -0.411 | 1 | |
| Momentum | -0.394 | -0.223 | 0.227 | 1 |
| State 2 | | | | |
| Market | 1 | | | |
| HML | -0.139 | 1 | | |
| SMB | -0.143 | -0.077 | 1 | |
| Momentum | 0.141 | -0.251 | -0.071 | 1 |

**Table 4.** The Determinants of State Probability

This table presents the estimation of the probit regression:
$P(D_t = 1) = F(\alpha + \beta_1 Default_t + + \beta_2 Term_t + \beta_3 DY_t + \beta_4 Illiquidity_t + \beta_5 ADS_t + \beta_6 Wvol_t + \beta_7 Wdisp_t)$,:
where $D_t = 1$ when the state probability is greater than 50% (regime 1) and $D_t = 0$ otherwise (regime 2). The probability of being in regime 1 is modeled as a function of the following financial and business cycle variables: (a) the default premium (Default) defined as the difference between the return of US BBB and AAA corporate bonds (b) the term spread (Term) defined as the difference between the ten-year USA treasury constant maturity yield and the three month T-Bill rate (c) the world market dividend yield (DY) (d) world stock market liquidity (Liquidity) using the liquidity measure of Pástor and Stambaugh (2003) (e) the business conditions index (ADS) which is designed to track real business conditions (f) world stock market volatility (Wvol) calculated using daily world stock market returns and (g) world stock return dispersion (Wdisp) defined as the cross-sectional standard deviation at time t using all G7 markets stocks covered by DataStream. In particular we calculate monthly return dispersion as $\sqrt{\sum_{i=1}^{N} w_{i,t} (r_i - r_m)^2}$ where $r_i$ is the stock return, $r_m$ is the return of the capitalization weighted market portfolio, N is the number of stocks and $w_{i,t}$ is the market capitalization weight of stock $i$ in month $t-1$. Columns 1-7 show estimated coefficients, z-statistics (in parentheses) and McFadden R-squares for each variable using equation 2.

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| Constant | -2.487 | -0.500 | 0.086 | -0.815 | -0.832 | -2.046 | -7.186 | -7.511 |
| | (-10.219) | (-3.703) | (0.488) | (-9.800) | (-10.541) | (-9.579) | (-11.819) | (-9.262) |
| Default | 0.736 | | | | | | | 0.250 |
| | (7.755) | | | | | | | (1.489) |
| Term | | -0.107 | | | | | | 0.019 |
| | | (-1.751) | | | | | | (0.175) |
| DY | | | -0.321 | | | | | -0.031 |
| | | | (-4.615) | | | | | (-0.370) |
| Liquidity | | | | -3.883 | | | | 1.714 |
| | | | | (-3.284) | | | | (1.079) |
| ADS | | | | | -0.584 | | | -0.435 |
| | | | | | (-5.930) | | | (-2.447) |
| Wvol | | | | | | 36.347 | | -4.624 |
| | | | | | | (6.811) | | (-0.767) |
| Wdisp | | | | | | | 84.963 | 84.939 |
| | | | | | | | (10.646) | (8.641) |
| McFadden R-Sq | 0.132 | 0.008 | 0.058 | 0.035 | 0.094 | 0.220 | 0.522 | 0.552 |

**Table 5.** Global Factor Portfolios – Short Sales Allowed

Panel A presents the in-sample results for the market, optimal and regime-optimal portfolios. It shows the mean, standard deviation, Sharpe ratio, return loss (RL), information ratio (IR), turnover, and tracking error (TE), where: $RL_i = (SR_b - SR_i)\sigma_i$, $\mathrm{IR_i} = \frac{\overline{\mu_i - \mu_B}}{\sigma_{r_i - r_B}}$, $TE = \sigma_{r_i - r_B}$, $Turnover = 12 * \frac{1}{T-1}\sum_{t=1}^{T-1}\sum_{j=1}^{Assets}\left(\left|w_{j,t+1} - w_{j,t}\right|\right)$, Break Even Costs = (Portfolio Return – Benchmark Return)/Turnover. The benchmark for the optimal (regime-optimal) portfolio is the market index (optimal). All the statistics are on yearly basis. Panel B (C) presents the allocation to factor funds for the optimal single state and the regime-optimal when the tracking error is equal to 2% (5%). The sample period is from 07/30/1981 to 12/31/2012.

Panel A. Portfolio Descriptive Statistics

| | Unconstrained | | | TE2% | | TE5% | |
|---|---|---|---|---|---|---|---|
| | Market | Optimal | Regime-Optimal | Optimal | Regime-Optimal | Optimal | Regime-Optimal |
| Mean | 10.75% | 41.86% | 88.12% | 13.28% | 16.63% | 17.08% | 25.42% |
| Standard Deviation | 15.52% | 23.27% | 45.35% | 14.83% | 14.78% | 14.30% | 15.44% |
| Sharpe Ratio | 0.40 | 1.60 | 1.84 | 0.59 | 0.82 | 0.88 | 1.35 |
| Return Loss | | -28.03% | -10.86% | -2.94% | -3.37% | -6.82% | -7.34% |
| Information Ratio | | 1.27 | 1.53 | 1.27 | 1.67 | 1.27 | 1.67 |
| Turnover | | | 1117.8% | | 44.55% | | 111.81% |
| Break Even Cost vs Market | | | 6.92% | | 13.20% | | 13.12% |
| TE vs Benchmark | | 24.57% | 30.31% | 2.00% | 2.00% | 5.00% | 5.00% |
| TE vs Market | | | 45.51% | | 3.81% | | 9.52% |

Panel B. Market and Factor Fund Weights – Short Sales Allowed (tracking error 2%)

| | Market | HML | SMB | Momentum |
|---|---|---|---|---|
| Single State Optimal | 100 | 26 | 7 | 11 |
| Regime-optimal High Risk | 100 | 40 | 14 | 15 |
| Regime-optimal Low Risk | 100 | 60 | 14 | 35 |

Panel C. Market and Factor Fund Weights – Short Sales Allowed (tracking error 5%)

| | Market | HML | SMB | Momentum |
|---|---|---|---|---|
| Single State Optimal | 100 | 64 | 19 | 29 |
| Regime-optimal High Risk | 100 | 100 | 36 | 37 |
| Regime-optimal Low Risk | 100 | 149 | 37 | 89 |

**Table 6.** Global Factor Portfolios – No Short Sales Allowed

Panel A presents the in-sample results for the market, optimal and regime-optimal portfolios. It shows the mean, standard deviation, Sharpe ratio, return loss (RL), information ratio (IR), turnover, and tracking error (TE), where: $RL_i = (SR_b - SR_i)\sigma_i$, $\mathrm{IR_i} = \frac{\overline{\mu_i - \mu_B}}{\sigma_{r_i - r_B}}$, $TE = \sigma_{r_i - r_B}$, $Turnover = 12 * \frac{1}{T-1}\sum_{t=1}^{T-1}\sum_{j=1}^{Assets}(|w_{j,t+1} - w_{j,t}|)$, Break Even Costs = (Portfolio Return – Benchmark Return)/Turnover. The benchmark for the optimal (regime-optimal) portfolio is the market index (optimal). All the statistics are on yearly basis. Panel B (C) presents the allocation to factor funds for the optimal single state and the regime-optimal when the tracking error is equal to 2% (5%). The sample period is from 07/30/1981 to 12/31/2012.

Panel A. Portfolio Descriptive Statistics

| | | Unconstrained | | TE2% | | TE5% | |
|---|---|---|---|---|---|---|---|
| | Market | Optimal | Regime-Optimal | Optimal | Regime-Optimal | Optimal | Regime-Optimal |
| Mean | 10.75% | 15.76% | 15.80% | 12.31% | 13.96% | 14.64% | 15.80% |
| Standard Deviation | 15.52% | 14.45% | 14.49% | 14.81% | 14.52% | 14.26% | 14.49% |
| Sharpe Ratio | 0.40 | 0.78 | 0.78 | 0.52 | 0.65 | 0.71 | 0.78 |
| Return Loss | | -5.44% | -0.01% | -1.93% | -1.80% | -4.40% | -0.99% |
| Information Ratio | | 0.74 | 0.04 | 0.78 | 0.83 | 0.78 | 0.50 |
| Turnover | | | 21.0% | | 7.81% | | 20.95% |
| Break Even Cost vs Market | | | 24.05% | | 41.10% | | 24.11% |
| TE vs Benchmark | | 6.78% | 0.91% | 2.00% | 2.00% | 5.00% | 2.32% |
| TE vs Market | | | 6.54% | | 3.96% | | 6.54% |

Panel B. Market, and Factor Fund Weight – Short Sales Allowed (tracking error 2%)

| | Market | Value | Small | Momentum |
|---|---|---|---|---|
| Single State Optimal | 66 | 22 | 0 | 12 |
| Regime-optimal High Risk | 40 | 42 | 0 | 18 |
| Regime-optimal Low Risk | 23 | 42 | 0 | 35 |

Panel C. Market, and Factor Fund Weight – Short Sales Allowed (tracking error 5%)

| | Market | Value | Small | Momentum |
|---|---|---|---|---|
| Single State Optimal | 15 | 53 | 0 | 32 |
| Regime-optimal High Risk | 0 | 100 | 0 | 0 |
| Regime-optimal Low Risk | 0 | 76 | 0 | 24 |

**Table 7.** Global Factor Portfolios – Out of sample evidence

This table presents the out-of-sample evidence. We use an expanding window approach. More specifically we use monthly data over the period from July 1981 to December 2003 to estimate the parameters of the multivariate regime-switching model and calculate the optimal weights of the assets. To minimize turnover, the portfolio weights calculated at the end of the year are kept constant for the next twelve months and as forecast for next month's state of the market we use the next twelve months forecasts of state probability. We then add twelve more months in the dataset and repeat the described procedure. Panel A (B) presents the evidence when short sales are (not) allowed. The definitions of the descriptive statistics are given in tables 5 and 6. The out-of-sample period is from January 2004 to December 2012.

Panel A. Short Sales Allowed

| | | TE2% | | TE5% | |
|---|---|---|---|---|---|
| | Market | Optimal | Regime-Optimal | Optimal | Regime-Optimal |
| Mean | 6.50% | 8.22% | 11.70% | 10.76% | 19.34% |
| Standard Deviation | 15.93% | 15.82% | 16.01% | 16.07% | 17.44% |
| Sharpe Ratio | 0.30 | 0.41 | 0.62 | 0.56 | 1.01 |
| Return Loss | | -1.75% | -3.41% | -4.21% | -7.81% |
| Information Ratio | | 1.00 | 1.45 | 1.05 | 1.57 |
| Turnover | | 5% | 43% | 8% | 88% |
| Break Even Costs vs market | | 34.4% | 12.09% | 53.25% | 14.60% |
| TE vs Benchmark | | 1.71% | 2.39% | 4.07% | 5.46% |
| TE vs Market | | | 4.01% | | 9.18% |

**Panel B.** No Short Sales are allowed

| | | TE2% | | TE5% | |
|---|---|---|---|---|---|
| | Market | Optimal | Regime-Optimal | Optimal | Regime-Optimal |
| Mean | 6.50% | 7.24% | 8.33% | 8.34% | 9.21% |
| Standard Deviation | 15.93% | 15.56% | 15.29% | 15.26% | 16.11% |
| Sharpe Ratio | 0.30 | 0.36 | 0.43 | 0.44 | 0.47 |
| Return Loss | | -0.85% | -1.19% | -2.04% | -0.49% |
| Information Ratio | | 0.53 | 0.61 | 0.53 | 0.36 |
| Turnover | | 3% | 20% | 6% | 7% |
| Break Even Costs vs Market | | 24.66% | 9.15% | 30.67% | 36.22% |
| TE vs Benchmark | | 1.40% | 1.80% | 3.49% | 2.37% |
| TE vs Market | | | 3.18% | | 4.98% |

**Table 8. Descriptive and Correlation Analysis of the DJ Thematic Indexes**

This table presents monthly descriptive statistics for the Dow Jones U.S. thematic market neutral size, value, and momentum indices and for the theoretical global factors. The data are from January of 2002 to December 2012.

| Descriptive Statistics | | | | | | |
|---|---|---|---|---|---|---|
| | DJ Thematic Indexes | | | Theoretical Global Factor Portfolios | | |
| | Value | Size | Momentum | Value | Size | Momentum |
| Mean | 0.63% | 0.50% | 0.14% | 0.69% | 0.04% | 0.35% |
| Median | 0.44% | 0.37% | 0.69% | 0.66% | -0.12% | 0.56% |
| Maximum | 14.26% | 11.98% | 11.20% | 6.89% | 4.53% | 9.92% |
| Minimum | -5.33% | -6.16% | -27.25% | -3.37% | -5.24% | -20.54% |
| Standard Deviation | 2.94% | 2.76% | 4.87% | 1.67% | 1.64% | 3.47% |
| Return to Risk | 0.21 | 0.18 | 0.03 | 0.41 | 0.02 | 0.10 |
| Correlation against the DJ Indexes | | | | 0.57 | 0.19 | 0.89 |
| Tracking error against the DJ Indexes | | | | 8.33% | 10.13% | 8.19% |

**Table 9.** Using the DJ Thematic Market Neutral Indexes as Proxies of the Theoretical Portfolios – Out of sample evidence

This table presents the out-of-sample evidence when our estimates of global premia are replaced with the DJ thematic market neutral indices. The definitions of the descriptive statistics are given in tables 5 and 6. The out-of-sample period is from January 2004 to December 2012.

| | | TE2% | | TE5% | |
|---|---|---|---|---|---|
| | Market | Optimal | Regime-Optimal | Optimal | Regime-Optimal |
| Mean | 6.50% | 8.14% | 11.24% | 10.80% | 19.41% |
| Standard Deviation | 15.93% | 16.53% | 17.65% | 17.88% | 21.19% |
| Sharpe Ratio | 0.30 | 0.39 | 0.54 | 0.51 | 0.84 |
| Return Loss | | -1.46% | -2.66% | -3.72% | -6.92% |
| Information Ratio | | 0.88 | 1.22 | 0.93 | 1.51 |
| Turnover | | 5% | 43% | 8% | 88% |
| Break Even Costs vs market | | 34.4% | 12.09% | 53.25% | 14.60% |
| TE vs Benchmark | | 1.86% | 2.53% | 4.63% | 5.71% |
| TE vs Market | | | 4.33% | | 9.67% |

**Table 10. Descriptive and Correlation Analysis of the MSCI Global Value, Size, and Momentum Indexes**

This table presents descriptive statistics for the MSCI Global Value, Size, and Momentum indices and for the theoretical global portfolios. The data are from January of 1997 to December 2012.

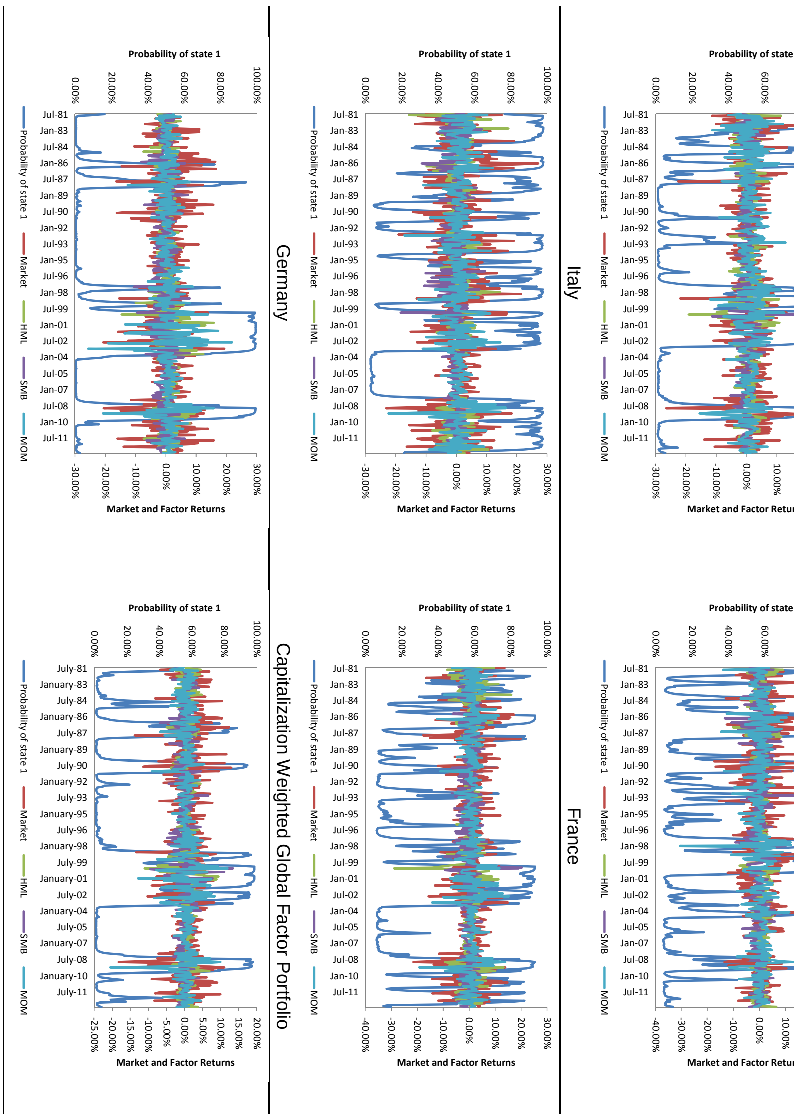
Descriptive Statistics

| | MSCI | | | Theoretical Global Factor Portfolios | | |
|---|---|---|---|---|---|---|
| | Value | Size | Mom | Value | Size | Mom |
| Mean | 0.61% | 0.75% | 0.93% | 1.02% | 0.70% | 0.96% |
| Median | 0.80% | 1.12% | 1.45% | 1.45% | 0.89% | 1.66% |
| Maximum | 14.27% | 17.22% | 17.96% | 12.35% | 13.75% | 10.44% |
| Minimum | -19.40% | -23.31% | -17.12% | -18.20% | -17.46% | -13.74% |
| Standard Deviation | 4.93% | 5.43% | 5.35% | 4.34% | 4.68% | 4.11% |
| Return to Risk | 0.12 | 0.14 | 0.17 | 0.23 | 0.15 | 0.23 |
| Correlation with MSCI Indices | | | | 0.91 | 0.94 | 0.88 |
| Tracking Error Against the MSCI Indexes | | | | 7.28% | 6.72% | 9.05% |

**Table 11** Using the MSCI Indices as Proxies of the Theoretical Long-only Factor portfolios – Out of sample evidence

This table presents the results from replacing for the out-of-sample study the returns of the global portfolios with that of the MSCI global indices (ACWA small cap, ASWI value standard and ACWI momentum standard). The definitions of the descriptive statistics are given in tables 5 and 6. The out-of-sample period is from January 2004 to December 2012.

| | | TE2% | | TE5% | |
|---|---|---|---|---|---|
| | Market | Optimal | Regime-Optimal | Optimal | Regime-Optimal |
| Mean | 6.50% | 7.04% | 7.91% | 7.88% | 10.54% |
| Standard Deviation | 15.93% | 16.24% | 16.69% | 16.78% | 18.73% |
| Sharpe Ratio | 0.30 | 0.33 | 0.37 | 0.37 | 0.47 |
| Return Loss | | -0.44% | -0.73% | -1.13% | -1.94% |
| Information Ratio | | 0.56 | 0.63 | 0.61 | 0.65 |
| Turnover | | 3% | 20% | 6% | 7% |
| Break Even Costs vs market | | 24.66% | 9.15% | 30.67% | 36.22% |
| TE vs Benchmark | | 0.96% | 1.39% | 2.29% | 4.10% |
| TE vs Market | | | 2.33% | | 4.31% |


Germany
Italy
Capitalization Weighted Global Factor Portfolio
France
Probability of state 1
Market and Factor Returns
Probability of state 1
Market
HML
SMB
MOM