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# ENGINEERING PROBLEMS IN MATHEMATICS LESSONS IN HIGHER EDUCATION

## Abstract:

It is a well-known problem that numerous disciplines and the connected labour market suffer from the lack of experts despite mass higher education. Subjects in engineering and natural sciences especially physics, chemistry and mathematics face a significant problem, namely interest of students turns away from these sciences. In engineering education in higher education, mathematics is a basic course in which students do not like to immerse deeply. Is there any way to recapture their interest in mathematics? A possible way could be teaching real-life problems, which can complete traditional education. As engineers solve real-life problems in their daily work their education should be practice-oriented, full of real-life problems.
In my presentation I would like to present some possibilities how can we connect mathematics with real life, what are the advantages and difficulties of using real life problems in lessons, what kind of technical mediators can help teachers to illustrate mathematical problems, what is the role of visualization in calculus, how can we make relations between abstract science and real world.

## Keywords:

real-life problems, word problems, visualization, higher education, mathematics teaching

## Introduction

It is a well-known problem that numerous disciplines and the connected labour market suffer from the lack of experts despite mass higher education. Globally, the shortage of professionals increased year by year in the last decade. Based on statistical numbers Japan, Peru and Brasilia faces the biggest lack. In the shortage occupation list we can find workman, sales representative and engineer in the top three in 2015 (10th Annual Talent Shortage Survey). Employers try the existing employees to participate in further trainings or recruit new employees so education has a great responsibility to release well-trained employees for the labour market.

In education in the field of engineering and natural sciences especially physics, chemistry and mathematics faces the biggest problems, interest of students turns away from these sciences.

R. Sutherland and S. Pozzi (1995) reported for the Engineering Council: „There is unprecedented concern amongst mathematicians, scientists and engineers in higher education about the mathematical preparedness of new undergraduates." This concern didn't change in the last years and it is almost universal around the world. They reported that the mathematical knowledge of first year undergraduate engineers is weaker and more variable nowadays than it was 10 years ago.

They identified two main reasons for these changes:

- university entrance requirements change, students could enter to university from vocational schools

- curriculum changes in pre-university education.

This research highlights that the mathematics content and the low level of requirements brought negative changes but there are many other social reasons. The Engineering Council in UK found a solution to renew the education for engineers. They distinguished two possible ways to be an engineer: the first way with more practice for incorporated engineers and the other way with more academic knowledge for chartered engineers. In practice-oriented education engineers study basic mathematical techniques and methods supported by the technologies. Furthermore research engineers study applied mathematics in connection with their specialization.

On the other hand we mustn't forget the aim of teaching mathematics, which influences the content. We need to consider mathematics from a broader perspective that Schoenfeld (1992) described as mathematical thinking. Mathematics courses often have the purpose to develop mathematical content knowledge but mathematical thinking includes problem solving strategies, metacognitive processes, resources, use of resources, beliefs and affects, and practices as well. Cardella's research (2008) proved that engineering students' mathematics education needs to contain all of these aspects.

## An aspect of mathematical knowledge: problem solving

Stanic and Kilpatrick (1988) distinguished three main themes in problem solving regarding its usage: problem solving as context, problem solving as skill, problem solving as art. In the first theme, problem solving is not the goal rather a process, which derives to reach other aims in the curricula. In this level problems can be identified:

- as a justification for teaching mathematics
- to provide specific motivation for subject topics
- as recreation
- as a means of developing new skills
- as practice.

In the second theme, skills work in hierarchy and the different kind of levels can be acquired one after another. Solving non-routine problems means a higher level skill than solving routine problems. The third theme means the real problem solving process. In engineering education it is useful to focus on all kind of theme that Stanic and Kilpatrick mentioned.

As engineers solve real-life problems in their daily work their education should be practice-oriented full of real-life problems. Real-life problems are drawn always as word problems. Word problems can be classified in several ways (Kulcsár, 2014):

- Problems to find and problems to prove (Polya, 1962)
- Qualitative and quantitative
- To prepare for a concept and to use a concept
- Well-structured and ill-structured (Simon, 1973)
- Open ended and closed
- One-step and multi-step problems
- Theoretical and practical

Polya – based on Euclid's Elements - classified the problems not by their subject but by their method of solution. The aim of a problem to prove is to decide whether the statement is true or false, to prove it or to refute it. The aim of a problem is to find an uncertain object based on conditions and data. Based on the subject the theoretical and practical classes are distinguished. The former is drawn in the language of mathematics and it contains only mathematical concepts. The latter can be divided into three more subclasses based on the realism of the problems:

- fiction
- lifelike
- real-life problems.

Teachers often use word problems which are drawn up in fictional situations with fictive figures and content. To solve these problems we need no special knowledge from different kind of sciences (only mathematics knowledge). These problems are useful to practice new formulas, concepts and contexts easily with little time investment. I call these tasks basic practical examples.

The next level needs more specific knowledge to understand the text of the problem which shows that it is not a general problem but it is connected to a specific profession. The technical terms show that what kind of specialization can understand the text. In this level we can teach this kind of practical problems to adequate students who study on the specific area and can understand the technical terms. These tasks do not need too deep understanding in other sciences so teachers can concentrate on the mathematical background without too much explanation in other fields.

Problems in the last level are the most specialized. The solution of them needs a lot of specific knowledge from different disciplines. The aim of these problems is not only to verify the use of mathematics in real life but to connect the mathematical knowledge with different areas and to connect the acquired knowledge with real life situations. These complex problems are the most time-consuming problems therefore teachers rarely present them in traditional lessons. Cross-curricular lessons however provide great opportunity for these complex problems to be solved. These kind of problems cannot be solved with routine algorithms that is why they are the most suitable to improve the heuristic thinking for engineers. Polya's problem solving process is perceived easily in these problems. At first the original problem has to be translated to the mathematical version of the problem. Then we need to find the solution of the mathematical version which needs to be interpreted in order to answer the original problem. Finally, we have to check the answer to see if it is realistic or unrealistic.

The following engineering problem is a lifelike problem. To understand the text students need some knowledge from the physics of motions.

## An example: Angular acceleration of a slider-crank mechanism

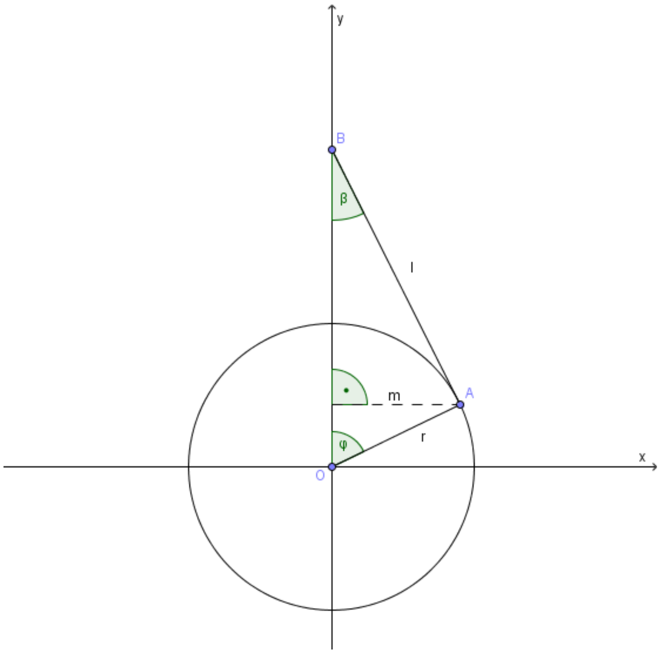
Take a slider-crank mechanism with a crank radius of $r$, the length of the connecting rod is denoted by $l$. The crank-shaft of the slider crank mechanism rotates with an angular velocity of $\omega = 1$ around the point $O$, point $B$ of the connecting rod slides along the $y$ axes. The connecting rod of the slider-crank mechanism performs a sweeping motion around point $B$, which results in large angular accelerations along the connecting rod. This angular acceleration multiplied by the mass of the connecting rod causes a distributed load perpendicular to the connecting rod. This distributed load tries to bend the connecting rod.

Find the extrema of the angular acceleration of the connecting rod.

**Figure 1: Slider-crank mechanism**

*Source: https://www.cgtrader.com/free-3d-models/vehicle/part/crank-piston-mechanism*

**Figure 2: Mathematical model of the problem**




*Source: own work in GeoGebra*

Based on Figure 2

$$m = l\ sin\beta = r\ sin\varphi \tag{1}$$

further

$$sin\beta = \frac{r}{l}\ sin\varphi. \tag{2}$$

$r$ and $l$ are constant, let

$$\frac{r}{l} = \lambda \tag{3}$$

further

$$sin\beta = \lambda\ sin\varphi.. \tag{4}$$

We know

$$\varphi = \omega\ t \tag{5}$$

$$\varphi = 1\ t \tag{6}$$

$$sin\beta = \lambda\ sint \tag{7}$$

$$\beta = arcsin(\lambda\ sint) \tag{8}$$

where $\left(-\frac{\pi}{2} < \beta < \frac{\pi}{2}\right)$.

Angular velocity of the connecting rod:

$$\omega = \frac{d\beta}{dt} = \frac{\lambda\, cost}{\sqrt{1-\lambda^2 sin^2 t}} = \lambda\, cost\, (1-\lambda^2 sin^2 t)^{-\frac{1}{2}}. \qquad (9)$$

Angular acceleration of the connecting rod due to the product rule:

$$\alpha = \frac{d\omega}{dt} = -\lambda\, sint\,(1-\lambda^2 sin^2 t)^{-\frac{1}{2}} - \lambda\, cost\,\frac{1}{2}(1-\lambda^2 sin^2 t)^{-\frac{3}{2}}\ \lambda^2 2\, sint\, cost =$$
$$\lambda\, sint\,(1-\lambda^2 sin^2 t)^{-\frac{3}{2}}\,(-(1-\lambda^2\, sin^2 t) - \lambda^2\, cos^2 t) = \lambda\, sint\,(1-\lambda^2\, sin^2 t)^{-\frac{3}{2}}\left(-1+\right.$$
$$\left.\lambda^2\,(sin^2 t + cos^2 t)\right) = \frac{\lambda\,(\lambda^2-1)\, sint}{(1-\lambda^2\, sin^2 t)^{3/2}}. \qquad (10)$$

To calculate the extrema of the angular acceleration we need the third derivative using the product rule again:

$$\frac{d\alpha}{dt} =$$
$$-\lambda\, cost\,(1-\lambda^2)(1-\lambda^2 sin^2 t)^{-\frac{3}{2}} - \lambda\, sint\,(1-\lambda^2)\left(-\frac{3}{2}\right)(1-\lambda^2 sin^2 t)^{-\frac{5}{2}}(-\lambda^2)2\, sint\, cost$$
(11)

$$-\lambda\, cost\,(1-\lambda^2)(1-\lambda^2 sin^2 t)^{-\frac{5}{2}}\,(1-\lambda^2\, sin^2 t + sin^2 t\, 3\lambda^2) = \frac{-\lambda\, cost(1-\lambda^2)(1+2\lambda^2\, sin^2 t)}{(1-\lambda^2\, sin^2 t)^{\frac{5}{2}}}. \qquad (12)$$

Numerator must be equal with 0:

$$-\lambda cost(1-\lambda^2)(1+2\lambda^2 sin^2 t) = 0. \qquad (13)$$

Hence $\lambda < 1$ $(1-\lambda^2) \neq 0$

further

$$1 + 2\lambda^2 sin^2 t \neq 0 \qquad (14)$$

than

$$cost = 0. \qquad (15)$$

Extrema of the angular acceleration are:

$$t = \pm\frac{\pi}{2} \qquad (16)$$

when the crank lines up with the $x$ axes.

When $t = \frac{\pi}{2}$

$$\frac{d^2\beta}{dt^2} = \frac{\lambda(\lambda^2-1)}{(1-\lambda^2)^{3/2}} = -\frac{\lambda}{\sqrt{1-\lambda^2}}. \qquad (17)$$

is the local minimum,

when $t = -\frac{\pi}{2}$

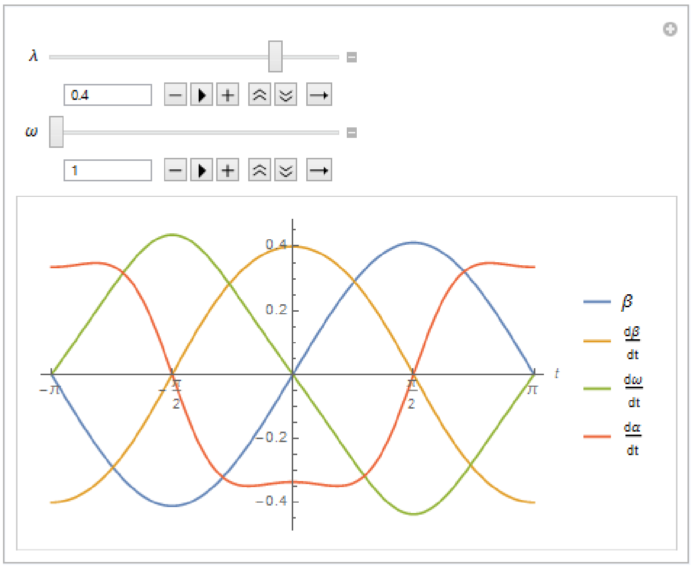
$$\frac{d^2\beta}{dt^2} = \frac{\lambda}{\sqrt{1-\lambda^2}} \qquad (18)$$

is the local maximum.

In theory $\lambda$ must be $0 < \lambda < 0.5$ based on Figure 2.

We can demonstrate the graphs of the function β and its first, second and third derivatives (Figure 3). Visualization help us to draw a conclusion from the zeroes about where local extrema are. In order to discover new correlations we can make an interactive manipulation of the values $\lambda$ and ω.

**Figure 3: Graphs of the function β and its first, second and third derivatives ($\lambda = 0.4$, $\omega = 1$)**




*Source: own work in Mathematica*

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