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CALCULATION OF CAPITAL-TO-OUTPUT RATIO IN A PRODUCTION FUNCTION BY ESTIMATING POTENTIAL OUTPUT AND OUTPUT GAP: THE CASE FOR THE CZECH REPUBLIC AND SLOVAKIA

Abstract:

One of the most used methods of estimation of potential output and output gap, used by many national and international organisations, is a production function. The aim of this paper is to study the impact of method of computation capital-to-output ratio on results of estimation of output gap and potential output, which are very important, but not measurable. We used two methods of computation. The first one was simple: we set it up constant. The second one was calculated according to a sophisticated model. The results of this paper have shown that using variable capital-to-output ratio will bring not very different results from using a constant one. These results were confirmed both for Czech economy and Slovak economy.

Keywords:

production function, capital-to-output ratio, potential output, HP filter

JEL Classification: E22, E24, E32

Introduction

Potential output, together with output gap, belong among important macroeconomic indicators which find a relative wide use by many macroeconomic calculations or predictions. Among the most frequently used utilization, we can mention prognosis of development of inflation in forthcoming quarters, when positive output gap indicates inflationary pressures and an increase in inflation, on the contrary a negative output gap indicates deflationary pressures and a decrease in inflation during forthcoming quarters. Potential product and output gap find use by evaluation of business cycle of analysed economy, too. All these calculations (or more precisely estimations) should be considered by decision making by many macroeconomic measures or recommendations, given by various national institutions like e.g. national banks of particular national economies, or various international institutions like European Central Bank, International Monetary Fund and OECD.

However it is important to be very careful with measuring of potential output and output gap, because these two variables are not measurable. The reason of this situation is easy: Both potential output and output gap are unobservable and thus cannot be measured with analogous accuracy to other macroeconomic variables.[1] Therefore by gaining of data of potential output and output gap, it is more considerate to talk about estimations than about measuring of these two variables.

From the end of the sixties up to now, there has been invented a relative large amount of methods of estimation of the potential output and output gap. In general, we can write that in the course of time relative simple methods of estimation of these two unobservable variables have been replaced by more complicated and sophisticated methods of estimation.

The oldest methods are characterized by ease of availability of required data and relative easy process of estimations, but their big disadvantage is their solely statistical character, which abstracts from any economic law, which characterizes economy and therefore it cannot explain any significant change in economy caused by various factors. Despite this negative fact, one method from this group, based only

[1] This paper does not deal with reliability of measuring other macroeconomic variables, like inflation or GDP. But in general, we can claim that reliability of measuring of these variables is much higher.

on statistical character, still belongs among the most used methods by many national and international organisations - Hodrick-Prescott filter (Hodrick and Prescott (1997)).

From the group of structural method of estimation the production function is the most often used method. Besides the Hodrick – Prescott filter, this method belongs among the most often used methods[2] for estimation of the two unobservable variables. Despite this method of estimation can explain some of economic laws which influence the economy, it still has its own disadvantages, e.g. there is still a necessity to use some statistical filter when we need to obtain potential values of important measures which we need for estimation of potential output. On the other hand, in these days, there are some sophisticated methods which belong among the most reliable methods of estimation (e.g. multivariate Hodrick-Prescott filter, multivariate unobserved component model or multivariate Beveridge – Nelson decomposition) that are not used very often. The answer to the question why is complicated, but one reason can be their high demands on required data and quite difficult process of estimation.

The aim of this paper is to analyse the production function as one of the most frequently used methods of estimation of potential output and output gap, more precisely the aim of this paper is to analyse the influence of capital-to-output ratio on results of estimation of potential output and output gap. Incorrectly estimated results of estimation can then negatively influence many macroeconomic calculations, which use data of output gap or potential product.

The analysis will be applied on economies of the Czech Republic and Slovakia. These economies are small open economies from Eastern Europe where there is a possibility, that these economies have not reached their steady state yet. Therefore there is a hypothesis, that capital-to-output ratio should not be considered as constant, but should be enabled to change during time periods.

The second chapter will deal with previous research of estimation of potential output and output gap for the Czech Republic and Slovakia. It will be shown, that there is only one older study that analyses the presumption of variable capital-to-output ratio for the Czech economy. The third chapter will introduce the used model, the fourth

[2] For example production functions (various types) is used for estimation Czech potential output and output gap by Ministry of Finance, Czech National Bank, OECD or IMF.

used data. The fifth chapter will show results of the research. It will be shown, that results of estimation of potential output with either constant or variable capital-to-output ratio for both analysed economies are not significantly different. In addition, our own calculated values of capital-to-output ratio will be compared with variable capital-to-output ratio calculated by OECD. It will be shown again, that the differences are not very significant, too.

2. Previous research

There is available quite wide range of analysis of potential output and output gap estimated for the large economies, but there is a substantially less amount of papers dealing with estimation of potential output and output gap for the Czech Republic and Slovakia. One from the first studies is the one from Hájek and Bezděk (2000), who used quite simple, but very often used methods for estimation of these two variables: Hodrick-Prescott filter and production function. They did not deal with the capital-to-output ratio, they only simply solved this problem with using the constant value for this variable. From other studies dealing with this problem, we can mention e. g. Beneš and N´Diaye (2004), who used a multivariate unobserved components model, then a study from Hurník and Navrátil (2005) or Dybczak, Flek, Hájková and Hurník (2006). From the newer studies analysing Czech potential output and output gap, we can mention a study from Plašil (2011), who analysed Hodrick-Prescott filter or Kloudová (2013), who tested some unobserved components model or Kloudová (2013), who analysed the ability of output gap to indicate inflation development in forthcoming quarters. On the contrary, there is only a few studies dealing with the influence of calculation of capital-to-output ratio on the estimation of potential output and output gap, especially for the Czech Republic. According to the author of this paper, only the study from Hájková a Hurník (2007) abandoned the simplifying presumption of constant capital-to-output ratio, because they considered an idea, that the Czech economy (like others economies from the Eastern Europe) did not reach its steady state.[3]

[3] An essay on suitability of setting the value of labour share to output and capital share to output on the value α = 2/3 and 1/3 for (1-α), which is generally accepted for the economy of United States of America, for others economies, too, is not solved in this paper. On the other hand, it is clear, that setting of the more suitable values would bring the more precise results of estimation of potential product and output gap.

The amount of studies dealing with the estimation of potential output and output gap for the Slovakian economy is even less than for the Czech economy. We can mention a study from Galabová (2005), who used unobserved component models and production function, but she simplified significantly the problem with the value of capital-to-output ratio with the method of calculation of average wage costs and value added (GDP). She did not even mention the possibility that Slovakian economy has not reached its steady state or the possibility of variable capital-to-output ratio. From other studies dealing with the Slovakian output gap and potential output we can cite Zimková and Bachorovský (2007) who used again relatively simple methods of estimation: a production function with constant returns to scale and simple univariate Hodrick- Prescott filter. More sophisticated methods for estimation of the Slovakian potential output and output gap used Kloudová (2013), who analysed these variables with several structural VAR models (so called SVAR models). According to the author, there is no study dealing with the variable capital-to-output ratio, which can influence results of estimation of output gap or potential output.

3. The model

For calculations in this research, the standard Cobb – Douglas production function was chosen, where there was considered a simplifying assumption of constant returns to scale. The potential output will be defined as a variable dependent on a product of total factor productivity A_t, capital stock K_t and total worked hours L_t. So, potential output can be written as follows:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

The forthcoming step will be to set up the determination of particular components to the growth of the potential output, where for this purpose a logarithmic version will be used in the forthcoming version:

$$\ln Y_t - \ln Y_{t-1} = \left(\ln A_t - \ln A_{t-1}\right) + \left(\ln K_t^{\alpha} - \ln K_{t-1}^{\alpha}\right) + \left(\ln L_t^{1-\alpha} - \ln L_{t-1}^{1-\alpha}\right) = A' + K' + L' \tag{2}$$

Then capital contribution to the growth of potential product can be defined as follows:

$$K_1' = \ln K_t^{\alpha_1} - \ln K_{t-1}^{\alpha_1} \tag{3}$$

And subsequently, we can write that capital share on the growth of potential output is equal to the ratio of two parameters α.

$$\frac{K_1'}{K_2'} = \frac{\alpha_1}{\alpha_2} \tag{4}$$

Logically, then we can write for the labour share on the growth of the potential output a following mathematical relationship:

$$\frac{L_1'}{L_2'} = \frac{1-\alpha_1}{1-\alpha_2} \tag{5}$$

The variable total hours worked L_t is rewritten and production function will have a following form, where under E_t unemployment will be understood and HW_t will be mean amount of worked hours on employment.

$$Y_t = A_t K_t^{\alpha} \left(E_t \frac{HW_t}{E_t} \right)^{1-\alpha} \tag{6}$$

Variable unemployment E_t will be divided into a population in productive age (15-64 years), participation rate of this age group *part* and unemployment rate non-accelerating inflation $NAIRU_t$, related to the population age between15-64 years.

$$E_t = pop_t \cdot part_t \cdot \left(1 - NAIRU_t\right) \tag{7}$$

Capital stock will be calculated according to Mourré (2009) with the method of continuous inflatory method, which is equal to the sum of capital stock from the previous year adjusted from depreciation rate, which responds to the value of 5%.

However this method of estimation of potential output and output gap has a disadvantage, because to obtain the potential levels of variable from the relationship (6), it is important to use some statistical filter. The most frequently used statistical filters are Hodrick-Prescott filter and then band-pass filters, too, especially Baxter-King filter and Christiano-Fitzgerald filter. For our purposes, we used Christiano-Fitzgerald filter.

To gain the capital-to-output ratio, there were used two methods. The first one was that we have chosen constant value of this variable. It was set to 0.35 for the entire

length of analysed time series, according to d´Auria et al. (2010). The second choice was calculation of capital-to-output ratio according to Freedman (2011), which enabled us to make capital-to-output ratio variable during the analysed time series.

The used formula according to Freedman (2011) is:

$$\alpha_t = 1 - \frac{NZ_t}{HDP_t} \cdot \frac{Z_t + S_t}{Z_t} \tag{8}$$

Where GDP_t is gross domestic product, NZ_t is compensation of employees, Z_t amount of employees and S_t is amount of self-employees.

Results of our research are shown in the chapter 5.

4. Data

All data used for calculation of capital-to-output ratio were downloaded from the statistical database of EUROSTAT, only the data of the variable capital-to-output ratio, which were used for comparison with the own calculated data, were downloaded from the database of OECD. The length of analysed time series was 19 years between 1996 and 2013, where the length of time series was chosen mainly due the availability of required data for this research. All data were used with the quarterly periodicity.

5. RESULTS

Results of calculation of capital-to-output ratio based on the above mentioned model are shown in the figure No.1. For the possibility of comparison, capital-to-output ratio calculated by OECD was added into the figure for the same time period. If we took a look at the figures, we would see that our own method for calculation has brought similar but not exactly the same results for the whole length of chosen time period. The question however remains unanswered, which of the two methods , has brought more accurate results. The same situation revealed in both Czech and Slovak economies.

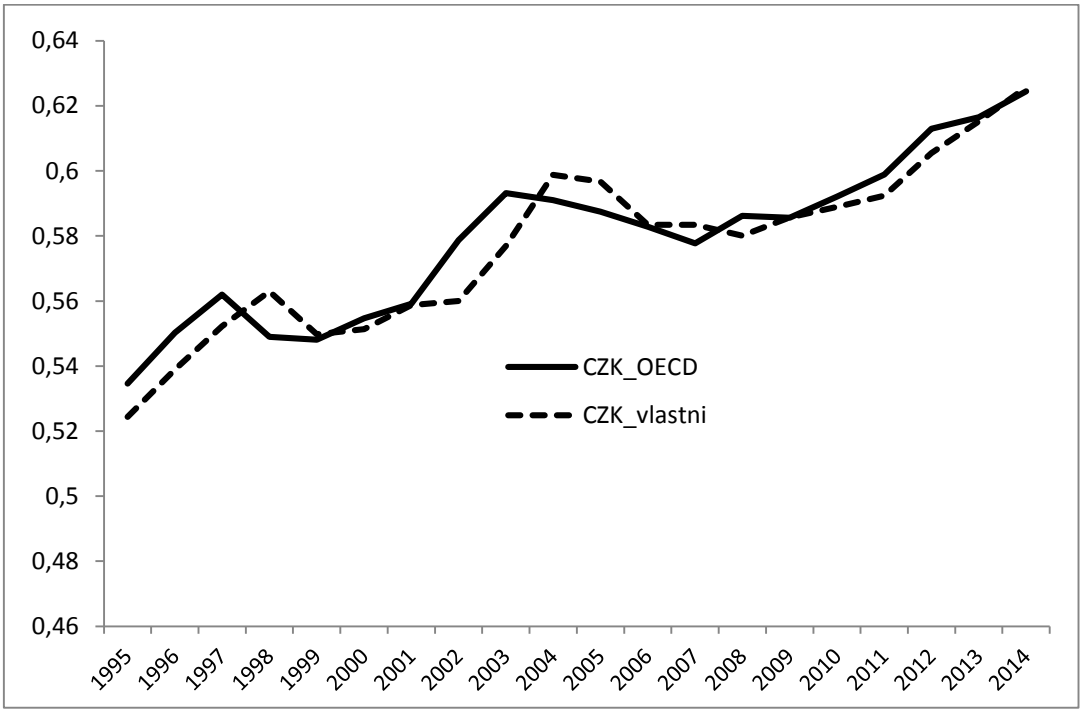
At the end of the nineties, there occurred the growth of the capital-to-output ratio, where our own calculation has brought higher results than data from OECD. At the end of 20th century and in the beginning of 21st century, capital-to-output ratio

decreased, where OECD calculated lower numbers again. Between 2001 and 2002, capital-to-output ratio started to increase and this situation lasted to 2004. In these years, OECD calculated higher values than our research. From 2004 to 2008, capital-to-output ratio had a decreasing tendency, which was replaced with the increase in 2009 and this trend lasted to the end of the analysed time series. In general, we can conclude that although these two different methods of calculation of capital-to-output ratio calculated different values, both methods calculated if not the same, then very similar trend. Only for a very short time there occurred situation when one method calculated opposite trend than the other.

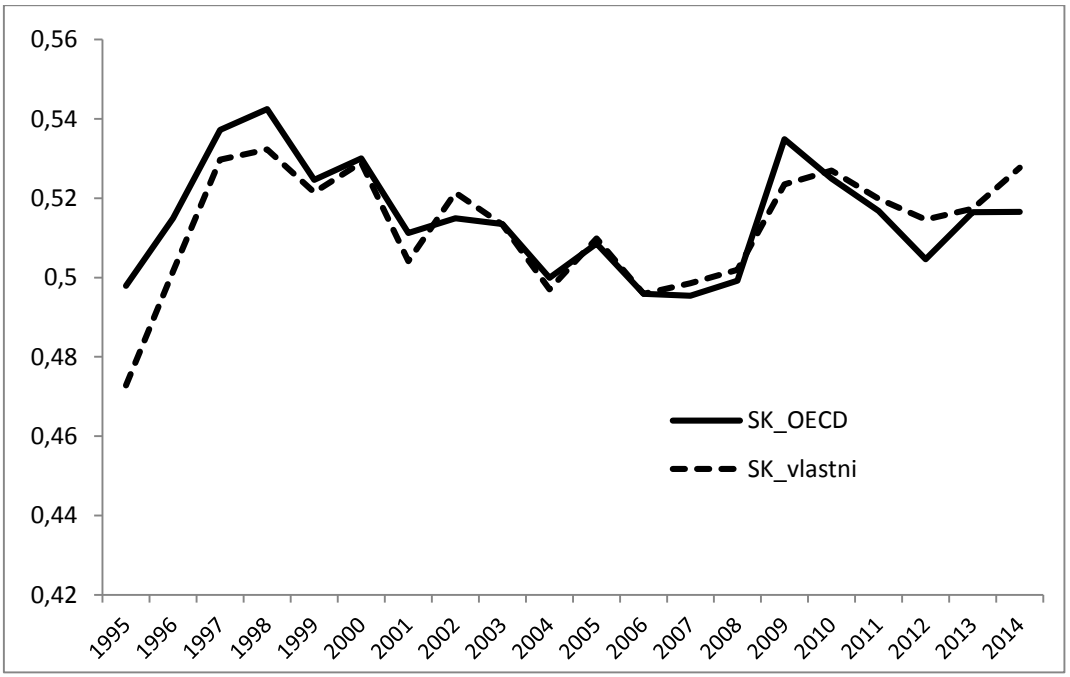
Very similar situation occurred in Slovakia, too. Both methods calculated different values for the same time period, but rarely calculated strictly opposite trend. The difference from the Czech economy is more frequent decrease and increase in the values, whereas capital-to-output ratio in the Czech economy has not so significant fluctuation. Another difference between Czech and Slovakian economies is the lower maximum and minimum, which occurred in Slovakia. If we took a look at the figures No. 1 and 2, we would see, that the lowest value for the Slovakian economy was 0.48, whereas in the Czech economy it was 0.52. The difference between the highest value in the Czech and Slovakian economy is quite significant (approximately 10 p.p.)

Table No. 1: Capital-to-output ratio: comparison of our values with OECD

a) The Czech Republic



b) Slovakia



Source: author´s own figures, data from OECD and EUROSTAT

The comparison between values obtained from the calculation with the constant and variable capital-to-output ratio for the Czech Republic is shown by tables No. 1. The table a) shows the growth of potential output and contributions of total factor productivity, labour and capital to this growth. If we looked at both tables and compared the values for each year, we would conclude, that these values are not significantly various. For example, in 2006 the growth of potential output with constant value of capital-to-output ratio, calculated growth of this variable is 4.8%. On the other side, the growth of potential output with variable capital-to-output ratio is 4.9%. Total factor productivity is the same for both calculations, 3.8%, contribution of labour (calculated with constant parameter α) is 0.3%, whereas with variable parameter α it is 0.5%. A small difference is by the capital, too (0.7% with the constant parameter α and 0.8% with variable parameter α). The same growth of potential output was calculated for years 2007, 2008, 2011, 2013 and 2014. If we made an average of calculated values for the particular years, we would get the same values for the variable and constant capital-to-output ratio, too.

Table No. 1: Average annual growth of potential product and particular contributions to the growth for the Czech economy

a) Constant capital-to-output ratio

	PP (%)	TFP (%, p.b.)	L (p.p.)	Capital (p.p.)
2006	4,8	3,8	0,3	0,7
2007	3,9	2,6	-0,2	1,1
2008	3,2	1,7	0,1	1,4
2009	2,2	0,9	0,4	0,9
2010	0,8	0,4	0,3	0,1
2011	0,6	0,1	0,5	-0,1
2012	0,7	0,1	0,5	0,1
2013	0,6	0,2	0,5	0,2
2014	0,7	0,1	0,5	0,1

b) Variable capital-to-output ratio

	PP (%)	TFP (%, p.b.)	Prácovní síla (p.p.)	Zásoba kapitálu (p.p.)
2006	4,9	3,8	0,5	0,8
2007	3,9	2,6	-0,3	1,1
2008	3,2	1,7	0	1,5
2009	2,1	0,9	0,3	0,9
2010	0,8	0,4	0,4	0,1
2011	0,7	0,1	0,5	0
2012	0,6	0,1	0,4	0,1
2013	0,6	0,2	0,5	0,2
2014	0,7	0,1	0,5	0,1

Source: author´s own tables, data from OECD and EUROSTAT

Quite similar situation occurred for the Slovakian economy. The Table 2 a) shows values obtained with calculations with constant capital-to-output ratio and table 2 b) shows values calculated with variable capital-to-output ratio. The only small difference from the Czech economy is the fact, that these both methods calculate less years with the equal value of the growth of potential output: if we looked at the tables, we would see, that this situation occurred only in the last two years, 2013 and 2014 (the same values 0.9 and 1.1 for both calculations). But still the differences are not very significant; for example, in 2006 calculations with the constant capital-to-output ratio for the growth of potential output bring value 4.17%, whereas for the constant parameter α it was only 4.09%. The contribution of total factor productivity with the constant parameter α was 2.7% and with the variable parameter it was 2.5%. Labour contribution for year 2006 was 1.09 % for constant parameter and 1.2% for variable parameter. The difference between capital contribution in this year was only 0.01 p.p. If we looked at all the years, we could conclude, that calculations with constant and variable capital-to-output ratio brought not very different results. At that, if we calculated the average of calculated values, we would obtain the same values (this holds for all variables: the growth of the potential output, contributions of total factor productivity, capital and labour, too).

Table No. 2: Average annual growth of potential product and particular contributions to the growth for the Slovak economy

a) Constant capital-to-output ratio

	PP (%)	TFP (%, p.b.)	Prácovní síla (p.p.)	Zásoba kapitálu (p.p.)
2006	4,17	2,7	1,09	0,38
2007	4,03	2,9	0,5	0,63
2008	2,7	1,9	0,3	0,5
2009	1,8	0,9	0,4	0,5
2010	1,1	0,7	0,3	0,1
2011	0,9	0,3	0,6	0
2012	0,8	0,3	0,4	0,1
2013	0,9	0,1	0,5	0,3
2014	1,1	0,6	0,2	0,3

b) Variable capital-to-output ratio

	PP (%)	TFP (%, p.b.)	L (p.p.)	Capital stoc (p.p.)
2006	4,09	2,5	1,2	0,39
2007	4,1	2,4	0,8	0,9
2008	2,5	1,7	0,5	0,3
2009	1,7	0,8	0,4	0,5
2010	1,3	0,8	0,5	0
2011	0,8	0,5	0,2	0,1
2012	0,7	0,5	0,2	0
2013	0,9	0,2	0,6	0,1
2014	1,1	0,4	0,5	0,2

Source: author´s own tables, data from OECD and EUROSTAT

Conclusion

Potential output and output gap belong among very important macroeconomic indicators which can find their relative wide use by many macroeconomic calculations and decision making. Therefore it is very important to obtain the most reliable data, because the wrong data can influence other calculations negatively.

Although there is a relatively wide range of possibilities how to estimate potential output and output gap, many national and international institutions prefer the relatively simple methods of estimation, primarily Hodrick-Prescott filter or production function. The reason why these institutions prefer these methods, although they are not the most reliable methods, was not the aim of this paper.

The aim of this paper was only to analyse the impact of capital-to-output ratio on calculation or more precisely the estimation of production function, because we considered the idea, that both Czech and Slovakian economies have not reached their steady state yet. So we tested, if this consideration will significantly influence the results.

For this purpose, we have chosen standard Cobb-Douglas production function with the constant returns to scale (the best type of production function has not been selected yet either). The first calculations enabled capital-to-output ratio to be variable and then we calculated with the constant capital-to-output ratio.

If we should answer shortly, whether the variable capital-to-output ratio influences the results significantly, we would say that it does not. For both Slovakian and Czech economies, the difference between calculations with constant and variable capital-to-output ratio were not very significant. On the other hand, for some years, the two methods calculated the same values of the growth of the potential output, for the Czech economy this situation occurred in the years 2007, 2008, 2011, 2013 and 2014, for the Slovakian economy this situation occurred only in the last two years, 2013 and 2014. Even if comparing the results only in the years with the different results, it is not possible to claim, that these differences were significant. Besides this, if we made an average of calculated values, we would get the same values for the variable capital-to-output ratio and the constant capital to ratio, too.

The less important aim of this paper was to compare our calculated values of capital-to-output ratio with the results from OECD, which used their own method for calculation of this variable. The results of comparison have shown that the calculated values were not the same in any year, but the values were very similar and rarely these both methods came with the strictly opposite trend. This situation occurred in both Czech and Slovakian economies. The only and just a little important difference between these two economies was, that in Slovakian economy, this variable fluctuated more significantly and reached lower values than in the Czech economy.

It is important to mention, that this paper did not analyse the reasons why the values of capital-to-output ratio are so high or low, but we believe this deserves a further separate research.

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