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ESTIMATING AND FORECASTING VALUE-AT-RISK USING THE UNBIASED EXTREME VALUE VOLATILITY ESTIMATOR

Abstract:

We provide a framework based on the unbiased extreme value volatility estimator (Namely, the AddRS estimator) to compute and predict the long position and a short position VaR, henceforth referred to as the ARFIMA-AddRS-SKST model. We evaluate its VaR forecasting performance using the unconditional coverage test and the conditional coverage test for long and short positions on four global indices (S&P 500, CAC 40, IBOVESPA and S&P CNX Nifty) and compare the results with that of a bunch of alternative models. Our findings indicate that the ARFIMA-AddRS-SKST model outperforms the alternative models in predicting the long and short position VaR. Finally, we examine the economic significance of the proposed framework in estimating and predicting VaR using Lopez loss function approach so as to identify the best model that provides the least monetary loss. Our findings indicate that the VaR forecasts based on the ARFIMA-AddRS-SKST model provides the least total loss for various x% long and short positions VaR and this supports the superior properties of the proposed framework in forecasting VaR more accurately.

Keywords:

Extreme value volatility estimator; Value-at-risk; Skewed Student t distribution; Risk management.

JEL Classification: C22, C53

1. Introduction

Value-at-risk (VaR) is widely used as a measure of market risk by financial institutions, regulators, business practitioners and portfolio managers. It is defined as the maximum potential loss that could be experienced by a portfolio at a given level of confidence over a given time horizon. The value-at-risk approach was first introduced by J.P.Morgan (Longerstaey and Spencer, 1996). It is used by financial institutions to determine the minimum capital requirement they need to deal with any catastrophic event in the market. It can be helpful in designing and implementing appropriate risk management policies against uncertain events. The literature provides various approaches to compute VaR which include non-parametric, semi-parametric and parametric approaches. The validity of any VaR approach is usually assessed by computing the number of exceptions by comparing the trading losses with the estimated VaR. The violation of the given VaR level by actual loss results in an 'exception'. This can also be related to predicting the tail probability. The literature also emphasizes the importance of the assumption of fat tails in estimating and predicting VaR (Bollerslev et al., 1992, Pagan, 1996, Palm, 1996). Various studies also highlight the importance of considering a possible asymmetry in the distribution of returns when estimating VaR (Barndorff-Nielsen, 1997, Giot and Laurent, 2004). In this study, we consider both asymmetry and fat tails in the estimation of VaR that is based on the conditional unbiased high-low volatility estimator to examine the dissimilar behavior of VaR for long and short positions.

The volatility estimators based on the high and the low have been acknowledged as being highly efficient in the finance literature. The daily open, high, low and close prices of most of the tradable assets are easily available. The different variants of the extreme value volatility estimators can be categorized as: method of moments estimators (Parkinson, 1980, Garman and Klass, 1980, Rogers and Satchell, 1991, Kunitomo, 1992, Yang and Zhang, 2000) and maximum likelihood (ML) estimators (Ball and Torous, 1984, Magdon-Ismail and Atiya, 2003, Horst et al., 2012). The ML estimators are efficient under ideal conditions; however, from a practical viewpoint, they suffer from a serious disadvantage. Among the method of moments estimators, the Rogers and Satchell (1991), hereafter referred as the RS estimator, stands out because it is the only one that is unbiased regardless of the drift parameter whereas all others are biased in one way or another if the mean return (drift) is non-zero. Kumar and Maheswaran (2013b) find that the RS estimator is severely downward biased when implemented in the data because of the random walk effect and propose the additive bias correction for the RS estimator, called herein the AddRS estimator, and show theoretically and empirically that it is unbiased. Kumar and Maheswaran (2013a) examine the statistical properties of the logarithm of AddRS (Log(AddRS)) estimator and find that its distribution is approximately Gaussian and hence based on the suggestion of Andersen et al. (2003) a linear Gaussian model can be applied to model the logarithm of AddRS estimator. We use the conditional volatility model based on the AddRS estimator for computing and predicting VaR in this study and compare the findings with various parametric alternative models.

In this paper, we provide a way of estimating and predicting long and short position VaR based on the unbiased AddRS estimator. We compare its forecasting performance with the forecasting performance of alternative models from the GARCH family and the range-based CARR model. We apply the Kupiec (1995) unconditional coverage test and the Christoffersen (1998) conditional coverage test to backtest the VaR models to assess their statistical accuracy in capturing upside and downside risk. Finally, we examine the economic significance of the proposed framework using Lopez (1998) loss function approach to select the best performing VaR model from among competing models that pass the unconditional coverage test and conditional coverage test as well as that provides the least monetary loss. Our findings are in favour of the VaR model based on the AddRS estimator in estimating and predicting more accurate VaR.

The remainder of this paper is organized as follows: Section 2 presents the theoretical background of the AddRS estimator. Section 3 describes the data and computational procedure to construct the AddRS estimator. Section 4 explains the methodology for estimation and evaluation of VaR models. Section 5 reports the empirical results and section 6 concludes with a summary of our main findings.

2. Data and procedure to construct AddRS estimator

2.1. Data

We use the daily open, high, low and closing prices of four global stock indices: Namely, Standard & Poor 500 (S&P 500), a free-float capitalization-weighted index of prices of 500 large cap stocks actively traded on United States stock exchanges; CAC 40, a capitalization-weighted index of the prices of 40 highest market cap stocks listed on the Paris Bourse (Euronext, Paris); IBOVESPA, an accumulation index of about 50 stocks traded on the São Paulo Stock, Mercantile & Futures Exchange covering 70% of the value of the stocks traded; and S&P CNX Nifty, the broad based benchmark of the Indian capital market. This covers the major developed markets (S&P 500 and CAC 40) from the United States and Europe and major emerging markets (IBOVESPA and S&P CNX Nifty) from Latin America and Asia. The sample period for all the indices is from January 1996 to May 2015. All the data have been collected from the Bloomberg database. In the following tables, we use Nifty to represent the S&P CNX Nifty index.

2.2. Constructing AddRS estimator

Suppose O_t, H_t, L_t and C_t are the opening, high, low and closing prices of an asset on day *t*. Define:

$$b_t = \log\left(\frac{H_t}{O_t}\right)$$

$$c_t = \log\left(\frac{L_t}{O_t}\right)$$

$$x_t = \log\left(\frac{C_t}{O_t}\right)$$

Let $u_t = 2b_t - x_t$ and $v_t = 2c_t - x_t$. Hence, the bias corrected extreme value estimators are given by:

$$Add\ ux = \frac{1}{2}(u_t^2 - x_t^2) + x_t^2.\mathbf{1}_{\{b_t=0\ or\ x_t=b_t\}} \qquad (1)$$

and

$$Add\ vx = \frac{1}{2}(v_t^2 - x_t^2) + x_t^2.\mathbf{1}_{\{c_t=0\ or\ x_t=c_t\}} \qquad (2)$$

Therefore, the unbiased AddRS estimator is given as:

$$AddRS = \frac{1}{2}[Add\ ux + Add\ vx] \qquad (3)$$

Kumar and Maheswaran (2013a) examine the distributional properties of the unconditional AddRS estimator and the logarithm of AddRS (Log(AddRS)) estimator and find that the logarithm of the AddRS estimator is approximately Gaussian. Based on the suggestion by Andersen et al. (2003), they made use of a ARFIMA model to generate forecasts for the AddRS estimator.

3. Methodology

3.1. Statistical approaches to compute Value-at-Risk

Suppose r_1, r_2, ……, r_n represent returns on a financial asset. Suppose FI = $Pr(r_t < r|\Omega_{t-1})$ is the cumulative distribution function conditional of the information set (Ω_{t-1}) at time t-1. Suppose the dynamics of returns follows the stochastic process:

$$r_t = \mu_t + \varepsilon_t = \mu_t + z_t\sigma_t \quad , \quad z_t \sim iid\ (0,1) \qquad (4)$$

where $\sigma_t^2 = E(z_t^2|\Omega_{t-1})$ and z_t has a conditional distribution function G(z), G(z) = $Pr(z_t < z|\Omega_{t-1})$. Suppose VaR(α) is the α quantile of the probability distribution of financial returns:

$$F(VaR(\alpha)) = Pr(r_t < VaR(\alpha)) = \alpha \quad or \quad VaR(\alpha) = \inf\{v \mid P(rt \le v) = \alpha\}$$
(5)

The α-quantile can be estimated in two ways. Firstly, by inverting the distribution of returns FI and it is given as:

$$VaR(\alpha) = F^{-1}(\alpha)$$
(6.1)

and secondly, by inverting the distribution function of the innovations G(z) which also requires the estimation of σ_t^2.

$$\text{VaR}(\alpha) = \mu_t + \sigma_t G^{-1}(\alpha) \qquad (6.2)$$

In this paper, we estimate VaR using the autoregressive fractionally integrated moving average (ARFIMA(p, d, q)) model for the logarithm of the AddRS estimator, the conditional autoregressive range (CARR) model for the trading range, the generalized autoregressive conditional heteroskedasticity (GARCH) model, the exponential GARCH (EGARCH) model, the fractionally integrated GARCH (FIGARCH) model and the Risk Metrics model based on the skewed Student-t distribution which the focus being on estimating $G(z_t)$. Moreover, we compute the VaR for the long position (the left tail of the probability distribution) as well as the short position (the right tail of the probability distribution). Suppose the VaR of a long position are given by equations (6.1) and (6.2), then the VaR of the short position can be computed by taking the α-quantile from the right tail, i.e., by taking 1-α in place of α.

$$\text{VaR}(1-\alpha) = F^{-1}(1-\alpha) \qquad (7.1)$$

$$\text{VaR}(1-\alpha) = \mu_t + \sigma_t G^{-1}(1-\alpha) \qquad (7.2)$$

3.2. ARFIMA based conditional volatility model for the AddRS estimator

The statistical and distributional properties of the logarithm of the AddRS estimator suggest that a long memory Gaussian autoregressive model would be appropriate to model the dynamics in the logarithm of the AddRS estimator. Hence, we consider a univariate autoregressive fractionally integrated moving average (ARFIMA(p,d,q)) for estimating and forecasting conditional the AddRS estimator with appropriate transformations. The general ARFIMA(p,d,q) model is given as:

$$\big(1-\phi(L)\big)(1-L)^d\,\big(log(AddRS)\big) = \big(1+\ \theta(L)\big)\varepsilon_t \qquad (8)$$

where ε_t is the residual term with mean zero and variance σ_ε^2, L is the lag operator, d is the fractional difference parameter which measures the degree of long memory and $0<d<1$, $\phi(L)$ and $\theta(L)$ are polynomials in the lag operator of orders p and q, respectively. The orders of the ARFIMA model are based on the Schwarz information criterion (SIC).

When $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, then by definition, $\exp(\varepsilon_t) \sim \text{logN}(0, \sigma_\varepsilon^2)$ (where logN denotes the log-normal distribution). Therefore, the conditional AddRS estimator is computed as:

$$AddRS_{t|t-1} = exp\left(log(AddRS_t) - \hat{\varepsilon}_t + \frac{1}{2}\hat{\sigma}_\varepsilon^2\right) \qquad (9)$$

where $\hat{\varepsilon}_t$ and $\hat{\sigma}_\varepsilon^2$ are estimated using the ARFIMA model given in equation (8).

We use the approach of Giot and Laurent (2004) in generating 1-day ahead forecasts of VaR based on the AddRS estimator. Let us assume that the conditional variance σ_t^2 of the return series r_t is proportional to $AddRS_{t|t-1}$, that is,

$$\sigma_t^2 = \rho Add\ RS_{t|t-1} \tag{10}$$

Let us assume that the conditional mean of the process follows some AR(p) process.

Hence, we need to estimate the following model:

$$r_t = \mu_t + \sqrt{\rho AddRS_{t|t-1}} z_t \tag{11}$$

Where ρ is the additional parameter to be estimated to make the variance of z_t equal to unity. In this specification, it needs to be noted that the dynamic characteristics of the conditional variance is captured by the ARFIMA model. Based on the recommendation by Giot and Laurent (2004), we assume that z_t follows the skewed Student-t distribution (SKST), i.e., $z_t \sim$ i.i.d. SKST(0,1,ξ,υ). The density of the skewed Student-t distribution is given as:

$$f(z_t|\xi,\upsilon) = \begin{cases} \frac{2s}{\xi + {}^1/_\xi} g(\xi(sz_t + m)|\upsilon), z_t < -m/s \\ \frac{2s}{\xi + {}^1/_\xi} g((sz_t + m)/\xi|\upsilon), z_t \geq -m/s \end{cases} \tag{12}$$

where g(·|υ) is the density of symmetric Student-t distribution, υ is the scale coefficient, ξ is the coefficient of asymmetry and m and s are the mean and the standard deviation of the nonstandard skewed Student-t distribution, respectively.

Hence, based on the 1-day ahead forecasts of μ_t and σ_t, the VaR forecasts for the long and the short position can be generated by using equations (6) and (7) respectively.

3.3. Backtesting VaR models

Backtesting is a method to identify whether a given VaR model adequately captures the real losses that the asset is exposed to. First, we apply the unconditional coverage test proposed by Kupiec (1995) to examine whether the frequency of exceedances satisfy the confidence level of VaR. Next, we apply the conditional coverage test proposed by Christoffersen (1998) which additionally looks at whether the violations are randomly distributed.

3.3.1. Unconditional coverage test

We utilize the Kupiec (1995) likelihood ratio test which considers the cases when the asset return exceeds the estimated VaR as an independent event arising from a binomial distribution. Suppose the confidence level is 1 – α, the sample size is T, the number of days of failure is N, the frequency of failure is given as *f* = N/T. A significant difference between *f* and α indicates a misspecification of the VaR model and this hypothesis can be tested by a likelihood ratio test statistic:

$$LR_{uc} = 2\ln[(1-f)^{T-N} f^N] - 2\ln[(1-\alpha)^{T-N}\alpha^N] \tag{13}$$

Under the null hypothesis, $LR_{uc} \sim \chi^2(1)$, and its critical value at 95% confidence level is 3.84. If LR_{uc} is larger than this critical value, the null hypothesis is to be rejected, which would indicate that the VaR model is inadequate.

3.3.2. Conditional coverage test

The conditional coverage test jointly tests whether (1) the frequency of failure is in line with α, and (2) the VaR violations are independently distributed over time (Christoffersen, 1998). The null hypothesis that the VaR violations are independent and the expected frequency of violation is equal to α can be tested by the following likelihood ratio test statistic:

$$LR_{cc} = -2\log[(1-\alpha)^{T-N}\alpha^{N}] + 2\log\left[(1-\pi_{01})^{n_{00}}\ \pi_{01}^{n_{01}}\ (1-\pi_{11})^{n_{10}}\ \pi_{11}^{n_{11}}\right] \qquad (14)$$

where $LR_{cc} \sim \chi^2(2)$, n_{ab} is the number of observations with value a followed by b, for a, b, = 0, 1 and $\pi_{ab} = n_{ab}/\sum_b n_{ab}$ is the probability of observing an exception, conditional on state a. The values of a, b = 1 indicates that a failure has occurred. The critical value of LR_{cc} at 95% confidence level is 5.99.

4. Empirical results

4.1. The ARFIMA-AddRS-SKST model

Based on the statistical properties of the Log(AddRS) estimator as shown in Kumar and Maheswaran (2013a) for the same data series, we first estimate the ARFIMA(*p,d,q*) model to capture the dynamics in the Log(AddRS) estimator of the chosen indices. We make use of the Schwarz information criterion (SIC) to select the appropriate orders of ARFIMA model for Log(AddRS) of all the indices under study. We find (0,*d*,1), (1,*d*,1), (2,*d*,0) and (1,*d*,1) being appropriate orders for S&P 500, CAC 40, IBOVESPA and S&P CNX Nifty, respectively. In the second step, we first identify the appropriate orders for the conditional mean equation for r_t using the Schwarz Information Criterion. Table 1 reports the estimation results of the ARFIMA-AddRS-SKST model. Panel I in Table 1 reports the ARFIMA(p,d,q) estimates based on the first step and panel II reports the results for the conditional mean and conditional variance equations as explained above in the second step.

Table 1: Results of ARFIMA-AddRS-SKST model

	S&P 500	CAC 40	IBOVESPA	Nifty
Panel I:				
θ_0	-0.626	-0.304	0.637	-0.067
	(2.255)	(2.720)	(0.592)	(1.184)
θ_1	-	$0.231^{\#}$	$-0.226^{\#}$	$0.172^{\#}$
	-	(0.041)	(0.023)	(0.059)
θ_2	-	-	$-0.076^{\#}$	-
	-	-	(0.018)	-
ϕ_1	$-0.373^{\#}$	$-0.512^{\#}$	-	$-0.397^{\#}$
	(0.019)	(0.037)	-	(0.067)
d	$0.491^{\#}$	$0.495^{\#}$	$0.428^{\#}$	$0.471^{\#}$
	(0.012)	(0.007)	(0.018)	(0.023)
Panel II:				
μ_0	$0.022^{\dagger}$	0.020	0.054^{*}	0.047^{*}
	(0.012)	(0.015)	(0.024)	(0.020)
μ_1	$-0.061^{\#}$	-0.029^{*}	0.024	$0.097^{\#}$
	(0.014)	(0.014)	(0.015)	(0.015)
ρ	$1.034^{\#}$	$1.402^{\#}$	$1.047^{\#}$	$1.186^{\#}$
	(0.026)	(0.032)	(0.027)	(0.034)
$Log(\xi)$	$-0.130^{\#}$	$-0.113^{\#}$	$-0.062^{\#}$	$-0.055^{\#}$
	(0.018)	(0.020)	(0.021)	(0.021)
υ	$8.573^{\#}$	$13.492^{\#}$	$10.076^{\#}$	$7.050^{\#}$

	(1.004)	(2.348)	(1.415)	(0.680)
LLF	-6688.868	-6025.392	-6355.947	-6233.267
AIC	2.727	2.431	2.637	2.567
Q(20)	20.751	27.574†	25.266	17.769
	[0.292]	[0.092]	[0.152]	[0.404]
Qs(20)	26.462	24.448	25.056	0.329
	[0.118]	[0.223]	[0.199]	[1.000]

$^{\#}$, * and † means significant at 1%, 5% and 10% level of significance, respectively. The terms in parenthesis (.) represent standard error and the terms in square braces represent p-value of the statistic.

Based on the insignificant values of the Ljung Box statistics for the residuals and the squared residuals (*Q(20)* and *Qs(20)*) at conventional level of significance, we take it as indicating that the ARFIMA-AddRS-SKST model appropriately captures the dynamics in the first and second conditional moments of all the time series under study. It may also be noted that the coefficient estimates are significant at conventional level of significance for all the indices under study with few exceptions. The significant values of the degree of freedom parameter (υ) for all the indices indicate that the standardized returns based on the AddRS estimator are leptokurtic. Moreover, the standardized returns of all the indices are left skewed (negative value of $Log(\xi)$). This impacts the estimation of VaR for the long and short positions. In addition, the long memory parameter (*d*) is significant and less than 0.5 for all the indices thereby indicating stationarity of the Log(AddRS) estimator.

We evaluate the out-of-sample performance of the models in forecasting VaR. Using a fixed-length rolling sample and by leaving out the last 1000 observations, we generate 1000 1-day ahead long and short positions VaR forecasts for each of the indices for further analysis. The rolling window approach allows us to capture the dynamic nature of data across different time periods. We consider {5%, 1%, 0.5% and 0.25%} VaR quantiles in our analysis. We consider the CARR model, the GARCH model, the FIGARCH model, the EGARCH model and the RiskMetrics model with innovations drawn from the skewed Student-t distribution as alternative models to compare the performance of the ARFIMA-AddRS-SKST model in forecasting VaR.

4.2. Back testing VaR models: Comparison of VaR models

4.2.1. Rate of failure

First off, we examine the relative performance of VaR models in term of the rate of failure implied by them. A failure is said to occur by a given VaR model if the absolute value of the realized return is greater than the absolute value of the predicted VaR. Suppose n is the number of returns observations and n_v represents the number of VaR violations. Then, the rate of failure of a VaR model is given as:

$$Rate\ of\ failure = \frac{n_v}{n}$$

Table 2 (not shown here and will be available upon request) reports the failure rate of the various VaR models under consideration for both long and short positions. Out of 32 cases (((4 cases for long position) + (4 cases for short position)) x 4 indices = 32 cases): the ARFIMA-AddRS-SKST model is ranked as number 1 for 27 cases, the CARR model is ranked as number 1 for 8 cases, the FIGARCH model is ranked as number 1 for 6 cases, the EGARCH model is ranked as number 1 for 4 cases and the RiskMetrics model is ranked as number 1 for only 4 cases. Hence, the ARFIMA-AddRS-SKST model outperforms the alternative models in appropriately capturing the failure rates.

4.2.2. Unconditional and conditional coverage tests

Tables 3 and 4 (not shown here and will be available upon request) report the p-value for the Kupiec (1995) unconditional coverage test and the Christoffersen (1998) conditional coverage test, respectively. The level of significance is set at 5%, that is, if the p-value of the model for unconditional and conditional coverage tests is greater than 5%, then the given model adequately captures the VaR. Once again, here too we have 32 cases (((4 cases for long position) + (4 cases for short position)) x 4 indices = 32 cases) to evaluate for each coverage test.

The ARFIMA-AddRS-SKST model based VaR forecasts provide desirable unconditional and conditional coverage tests p-value with a success rate of 100% for both the tests. The success rate of the CARR model based VaR forecasts is 84.38% (successful for 27 cases out of 32 cases) for the unconditional coverage test and 87.5% (successful for 28 cases out of 32 cases) for the conditional coverage test. The success rate for the GARCH model is 75% for the unconditional coverage test and 81.25% for the conditional coverage test. Other models do not perform better than ARFIMA-AddRS-SKST model. Overall, it can be seen that the ARFIMA-AddRS-SKST model outperforms the alternative models in capturing VaR more accurately and does not exhibit any bias towards emerging or developed markets.

Table 3: Unconditional coverage test

	VaR for long position				VaR for Short position			
	5%	1%	0.5%	0.25%	5%	1%	0.5%	0.25%
S&P 500								
ARFIMA	0.885	1.000	1.000	0.743	0.885	0.746	0.642	0.743
CARR	0.666	0.231	1.000	0.743	0.159	0.362	0.398	0.743
GARCH	0.475	0.362	1.000	0.743	0.475	0.314	0.333	0.279
FIGARCH	1.000	1.000	0.333	0.743	0.299	0.170	0.333	0.279
EGARCH	0.093	0.011	0.398	0.020	0.885	0.314	0.333	0.279
RiskMetrics	0.069	0.011	0.049	0.061	0.093	0.746	0.642	0.759
CAC 40								
ARFIMA	0.393	1.000	0.642	0.743	0.792	0.754	0.642	0.743
CARR	0.001	0.139	0.216	0.164	0.557	0.139	0.049	0.383
GARCH	0.037	0.538	0.642	0.383	0.885	0.362	0.216	0.383
FIGARCH	0.204	0.538	0.642	0.383	0.557	0.754	0.398	0.383
EGARCH	0.122	0.510	0.642	0.383	0.233	0.754	0.642	0.383
RiskMetrics	0.027	0.362	0.664	0.383	0.566	0.362	0.398	0.383
IBOVESPA								
ARFIMA	0.770	1.000	1.000	0.759	0.375	0.746	0.642	0.743
CARR	0.037	0.754	1.000	0.759	0.002	0.170	0.126	1.000
GARCH	0.233	0.314	0.333	0.759	0.000	0.030	0.126	0.279
FIGARCH	0.770	0.746	0.664	0.743	0.000	0.030	0.333	0.279
EGARCH	0.001	0.079	0.126	1.000	0.000	0.030	0.029	1.000

RiskMetrics	0.019	0.754	0.216	0.164	0.178	0.314	1.000	0.759
S&P CNX Nifty								
ARFIMA	0.666	1.000	0.642	0.759	0.884	1.000	1.000	0.743
CARR	0.004	0.538	0.642	0.743	0.475	1.000	1.000	0.759
GARCH	0.133	0.009	0.029	1.000	0.048	0.009	0.029	1.000
FIGARCH	0.461	0.009	0.126	1.000	0.375	0.314	0.126	1.000
EGARCH	0.009	0.009	0.029	0.279	0.022	0.030	0.029	1.000
RiskMetrics	0.159	0.754	0.333	1.000	0.257	0.362	0.642	0.743

4.3. Economic significance analysis using Lopez (1998) loss function approach

To examine the economic significance of the findings, we us the loss function approach, proposed by Lopez (1998), and this provides a way to examine the magnitude of the exceedance which can be related to the monetary loss. We apply the loss function only on those VaR models which qualify the unconditional coverage test and the conditional coverage test.

For long position VaR, the results indicate that out of 16 cases (4 indices x 4 levels of VaR), 12 cases are in favor of the ARFIMA-AddRS-SKST model in capturing appropriate VaR with minimum total loss. This include: (1) cases related to 1%, 0.5% and 0.25% long position VaR of S&P 500, CAC 40 and IBOVESPA (9 cases); (2) 5% VaR of CAC 40 and IBOVESPA (2 cases); and (3) 1% VaR of S&P CNX Nifty (1 case). The remaining 4 cases are in favor of remaining models which include one case of CARR model (5% VaR) for S&P 500, and three cases from S&P CNX Nifty for 5% VaR (GARCH model), 0.5% VaR (FIGARCH model) and 0.25% VaR (EGARCH model). This indicates that the ARFIMA-AddRS-SKST model perform better than other models in capturing long position VaR .

For short position VaR, out of 16 cases, 14 cases are in favor of the ARFIMA-AddRS-SKST model in capturing appropriate short position VaR with minimum total loss. The remaining two cases include: one case is from the FIGARCH model for 5% VaR of S&P 500 and the second case is from the FIGARCH model for 0.5% VaR of S&P CNX Nifty. Here also, the results indicate that the ARFIMA-AddRS-SKST model better than other models in capturing short position VaR.

Overall, the results indicate that out of 32 cases (for both long and short position VaR): 26 cases are in favor of the ARFIMA-AddRS-SKST model in capturing appropriate

VaR with minimum total loss. Overall, our results indicate that the ARFIMA-AddRS-SKST model based VaR forecasts outperforms the VaR forecasts from the alternative models.

Table 6: Loss function

	VaR for long position				VaR for short position			
	5.00%	1.00%	0.50%	0.25%	5.00%	1.00%	0.50%	0.25%
S&P 500								
ARFIMA	108.206	16.916	5.976	2.007	69.600	7.001	3.116	1.000
CARR	96.118	19.151	-	2.039	80.046	14.919	-	2.078
GARCH	108.044	-	-	-	73.485	8.250	3.164	1.000
FIGARCH	96.254	19.270	-	-	59.152	7.021	3.171	1.012
EGARCH	113.992	-	-	-	65.944	7.705	-	1.005
RiskMetrics	-	-	-	-	-	10.908	4.557	3.027
CAC 40								
ARFIMA	95.359	11.637	2.410	1.055	56.596	5.598	1.348	7.519
CARR	-	-	11.240	6.240	111.571	34.308	20.515	9.151
GARCH	-	-	6.861	5.025	124.808	36.731	22.282	12.362
FIGARCH	112.991	16.972	6.065	4.579	119.347	37.077	23.997	15.075

EGARCH	-	14.643	7.666	5.932	107.732	29.368	14.337	9.120
RiskMetrics	-	-	9.668	5.594	-	38.611	24.016	15.403
IBOVESPA								
ARFIMA	73.938	7.052	2.082	1.001	23.155	1.275	2.141	1.004
CARR	-	23.065	-	3.814	-	7.410	2.175	1.154
GARCH	104.482	18.462	-	3.491	-	-	2.495	1.033
FIGARCH	-	20.757	-	3.100	-	-	3.296	1.010
EGARCH	-	-	2.133	1.152	-	-	-	1.051
RiskMetrics	-	-	-		73.069	11.911	6.629	3.420
S&P CNX Nifty								
ARFIMA	70.417	11.283	2.345	1.326	52.281	7.524	4.722	1.403
CARR	-	17.110	5.290	2.061	78.667	12.544	-	-
GARCH	68.765	-	-	2.004	66.561	-	-	1.752
FIGARCH	72.576	-	2.054	1.531	60.550	8.630	2.018	1.661
EGARCH	-	-	-	1.004	-	-	-	1.541
RiskMetrics	93.924	-	3.031	1.428	84.164	-	-	2.340

5. Conclusion

In this study, we provide a framework to estimate and forecast the long position as well as the short position VaR using the unbiased extreme value volatility estimator (The AddRS estimator). Our framework also incorporates the impact of asymmetry and leptokurtosis by assuming the skewed Student-t distribution for the innovation term. This framework is referred as the ARFIMA-AddRS-SKST model. Using a rolling sample approach based on daily data, we generate 1000 1-day ahead VaR forecasts for both long as well as short positions for four global indices (S&P 500, CAC 40, IBOVESPA and S&P CNX Nifty). We assess the performance of the ARFIMA-AddRS-SKST model in predicting accurate long position and short position VaR using the rate of failure approach, the Kupiec (1995) unconditional coverage test and the Christoffersen (1998) conditional coverage test and compare them with the corresponding results from alternative models with innovations drawn from the skewed Student-t distribution. We select the best performing VaR models for further analysis. Our findings from these tests support the superiority of the ARFIMA-AddRS-SKST model in predicting VaR more accurately. Finally, we undertake an analysis based on the Lopez (1998) loss function approach to study the economic significance of the ARFIMA-AddRS-SKST model. Our findings indicate that the ARFIMA-AddRS-SKST model outperform other models in forecasting more accurate VaR. In this context, this study highlights the importance of incorporating more information, in the form of open, high, low and close prices of asset, in computing and predicting VaR more accurately.

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