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ASSET ALLOCATION BREWED ACCROSS AFRICAN STOCK MARKETS

Abstract:

Using data from eleven African stock markets between the years 2000-214 and adopting the Markowitz optimization technique, we construct the optimum portfolio and the minimum variance portfolio across eleven African stock markets. We find that, efficient allocation of assets across the African continents can offer better risk-return trade off than an investment that is country specific. The result is robust, as the bootstrap technique adopted did not significantly vary the results. Finally, comparing the risk and return of the optimum portfolio to the Standard and Poor Dow Jones index shows there is a reward for bearing extra risk to invest in Africa.

Keywords:

Asset Allocation, African Stock Markets, investments, portfolio theory, optimal portfolio, bootstrapping

JEL Classification: G10, G11, G00

I. INTRODUCTION

Recently, the power and advanced features of computers have made the application of the Portfolio Theory (PT) developed by Markowitz (1952) prominent in the modeling and allocating of assets in both academia and industry (Wang and Forsyth, 2011; Huang and Lee, 2010; Huang, 2008; Elton and Gruber, 2000; Markowitz, 1999; Evans and Archer, 1968). The portfolio theory stipulates that investors seek to maximize their expected returns and minimize the risks associated with their portfolios. In effect the PT allows the quantification of risks and expected returns and selections of optimal portfolios by investors in financial markets (Hibiki, 2006). Strategic asset allocation is mostly the optimal allocation of risky assets (bonds, lease financing, stocks) and risk free (treasury bills, government bonds) assets to an investor over his or her investment horizon (Cesari and Cremonini, 2003). The popularly known strategies are the static, also known as the tactical asset allocation and the dynamic strategies. With the static or tactical asset allocation the investor defines the mean-variance strategy of optimization over a single period (Brennan, Schwartz, and Lagnado, 1997). However, the dynamic asset allocation assumes either a continuous or a multiple period model (Yao, Li, and Chen, 2014). This study employs a static asset allocation, thus the classic Markowitz mean-variance optimization technique and builds up on previous literature in the context of the study. To the best of my knowledge most of the studies that have proposed a dynamic asset allocation did not apply it in a real world situation as a result of its complex nature (Liu, Zhang, and Xu, 2012; Calafiore, 2008). In addition, the application of a dynamic model would require a huge data set, since the time period would be sub-divided into several time periods to obtain a multi-period situation (Mossin, 1968). This is difficult to attain, especially in the context of Africa where the data is limited as a result of the age of the stock markets. Hence, this study employs a static asset allocation. The basic allocation problem is to decide which asset classes to include in a portfolio and in what proportions (Gratcheva and Falk, 2003). This is because the asset allocation decision has a cumulative influence on the portfolios overall performance than any other decision. The asset allocation decision may not necessarily imply diversification in one market, but cross country diversification may also apply. Cross border diversification of products and processes has the tendency of improving the risk-expected return performances of portfolios and securities. This is on the premise that, macroeconomic factors of the various countries do not cause stock returns to be strongly correlated (Obstfeld, 1994). However, investors prefer to invest locally or in countries close by or familiar, thus investors would prefer to invest in markets they are conversant with (Coval and Moskowitz, 1999).

Recent studies have indicated that frontier markets such as most markets in Africa are becoming a lucrative place to invest since the correlation between African markets and the other developed markets is still low. Furthermore, over the years, developing markets have illustrated rapid financial, economic and developmental growths (Groot, Pang, and Swinkels, 2012; Li, Sarkar, and Wang, 2003). Groot et al. (2012) found that investing in a frontier markets tend to significantly extend the mean-variance efficient frontiers and provides investment opportunities which yield higher returns. We also believe cross border diversification on the African stock markets can serve as an ideal out of sample investment display, since most studies on cross country portfolio selection and diversification focused more on developed economies (Guidolin and Hyde, 2012; Horneff, Maurer, Mitchell, and Stamos, 2009).

Klassen and Yoogalingam (2013) have also argued that to determine whether or not a technique, strategy or model employed in the allocation and selection of the optimal portfolio would be able to withstand any variations in macroeconomic variables, as well as any other idiosyncratic errors in measurement, the simulation process must be adopted. This is because the simulation process addresses the possible stochastic factors of the sampling process while simultaneously determining whether there exist significant differences between the simulated results to that of the original (Aslanidis and Casas, 2013; Adachi and Gupta, 2005). The bootstrapping technique would therefore be employed as a simulation technique. Studies that employed bootstrapping as a simulation technique used it mainly because of its independence to distribution of stock returns (Jacob et al., 2014). Bootstrapping can depict various and probable situations that could result in variations in stock returns in a given economy. That is, bootstrapping has the ability to provide various datasets (replicates) that can result from variations such as changes in business cycles, inflation and other diversifiable errors in the economy (Assaf, Barros, and Matousek, 2011; Simar andWilson, 2007; Simar and Wilson, 2000; Simar andWilson, 1998). Also the bootstrap is a more effective and efficient way of obtaining possible datasets that can result from variations in the economy and will be difficult to gather and collect for this purpose (Tortosa-Ausina, Armero, Conesa, and Grifell-Tatje, 2012). Therefore, this study goes beyond the literature on cross-border asset allocation and portfolio diversification by including bootstrapping as a simulation technique to cater for any variations that can occur in stock returns.

Another important observation from the literature on asset allocation is that, the studies turn to concentrate primarily on developed economies to the neglect of emerging and developing economies. For example, Aslanidis and Casas (2013), Chu (2011), Cesari and Cremonini (2003) and Campbell, Huisman and Koedijk (2001) have all focused on developed economies. Even the few papers that conducted cross-country assessments used samples dominated by developed countries. For example, whereas Chen et al. (2014) only focused on 8 developed countries, Driessen and Laevan (2007) considered 52 countries, only 18 were developing economies of which 6 were African countries. To the best of our knowledge, only Mensah et al. (2013) have considered country specific optimized portfolio choice in Africa, and specifically Ghana. It is important for more insights on investment portfolios in developing economies, especially Africa. This is because Africa is gradually becoming a lucrative investment destination. Forbes Report (2014) suggests that Africa is a two trillion dollar economy with about a third of its fifty-four countries having Gross Domestic Product (GDP) growth rates of at least 6% annually. Out of the top ten fastest growing economies in the world, six are in Africa. This lays a strong foundation for studies targeting developing economies, especially in Africa.

Subsequent sections of the paper is organized as follows; section II review both the theoretical and empirical literature related to the topic understudy. Section III discusses the source of data and methodology. In sections IV, we present the results, and finally conclusions and recommendations are discussed in section V.

II. LITERATURE REVIEW

Diversification is a fundamental principle in finance, which aims at minimizing the risk faced by investors (Gaudecker, 2015; Amenc and Martellini, 2011; Goetzmann and Kumar, 2008; De Santis and Gerard, 1997; Meric and Meric, 1989; Klein and Bawa, 1976; Lessard, 1973). This is done by investing in different assets, asset classes and markets that have low, negative or possibly no correlations between their returns, thereby reducing the risk subject exposed to investors. The Markowitz mean-variance criterion incorporates the benefits of portfolio diversification (García-Herrero and Vázquez, 2013; Driessen and Laeven, 2007). That is, the theory postulates that, assets cannot be chosen based on only attributes specific to a security. Instead, investors are required to take into consideration the correlation between the various assets (Kisaka et al., 2015). In other words, investors should allocate their wealth among securities that are not highly correlated so that events such as changes in business cycles and macroeconomic conditions, they can benefit from the diversification (Gökgöz and Atmaca, 2012).

There has been several studies on the need and benefits of portfolio diversification (Gaudecker, 2015; Levy and Levy, 2015; Zhou and Nicholson, 2015; Brandtner, 2013; Hung, Liu, and Tsai, 2012; Amenc and Martellini, 2011; Goetzmann and Kumar, 2008; De Santis and Gerard, 1997; Meric and Meric, 1989). Zhou and Nicholson (2015) constructed a diversified portfolio across 3 asset classes in the US economy. They found that by modelling a covariance asymmetry as a result of the asymmetric response correlation and volatility has possible shocks that could occur in returns, US investors tend to obtain significant gains on a diversified portfolio across these asset classes. Brandtner (2013) also examined the optimal choices for a mean-variance technique as compared to a value at risk (VaR) technique, specifically the spectral risk approach. He found that the benefits of diversification are not optimized when employing this risk (spectral risk) measure since it deviates from the typical trade-off between risk and expected return captured in Markowitz's optimization technique.

However, all the studies above focused on the benefits of portfolio diversification within an economy. This is because, practically most investors prefer local investment to cross border investment (Driessen and Laevan, 2007). Studies such as, that of Miralles-Marcelo, Miralles-Quiros, and Miralles-Quiros (2015) as well as García-Herrero and Vázquez (2013) have indicated the possible benefits of cross-border portfolio diversification when the stock returns of the countries are not highly correlated. Miralles-Marcelo et al. (2015) examined how to increase the benefits and make cross borders diversification more attractive to prospective US investors using stocks from the Japan, UK and US markets. They found that the benefits of cross border portfolio diversifications are more significant and realized when investors invest in the US currency (dollar) instead of their individual countries' currencies. García-Herrero and Vázquez (2013) found that the benefits of diversification are also realized in the banking industry when banks have subsidiaries by employing a sample of 38 banks from 8 developed economies (US, UK, Spain, France, Germany, Canada, Japan and Italy). They also found that, subsidiaries of banks situated outside their respective home countries, specifically developing economies tend to offer better risk-expected return trade-offs than their parent banks. This, therefore further indicates the substantial benefits of cross border diversification. Despite, the substantial studies on the benefits of portfolio diversification within an economy and a few cross borders, most of the samples of these studies focused either only on or dominated by the developed economies to the neglect of developing economies. For example, Chen et al. (2014) modeled the stock and bond returns of 8 developed economies (US, UK, Australia, France, Germany, Canada, Japan and Italy) using a semi-parametric copula technique. This model was then used to estimate two risk measures (expected shortfall and value at risk) in order to construct diversified portfolios across the two asset classes.

III. SOURCE OF DATA AND METHODOLOGY

The study formulates an optimized portfolio across eleven African stock markets (Cote D'Ivoire, Mauritius, Kenya, Nigeria, Tunisia, South Africa, Morocco, Botswana, Ghana, Namibia and Zambia) by employing the Makowitz Portfolio Optimization technique and compares it to that of the SandP Dow Jones portfolio. Pairwise correlations are constructed to check for the correlations among the eleven stocks. The correlation matrix is constructed in order to check if there exist low and negative correlations among the markets. It mentioned earlier that in order to benefit from international and portfolio diversification, there should be low and possibly negative correlation between stock returns of the markets (Gerstner, Griebel, Holtz, Goschnick, and Haep, 2008). To construct an optimal portfolio, there is the need to gather information on dividend-adjusted market returns and treasury bill rates of the countries under consideration. Market indices are sourced from the Thomson Reuters Datastream database and computed by Standard and Poor's. Standard and Poor's has been considered to be one of the largest source of indices on capital markets across the globe. It gathers daily, weekly and monthly stock indices globally. Treasury bill rate is used as a proxy for the risk-free asset in the study. The treasury bill rates were derived from the World

Development Indicators database. These are computed as the difference between the yearly lending rates and risk premiums on each of the eleven countries. This method is used in obtaining the treasury bill rates since it may be difficult to obtain the treasury bill rates for all the eleven countries directly. Hence, for uniformity and prevention of bias this technique is employed.

The data is monthly price data between the years 2000 and 2014. Using data availability as criteria, only 11 countries were captured in our sample. This study also focuses on these 11 countries because they may offer better investment opportunities and portfolios with high returns (Groot et al., 2012; Jacobs et al., 2014). In computing the stock returns, the monthly stock price indices of each country are used. The asset allocation and portfolio optimization literature indicates that in order to take advantage of diversification in portfolio optimization and asset allocation, low and negative correlations need to exist between stock returns of markets or securities (Mensah et al., 2013; Alagidede and Panagiotidis, 2009). This study also performs hypothesis testing to determine if these correlations are significantly different from zero. The price indices obtained for the various countries are transformed into returns. The stock returns are then tested to determine if they are normally distributed since it is an assumption for the Markowitz optimization technique (Guo, Ye, and Yin, 2012).

The study employs the Markowitz Portfolio Optimization model, thus to determine the optimal risky portfolio that investors can hold by investing in stock markets indices across the eleven African stock exchanges mentioned above. Given the prcie indices for country i as P_{it}, where $t = 1, \cdots, T$ and $i = 1, \cdots, 11$. The return of country indices i at month t is therefore computed by the formula:

$$r_{it} = \left(\frac{P_{it} - P_{it-1}}{P_{it-1}} \right) \times 100\% \qquad (1)$$

Dividend payments are not included in the computation of returns since all the indices are divided adjusted. Markowitz's model is therefore formulated using the optimization formulation (2) and (3) below. Model (2) is a minimization problem that aids in computing the minimum variance portfolio MVP. Conversely, model (3) is a maximization problem aimed at maximizing the Sharpe ratio in order to generate the Tangency Portfolio (Mensah et al., 2013).

$$\begin{aligned} &\text{Minimize} \quad \sigma_P^2 = \sum_{i=1}^{11} w_i^2 \sigma_i^2 + 2\sum_{i,j=1}^{11} w_i w_j \operatorname{cov}(r_i, r_j) \qquad \text{, where } i \neq j \\ &\text{subject to:} \\ &\sum_{i,j=1}^{11} w_{ij} = 1 \\ &\sum_{i=1}^{11} w_i E(r_i) \geq r_p \\ &w_{i,j} \geq 0 \qquad i, j = 1,2,...,11 \end{aligned} \tag{2}$$

where w_i are the weights of individual indices from different countries, σ_i is the standard deviation of the country specific index returns, representing the total risk. $\operatorname{cov}(r_i, r_j)$ is the variance-covariance matrix of all the eleven country indices. The objective here is to determine how much to invest in the various markets so as to minimise the variance of this portfolio. However, this is subject to some constraints. The first constraint ensures that the total budget is invested in the markets (Farinelli et al., 2008). This means that the sum of the weights for the various markets should be equal to 1, not less or greater. The second constraint requires the optimal expected return of the portfolio to be either equal to or greater than the minimum portfolio return. The minimum portfolio return is defined as the expected return from investing equally across the markets. It is therefore expected that, the optimal solution is not dominated by this heuristic choice. The final constraint, the non-negativity constraint requires that either nothing or strictly positive weight is to be assigned to a market. It also shows the absence of short-selling. Short selling is not assumed since in developing markets like those in Africa, it is hardly practised due to the illiquid nature of stock markets (Mensah et al., 2013).

For the tangency portfolio (TP, optimal portfolio), we maximize the Sharpe ratio subject to the same constraints as in model (2). It is expressed as follows:

$$\begin{aligned} &\text{Maximize} \quad S_p = \left(\sum_{i=1}^{11} w_i E(r_i) - r_f \right) \sigma_p^{-1} \text{, where } i \neq j \\ &\text{subject to:} \\ &\sum_{i,j=1}^{11} w_{ij} = 1 \\ &\sum_{i=1}^{11} w_i E(r_i) \geq r_p \\ &w_{i,j} \geq 0 \qquad i, j = 1,2,...,11 \end{aligned} \tag{3}$$

It must be noted that $\sum_{i=1}^{11} w_i E(r_i)$ is the expected return of the portfolio. r_f is the risk-free rates of the markets under study. σ_p^{-1} denotes the inverse of the portfolio's risk which is measured using the standard deviation of the portfolio obtained from the combination of all the African market indices.

Bootstrapping is related to simulation, except that with simulation data is completely artificially generated for the purpose of making statistical inferences. However, in bootstrapping empirical estimators are generated using data points sampled repeatedly (with replacement) from the sample. The basic assumption underlining the bootstrapping algorithm is that, the sample if well constructed is a good representative of the actual population. Therefore, by resampling randomly from the sampling distribution, we can get an approximation of the exact nature of the population distribution by correcting the bias in the sampling distribution.

Given that sample data of $R = (r_1,...,r_T)$ are available to estimate a desired parameter $\theta(R)$. Since the true population F is unknown, the true parameter $\hat{\theta}(F)$ is also unknown. Therefore, we draw B samples of $R^{(b)} \big| b = 1,...,B$ from the actual data. This provides B number of pseudo samples each of the size T: $R^{(b)} = (R_1^{(b)},...,R_T^{(b)})$. We can therefore compute $\theta^*\left(R^{(b)}\right)$ for each $i = 1,...,T$. Therefore the relationship between the true and sample estimates can be computed using the relationship between the sample and bootstrapped estimates (de Borger, Kerstens and Staat, 2008)

For example,

$$\left(\hat{\theta}(F) - \theta(R)\right| X \approx \left(\theta(R) - \theta^*(R^{(b)}\right| R^{(b)}$$

This means that, the difference between the true (but unknown) estimate $\hat{\theta}(F)$ and the sample estimate $\theta(R)$, is approximately equal to the difference between the sample estimate $\theta(R)$ and the bootstrapped estimate $\theta(R^{(b)})$. The bias in the estimate is therefore computed as:

$$Bias(\theta^*) = \frac{1}{B}\sum_{b=1}^{N} \theta^*\left(R^{(b)}\right) - \theta(R)$$

It should be noted that $\frac{1}{B}\sum_{b=1}^{B}\theta^*\left(R^{(b)}\right)$ represents the mean of the bootstrapped estimate.

The variance for the estimate is also computed by $var(\theta^*) = \mathrm{E}[\theta^* - E(\theta^*)]^2$. Rule of thumb, if $\frac{|Bias(\theta^*)|}{Std(\theta^*)} \leq 0.25$, then deviation of the bootstrapped estimates from the actual sample is insignificant (Efron and Tibshirani, 1993), where $Std(\theta^*)$ represents the standard deviation of the bootstrapped estimate θ^*. If the biases found are significant, the bias corrected estimates are computed as the actual estimate from the original sample minus the bias computed. (Efron and Tibshirani,1993). Mathematically, this is represented as follows;

$$\theta(X_*^*) = \theta(X) - Bias(\theta^*)$$

where $\theta(X_*^*), \theta(X)$ and $Bias(\theta^*)$ represents the bias corrected estimate, the actual estimate from the original sample and the bias estimate respectively.

IV. RESULTS

Table 1 below displays the yearly average returns, standard deviations, minimum and maximum returns and the Sharpe ratios of the eleven African stock markets. The descriptive statistics of the SandP Dow Jones stock returns are also reported. The average returns and their associated risks are also indicated. There seem to be a positive relationship between risk and return on the African markets

Table 1:Descriptive Statistics of Stock Returns for the Eleven African Countries

Country	Mean	Standard Deviation	Minimum	Maximum	SR
Botswana	0.0940	0.1786	-2.1942	2.1920	0.5261
Namibia	0.0747	0.1965	-2.1643	1.6680	0.3800
Morocco	0.0745	0.1689	-1.7198	2.0814	0.4410
Tunisia	0.0291	0.1648	-2.1858	1.6594	0.1765
Ghana	0.0183	0.2298	-2.5131	2.9120	0.0795
Nigeria	0.0902	0.3098	-5.7045	4.7378	0.2911
Kenya	0.1493	0.2940	-5.8810	2.7395	0.5079
Mauritius	0.0869	0.2223	-4.2539	2.6483	0.3910
Zambia	0.1764	0.2638	-4.3695	3.6432	0.6689
SA	0.0740	0.2758	-4.2672	2.6416	0.2684
CD	0.1499	0.2436	-2.4760	3.0772	0.6152
SPDJ	0.0497	0.1450	-1.8336	1.1783	0.3426

*__SA-South Africa, CD-Cote D'Ivoire, SPDJ-SandP Dow Jones__

The lowest risk is 16.48%, which corresponds to an average return of 2.91%, whereas the highest risk is 30.98%, which is associated with an average return of 9.02%. Zambia's portfolio has the highest average return of 17.64% with a risk of 26.38%, whereas that of Ghana has the least average return of 1.83% with a risk of 22.98%. The average return and standard deviation of the SandP Dow Jones' portfolio is also reported in the Table with an average return of 4.97% with a risk of 14.50%. In comparison to the average returns of the eleven African portfolios, with the exception of the portfolios from Tunisia and Ghana, all the others have a higher average return than that of the SandP Dow Jones. However, the portfolio from SandP Dow Jones tends to have a lower risk than those from the eleven African countries. These results conform to the argument made by Bekaert and Harvey (1997) and Harvey (1995) that, stock returns of developing economies tend to have different characteristics as compared to the developed economies. The Sharpe ratios (SR) were also estimated using the average return and risk for the individual market portfolios of the eleven African countries and that of the SandP Dow Jones. These ratios indicate the reward to variability, thus the ratio of average return per risk taken, assuming a risk-free rate of zero (Mensah et al., 2013). Zambia's portfolio reported the highest reward to volatility of 66.89%, whereas Ghana's reported the least (7.95%). On the average, more than 50% of individual countries (African) portfolios have low Sharpe ratios (less than 50%) indicating that, investors investing in these individual country's portfolio are not being rewarded greatly for the excessive risk taken. The same can be said for the SandP Dow Jones, which has a Sharpe ratio of 34.26%.

A correlation matrix for the stock returns of the eleven stock markets and that of the SandP Dow Jones was constructed. The researcher further tested the hypothesis of no correlation, to check if these correlation coefficients are significantly different from zero. Table 2 reports the correlation coefficients of the stock returns between the various African countries and SandP Dow Jones. The least correlation coefficient (0.01) is reported between the stock returns of Ghana and Tunisia, whereas the highest (0.38) is reported between Kenya and Mauritius. It is realized from Table 2, that all the correlations are below 50%, indicating that the relationship between the returns of the eleven African stock markets are not strong (Cohen, 1988). These results therefore, do not warrant any of the eleven African countries to be eliminated. Hence portfolio diversification across these countries should result in higher returns or reduce the risk associated with these returns as compared to investing in any of the individual countries (Groot et al., 2012; Li et al., 2003; Harvey, 1995; Obstfeld, 1994).

Table 2: Correlation matrix between the Indices

	Country	1	2	3	4	5	6	7	8	9	10	11	12
1	**Botswana**	1											
2	**Namibia**	0.23*	1										
3	**Morocco**	0.25*	0.31*	1									
4	**Tunisia**	0.23*	0.18*	0.2*	1								
5	**Ghana**	0.04	-0.04	0.12	-0.01	1							
6	**Nigeria**	0.17*	0.08	0.18*	0.03	0.11	1						
7	**Kenya**	0.13	0.13	0.29*	0.1	0.11	0.09	1					
8	**Mauritius**	0.23*	0.16*	0.31*	0.21*	0.16*	0.27*	0.38*	1				
9	**Zambia**	0.07	0.09	0.02	0.06*	0.09	0.07	-0.02	0.17*	1			
10	**SA**	0.25*	0.06	0.04	0.09	0.1	0.15*	0.18*	0.22*	0.21*	1		
11	**CD**	0.24*	0.25*	0.33*	0.18*	0.13	0.21*	0.17*	0.29*	0.14*	0.07	1	
12	**SPDJ**	0.34	0.32	0.28	0.15	0.01	0.14	0.26	0.14	0.00	0.1	0.21	1

* **Correlation is significant at 5% significance level.**

It can also be observed that the correlation coefficients reported in Table 2 between each of the African countries and SandP Dow Jones index are not significance. This corresponds and affirms the argument that the stock returns of African (developing) economies and that of the developed, which is represented by SandP Dow Jones are not related to each other (Jacobs et al., 2014; Bekaert and Harvey, 1997a). Hence foreign investors may be exposed to higher investment opportunities if they invest and diversify in Africa.

The expected returns and the variance-covariance matrix of the eleven African stock markets are used as the inputs to the linear programming models. The first model aims at minimizing the variance and the second at maximizing the Sharpe ratio in order to generate minimum and tangency (optimal) portfolios respectively.

Table 3: The Expected Returns, Risks, Sharpe Ratios and Weights Allocations

	EQWP	MVP1	MVP2	TP
Expected Return	0.0925	0.0652	0.0925	0.1338
Standard Deviation	0.1184	0.0998	0.1066	0.1367
Sharpe Ratio	0.7810	0.6536	0.8671	0.9788
Weights Allocation to each Country				
Botswana	0.0909	0.1264	0.1526	0.1649
Namibia	0.0909	0.1143	0.0932	0.0338
Morocco	0.0909	0.1941	0.2014	0.1638
Tunisia	0.0909	0.2586	0.1446	0.0000
Ghana	0.0909	0.1634	0.0838	0.0000
Nigeria	0.0909	0.0173	0.0145	0.0003
Kenya	0.0909	0.0000	0.0351	0.1262

Mauritius	0.0909	0.0000	0.0000	0.0000
Zambia	0.0909	0.0876	0.1733	0.3054
SA	0.0909	0.0383	0.0252	0.0000
Cote D'Ivoire	0.0909	0.0000	0.0766	0.2057

*EQWP-Equally-weighted, TP-Tangency and MVP-Minimum-Variance Portfolios

Equally-weighted, minimum-variance and tangency (optimal) portfolios are constructed across the eleven African stock markets. The various expected returns, standard deviations and Sharpe ratios associated with these portfolios are reported in Table 3. The corresponding weight allocations required to construct the respective portfolios are also reported. The equally-weighted portfolio across the eleven stock markets reported an expected return of 9.25% with a risk of 11.84%. Corresponding to its name (equally-weighted portfolio), since there are eleven countries being considered in this study, investors would have to invest 9.09% of their entire wealth in each of these countries to obtain the risk and expected return associated with this portfolio.

Two minimum-variance portfolios were constructed. The second minimum-variance portfolio was subjected to an additional constraint, which restricts the expected return of this minimum-variance portfolio to be either equal to or greater than that of an equally-weighted portfolio. The first minimum-variance portfolio reported an expected return of 6.52% with a risk of 9.99%, whereas the second reported an expected return of 9.25% with a risk of 10.66%. The additional constraint placed on the second minimum-variance portfolio increased the risk and expected return in comparison to that of the first by 2.73%. Hence, this portfolio dominates the heuristic choice of investing equally among the eleven countries which was the purpose of including this additional constraint. This results show that with the extra constraint placed on the second minimum-variance portfolio, though it has the same expected return as that of the equally weighted (9.25%), the risk associated with it is lower than that of equally weighted portfolio by 1.18%.

Highly risk-averse investors would opt for the first minimum-variance portfolio, since it offered the least risk. Such investors would have to allocate 12.64%, 11.43%, 19.41%, 25.86%, 16.34%, 1.73%, 8.76% and 3.83% to Botswana, Namibia, Morocco, Tunisia, Ghana, Nigeria, Zambia and South Africa respectively, and nothing to Kenya, Mauritius and Cote D'Ivoire respectively so as to ascertain the expected return and risk associated with the first minimum-variance portfolio. Less risk-averse investors would opt for the second minimum-variance portfolio. These investors would, however, have to invest 15.26%, 9.32%, 20.14%, 14.46%, 8.38%, 1.45%, 3.51%, 17.33%, 2.52% and 7.66% to the portfolios of Botswana, Namibia, Morocco, Tunisia, Ghana, Nigeria, Kenya, Zambia, South Africa and Cote D'Ivoire respectively and nothing to Mauritius. It is noticed from Table 3 that, such investors have higher Sharpe ratios than the former. This is so, since less risk-averse investors are being rewarded by a margin of 21.35% for the extra risk taken. This also illustrates the positive association between expected return and risk, hence, the higher the risk, the higher the expected return on the portfolios.

The tangency portfolio reported an expected return of 13.38% with a risk of 13.67%. It is observed that the tangency portfolio provides an optimum solution to the asset allocation problem most investors are faced with, on how much to allocate to each asset or market in order to maximize their wealth at the possible minimum risk (Zhu and Zhou, 2009; Gratcheva and Falk, 2003). This is because this portfolio offers the best blend of risky securities that gives the best risk-expected return trade-off on a portfolio (Xie, 2009; Jorion 1992). The optimal portfolio also had the highest Sharpe ratio of 97.88%, indicating that investors, who invest in the optimal portfolio, would attain a compensation of 97.88% for taking a risk of 13.67% to obtain a higher expected return of 13.33% as compared to the equally weighted and minimum-variance portfolios.

Investors who choose a portfolio beyond the tangency portfolio would not be rewarded for the excessive risk taken. This is because the tangency portfolio is the optimum choice, in that, this portfolio gives the best risk-return trade-off investors can obtain (Tepla, 2000). It is realised from Table 4 that, beyond the tangency portfolio (the nineth column highlighted), the Sharpe ratio begins to fall.

In order to determine if the optimization technique employed in the selection of the optimal portfolio can withstand any variations in stock returns that may come about because of different macroeconomic shocks; the bootstrapping algorithm is use as a simulation technique. The sample of stock prices for the eleven stock markets were bootstrapped 2000 times to obtain 2000 replicates of stock returns for each country. The means, standard deviations, bias estimates and the significances of the biases are shown in Table 5 panels A, B and C. The means and standard deviations for bootstrapped estimates are also reported in Table 5 panel A, B, and C. These are used as inputs in computing the biases. The biases reported in Table panels A, B and C indicates the deviations of the various bootstrapped estimates from the actual estimates computed using the original sample. These biases were then tested to determine if they were significant.

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Table 4: The Expected Returns, Risks, Sharpe ratios and Weights Allocated to a given Portfolio on the Efficient Frontier

ER	0.0652	0.0711	0.0769	0.0828	0.1062	0.1121	0.1296	0.1338	0.1355	0.1413	0.1530	0.1647
SD	0.0998	0.1002	0.1013	0.1030	0.1147	0.1186	0.1326	0.1367	0.1384	0.1450	0.1605	0.1827
SR	0.6536	0.7092	0.7592	0.8036	0.9261	0.9445	0.9773	0.9788	0.9786	0.9745	0.9535	0.9014
Weights Allocated to each Country												
Botswana	0.1264	0.1354	0.1411	0.1454	0.1626	0.1669	0.1738	0.1649	0.1607	0.1457	0.0950	0.0000
Namibia	0.1143	0.1117	0.1075	0.1022	0.0812	0.0760	0.0511	0.0338	0.0265	0.0009	0.0000	0.0000
Morocco	0.1941	0.2000	0.2020	0.2018	0.2012	0.2011	0.1845	0.1638	0.1548	0.1230	0.0254	0.0000
Tunisia	0.2586	0.2307	0.2056	0.1825	0.0901	0.0669	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Ghana	0.1634	0.1445	0.1274	0.1109	0.0451	0.0287	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Nigeria	0.0173	0.0175	0.0168	0.0159	0.0124	0.0115	0.0056	0.0003	0.0000	0.0000	0.0000	0.0000
Kenya	0.0000	0.0000	0.0046	0.0161	0.0620	0.0735	0.1126	0.1262	0.1317	0.1510	0.1901	0.1752
Mauritius	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Zambia	0.0876	0.1100	0.1284	0.1453	0.2128	0.2297	0.2863	0.3054	0.3130	0.3394	0.3946	0.5628
SA	0.0383	0.0359	0.0336	0.0304	0.0180	0.0149	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
CD	0.0000	0.0142	0.0331	0.0494	0.1146	0.1309	0.1861	0.2057	0.2133	0.2400	0.2948	0.2620

*ER-Expected return, SD-Standard deviation and SR-Sharpe ratio

Table 5

Panel A: The bootstrapped Estimates of the Equally-Weighted Portfolio

Estimate	Mean	SD	Bias	Significance
Expected return	0.0922	0.0307	-0.0003	0.0098
Standard deviation	0.1170	0.0129	-0.0014	0.1085

Panel B : The Bootstrapped Estimates of the Minimum-Variance Portfolio

Estimate	Mean	SD	Bias	Significance
Expected return	0.0682	0.0269	0.0030	0.1115
Standard deviation	0.0958	0.0078	-0.0040	0.5128

Panel C: The Bootstrapped Estimates of the Tangency Portfolio

Estimate	Mean	SD	Bias	Significance
Expected return	0.1592	0.0358	0.0254	0.2094
Standard deviation	0.1330	0.0190	-0.0037	0.1947

With the exception of the minimum-variance's risk, all the biases were statistically insignificant. This is because the significance which was computed as, $\frac{|Bias(\theta^*)|}{Std(\theta^*)}$ (Efron and Tibshirani, 1993) for all the estimates were less than 0.25, except that of the minimum-variance portfolio, which has a significance of 0.5128. Therefore, the bias corrected estimate was computed for the risk of this portfolio. This was estimated as the difference between the actual estimate and the bias of its corresponding bootstrapped estimate. This is shown in Table 6. With the exception of the Sharpe ratio and standard deviation of the minimum-variance portfolio, all the estimates' bias corrected values are equivalent to the actual estimates reported earlier.

Table 6: Bias Corrected Estimates

	EQWP	MVP	TP
Expected return	0.0925	0.0652	0.1338
Standard deviation	0.1184	0.1038	0.1367
Sharpe ratio	0.7810	0.6283	0.9788

V. CONCLUSION

This study found that a diversified portfolio across the eleven African stock markets offers a better investment opportunity than the individual countries' portfolios. The minimum-variance and the optimal (tangency) portfolios constructed across these eleven stock markets dominated most of the individual countries' portfolios. With the exception of Zambia market index, all the others were beneath the efficient frontier. This is not surprising, since Zambia had an average return of 17.64% with a risk of 26.38%, the other countries had a lesser average return but were highly risky. The Sharpe ratios of the individual countries' portfolios confirmed this, with Zambia having the highest Sharpe ratio of 66.89% as compared to that of the minimum-variance (65.36% and 87.61%) and the optimal portfolios (97.88%) constructed. Two minimum-variance portfolios were constructed. The first had an expected return of 6.52% with a risk of 9.98%, whereas the second (additional constraint placed on it) had an expected return of 9.25%with a risk of 10.88%. The optimal portfolio offered an expected return of 13.38% with a risk of 13.67%. These portfolios (minimum-variance and optimal portfolios) constructed also outperformed that of the SandP Dow Jones. The SandP Dow Jones portfolio had an average return of 4.57% with a risk of 14.50%.

Investors can invest and diversify across assets in their individual countries. However, they should sometimes move out of their comfort zone and invest across borders, since it offers higher and better investment opportunities as shown by this study. Though Africa's investment environment is risky, it offers a better investment opportunity than that of the US economy which is less risky. The portfolios constructed indicated that, though the individual African countries' portfolios are highly risky, a well-diversified portfolio can offer a better risk-return trade-off by reducing the risk and increasing the return. The outcomes of the study also indicated that possible variations that can affect macroeconomic environment resulting in differences in returns could not have a significant effect on optimal choices. Hence the study also gives some level of certainty to investors, since the bootstrapping performed caters for most of the possible variations that can occur on the stock markets.

The study can motivate foreign investors to invest in African markets. This is because they will be rewarded for the extra risk they would take for investing in Africa, as compared to their less risky environment. The bootstrapping technique adopted portrays the robustness of the results to the variations in returns. The results of this research also suggest that well diversified portfolio across the eleven stock markets offers very attractive investment opportunities as compared to that of the developed economy.

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