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COMPARISON OF TWO ENTROPY APPROACHES IN UNDERSTANDING STOCK MARKET DYNAMICS

Abstract:

Stock market is a typical complex system with a great number of agents interacting with each other. Recent global financial crisis has shown the importance of understanding the dynamics of the stock market in details with a close look at its complexity. In the complexity science, two entropy approaches have been widely used, i.e. maximum entropy principle and multiscale entropy, and in this paper I define the generalized entropy of the market with fully consideration of the complex interactions among various agents. Following the first approach, the whole market, as an open system, always has an optimization process so that the generalized entropy of the whole stock market is maximal under the given constraints. And I have derived the nonlinear dynamic equation for the stock market is accordingly. Following the second approach, I have been able to identify certain market patterns in different scales for different financial quantities. Using empirical data from both Chinese and US stock markets, simulations, profound discussions and comparison are provided. Thus, a new framework for studying the dynamics of stock market is obtained, which will be very useful for the market investors, analysts and regulators.

Keywords:

stock market, dynamics, complex system, entropy, maximum entropy principle, multiscale entropy

Introduction

Recent financial crisis have shown that financial market is complex with lots of instability and risk which is changing all the time. As the barometer of financial market, stock market is a key facility for trading and raising capital. With fast growing technology, nowadays the stock market has become more and more complex and people have been trying different approaches to understand its dynamics and to predict possible risk specifically and systemically.

To help study the complexity in various systems, physicists have put forward the concept of “entropy” which has played an important role in thermodynamics. Being defined as the sum of heat supplied divided by temperature originally, entropy has been the central topic to the Second Law of Thermodynamics, which helps measure the amount of order/disorder and chaos with a macroscopic system. Except for the definition in thermodynamics, entropy has been widely extended in statistics mechanics and information theory which was firstly developed by Ludwig Boltzmann in the 1870s.

There are two commonly widely used approaches for the entropy studies: one is the well-known maximum entropy principle, meaning subject to precisely stated prior data, the probability distribution which best represents the current state of knowledge is the one with largest entropy. And the other is the multiscale entropy, which has been developed from the Kolomogrov-Sinai(KS) entropy and put it into different scales when analyzing data series from experimental physics, geophysics, physiology, medicine and social sciences. Although mathematicians have developed dozens of methods to describe and predict the fast-change state of certain financial asset or market as a whole, they are still far away from understanding the market dynamics thoroughly with precise prediction. These two approaches on entropy exactly offer us another aspect of uncovering market dynamics and risk management upon that, helping us analyze the stock market from both macro and micro levels. Through detailed analysis and comparison, a possible new framework of studying the dynamics of stock market will be obtained.

This paper contains five parts including the introduction. The second part I will illustrate how the stock market operate as a complex system and how to quantify its complexity using the quantity “generalized entropy”. The third and fourth parts I will present the methodology and experimental work of market data for both entropy methods, showing their similarities and differences. Last part I will give a conclusion on my findings and possible future work on entropy analysis of financial market.

Complex economy/finance and generalized entropy

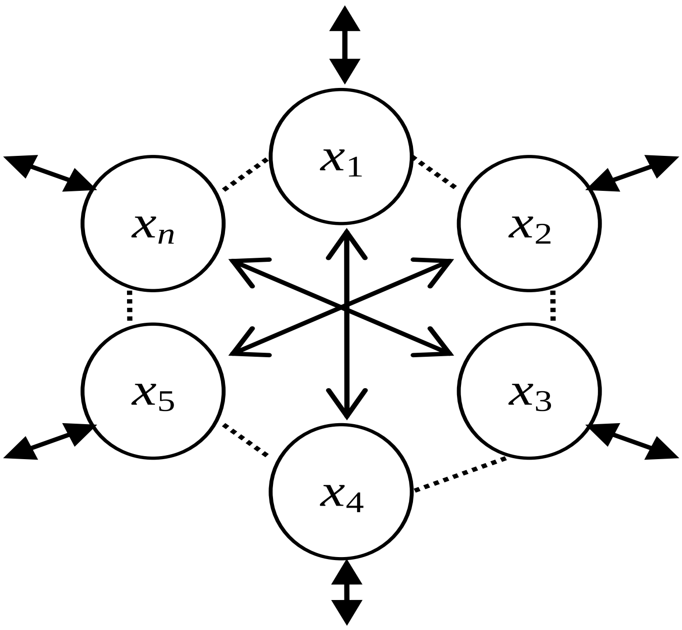
The financial market is a system composed of vast and complicated set of arrangements and actions wherein people and entities can compete and trade financial securities, commodities, and other fungible items of value. Economics and financial market is actually a complex system with massive parallel concurrent behavior within it and complexity economics/finance is a movement in the social sciences that studies how the interacting agents in a financial market create overall economic patterns, and how these overall patterns in turn cause the interacting agents to change and adapt (Arthur 1999). Complex economics/finance is in line with Adam Smith's idea that all those aggregate patterns forms come from individual behavior, and individual behavior in turn responds to these aggregate patterns. It is this recursive loop that connects economics and finance with the complexity. Very much alike the complexity, what we are trying to find out in the financial market is how about financial instruments and trading mechanism evolve, and how this formation affects the agents who causing it.

Complex economy/finance looks at the market not necessarily always staying in equilibrium, it is constantly subjected to shocks, both exogenous and endogenous, which affects its short-term movements. There are frequent local nonlinear interactions that lead to significant deviations of finance variables (volume, quantities, asset prices, etc.) from their equilibrium values even in the absence of strong or systematic perturbations to the system (Arthur 2014). We see such deviations in many occasions in the part human history, which often have the "fat tails" characteristics of the power laws of complex systems, as opposed to the Gaussian distributions of Neoclassical theory (Farmer, Lillo, 2004).

In this paper, we look into the financial market which consists of numerous influential factors which could be regarded as generalized interacting elements — "Agents", such as individual investors, domestic and foreign financial institutions, macroeconomic indexes, and governments etc. The agents are usually multiply scaled. Larger scales of agents are created by lower grade agents. They can form and separate into different scales but all the agents compete for seeking the greatest profits. Investors make investing decision based on the asset status, economic status, other investors' actions, the politics etc., which induces the capital and/or information flow in the stock market. The capital/information flows that connect investors are also reflected by the economic indices. All these capital flows and information flows are the representation of all kinds of interactions among agents in market. Agent interactions generate a huge interacting network mapping the real market.

Thus, we can establish a physical model for the stock market: a complex open system composed of n-agents from x_1 to x_n as we can see in Figure 1. x_n can be used to represent certain agent which is the key factor in the market, like foreign funds, exchange rate, interest rate and so on. The market maintains the flow of information and cash/capital as the main form of exchange with the external environment. At the same time a variety of information/capital flow drives the complex interactions between the agents. Capital/information flows induced by interactions among agents in the market network system include countless feedbacks, adjusting mechanisms, and restrictions, in complex nonlinear means. Therefore, the grasp of structural dynamics actually means grasping the features of the capital/information fluxes exchange mentioned above among the network of stock market systems.

Figure 1: Complex interactions among agents in the stock market



Entropy seems to be the best quantity to fulfill the function of explaining such complex dynamics. In accordance with the definition of entropy, entropy can be expressed as $S = k\ln(1/D)$ or $S = -\sum_i \rho_i \ln \rho_i$, where *k* is a coefficient, *D* represents all the possible permutations and combinations of the agents, and ρ_i represents the probability density which shows the extent to which these agents interact with each other. However, the statistical model of the classical entropy only considers the short-range correlation, which means that there is only weak nonlinear interactions between agents, making the classical non-equilibrium statistical mechanics restricted to the nearly equilibrium field, and not be used in many complex open systems, such as the stock market. Therefore, we must expand the physical meaning of entropy so that it can express the complex interactions (including linear, weakly nonlinear and strongly nonlinear) of the agents of stock market, and unifies the various driving forces to reflect the dynamics of the stock market.

Generally, we consider the a stock market is built up by enormous interacting agents numbered by X_1 to X_n, as shown in Figure. 1. The driving forces of elements X_1 to X_n, i.e., to get generalized fluxes (capital flow, information, etc.) can be expressed as x_1, x_2, . . . , x_n. The driving forces here may indicate the policy stimulation, investment opportunity, surplus funds or corporate announcements etc. The generalized entropy function J of the system can be generated as the relationship of the driving force x_1, x_2, …, x_n:

$$J = \eta + \sum_i \gamma_i x_i + \sum_{ij} \gamma_{ij} x_i x_j + \sum_{ijk} \gamma_{ijk} x_i x_j x_k + \sum_{ijkl} \gamma_{ijkl} x_i x_j x_k x_l + \cdots \tag{1}$$

Assuming that the flux for the state of the stock market existing within the volumetric unit dx at time t is J, the averaged flux or entropy functional over all possible microstates is

$$\overline{J} = \int \rho(x,t) J(x) \mathrm{d}x \tag{2}$$

As shown in Figure 1, the generalized entropy variables representing different ways of permutations and combinations of the agents can be generalized to reflect not only the effect that the environment does to the stock market, but also the constraints in the complex stock market. The constraints of the market including all kinds of conservation expressed by many macroscopic, nonlinear partial differential equations with temporal-spatial boundary conditions, which can be sufficiently transformed into one to four organism interaction momentum quadratures of the ensemble:

$$\langle x_i \rangle = f_1,\ \langle x_i x_j \rangle = f_2,\ \langle x_i x_j x_k \rangle = f_3,\ \langle x_i x_j x_k x_l \rangle = f_4 \tag{3}$$

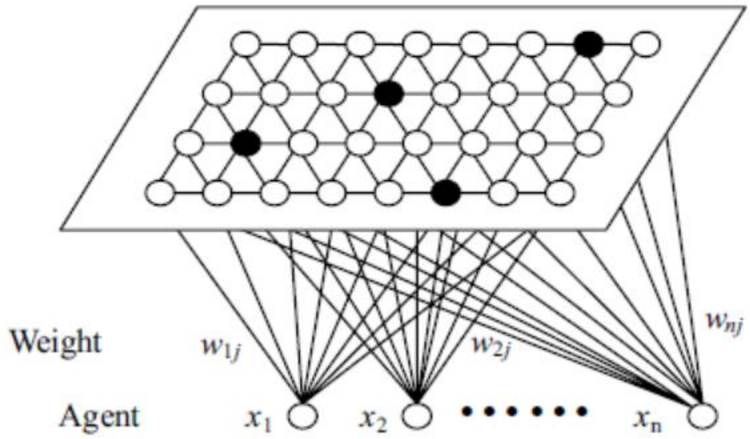
It can be seen that the physical model of the stock market proposed here is actually the carrier of general entropy. Developed from non-equilibrium statistical mechanics, the generalized entropy is a more comprehensive physical than the Newtonian mechanics of energy and the classical entropy. The generalized entropy not only expresses the correlation between agents, but also contains the complex factors that influence the system and accurately expressed the system's constraints. In the stock market, the factors affecting the market are complex, so the parameters of equation (1) and equation (3) are complex, but these parameters can be accurately determined if given the data. To analyze stock systems, we just need to deal with the interactions among agents and generalized entropy dynamics inside the ensemble, just like conservative mechanistic systems and equilibrium or near-equilibrium dissipative systems (Haken 2006).

Maximum entropy principle and SOM simulation

In physics and information theory, according to the Second Law of Thermodynamics, Maximum Entropy Principle is the driving force for the dynamics of the complex systems. As a typical complex open system, stock market should also obey such principle, meaning that stock market is always looking for an optimization process under the given constraints to make their “generalized entropy” to the greatest, in other word the stock market always tries to maximize their own development or to maximize the use of resources. Mathematically speaking, by using Lagrange multipliers, let us maximize Eq. (2) under the constraints of Eq. (3).

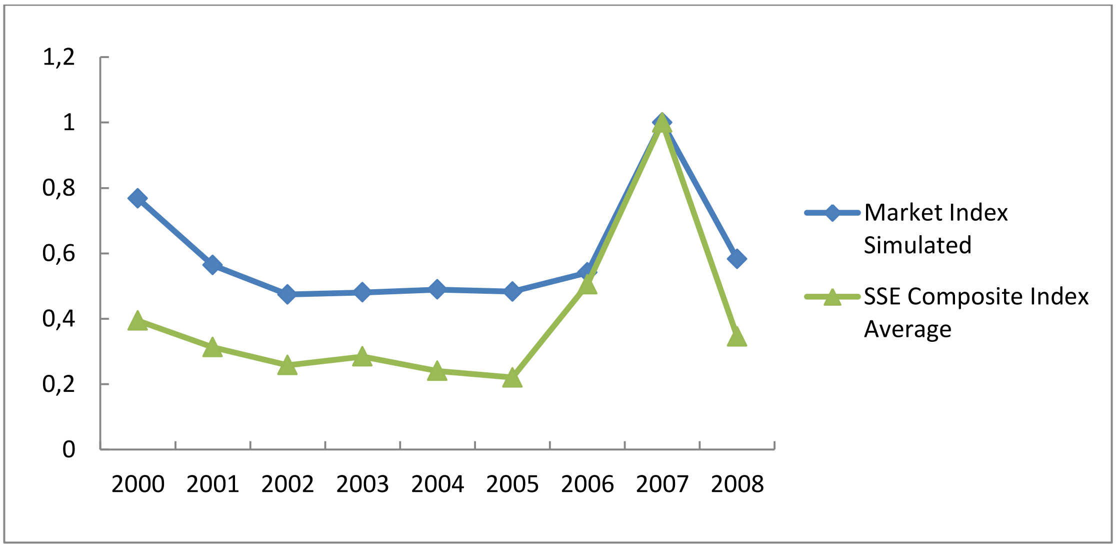
In order to maximize the generalized entropy of the stock market, only through mathematics of transforming the above equations under certain constrains is not possible. Instead the proper simulation method is applied here to realize the principle in the stock market. I notice that a kind of artificial neural networks (ANNs) called a Self-Organization Feature Map (SOFM or SOM) network algorithm is similar to our theoretical analysis of pattern dynamics in complex system, especially in stock market. In a self-supervised way, a SOM network extracts structural information from numerical data instead of memorizing it as is shown in Figure 2. The SOM can provide a topological ordering of the classes, thus automatically catching specific features in the input data and organizing them spatially (Chai, Li, 2007). The arithmetic principles of a SOM and our theoretical framework are utterly similar: to introduce the connection of competition and optimization in nonlinear systems. Thus, we can map our theoretical framework to the numerical method of the SOM, providing a basis for many applications of the Maximum Entropy Principle to future studies of the stock market.

Figure 2: How SOM network works (self-organizing through competition)



To have a good simulation of the stock market macro state, proper agents' data should be used and I have been able to collect 9 sets of agents' data on Chinse stock market from 2000 to 2008 including GDP, Interest Rate, Price Index, Exchange Rate, Stamp Duty, and Average Profit Rate of Listed Companies, the ratio of the investment in stock market and the total investment for individual investors, domestic investment institutions, and foreign investors.

Figure 3: Comparison between simulated market index with SSE Composite Index Average (yearly)



Source: Data from Yearbook of China's Finance and Banking 2009

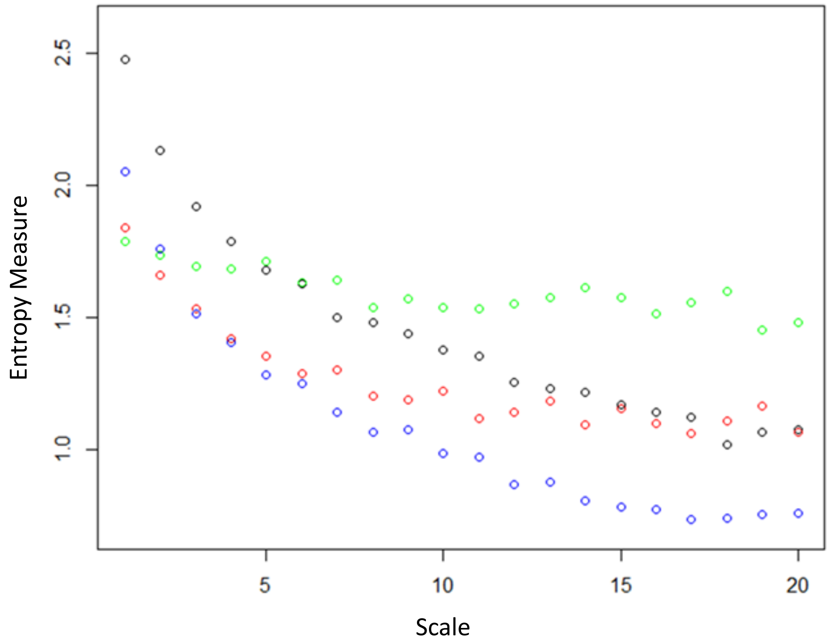
Following the maximum entropy principle, using the SOM simulation of the 10 input factors, I have been able to generate the above simulated market index which is clearly highly correlated with the SSE Composite Index Average which can be seen in Figure 3. After the 2000 "Internet bubble", Chinese Stock market has been declining ever since until 2005 it has gradually become a bull market and reach its peak in 2007 but followed by the effect of 2008 global financial crisis which heavily crashed the stock market in China. The maximum entropy simulated index perfectly catches these changes and the change of the index become even sharper than the normal SSE Composite Index when market fluctuate with bigger amplitude which gives better warnings of possible market risk. As an integrated measurement for stock markets macro states, maximum entropy principle and SOM simulation provide a more profound indicator for stock markets as higher value means much more complexity associated with it and the market pattern is at its optimum making a golden time for investors.

Multiscale Entropy and application

As we all know that complexity is associated with “meaningful structural richness” (Grassberger 1991), to better understand the structural change of the system, the entropy rate and Kolmogorov complexity have been raised up. However, when applied in data series from experimental physics, geophysics, physiology, medicine and social sciences which are finite length, the Kolomogrov-Sinai(KS) entropy cannot be estimated with reasonable precision. For the analysis of short and noisy time series in the ”real world”, Pincus (1991) introduced a family of measures termed approximate entropy (ApEn) and later another modified algorithm sample entropy (SampEn) has been proposed (Richman, Moorman 2000). Extending these methods to the analysis of structures on different scales, Zhang (1991) proposed a general approach to consider the multiple time scales in physical systems while Costa, Goldberger, and Peng (2002) went a step further by introducing a better technique called multiscale entropy (MSE) which proved to be well applied in physical and physiologic time series. Detailed multiscale entropy (MSE) methodology can be found in Costa, Goldberger, and Peng (2005).

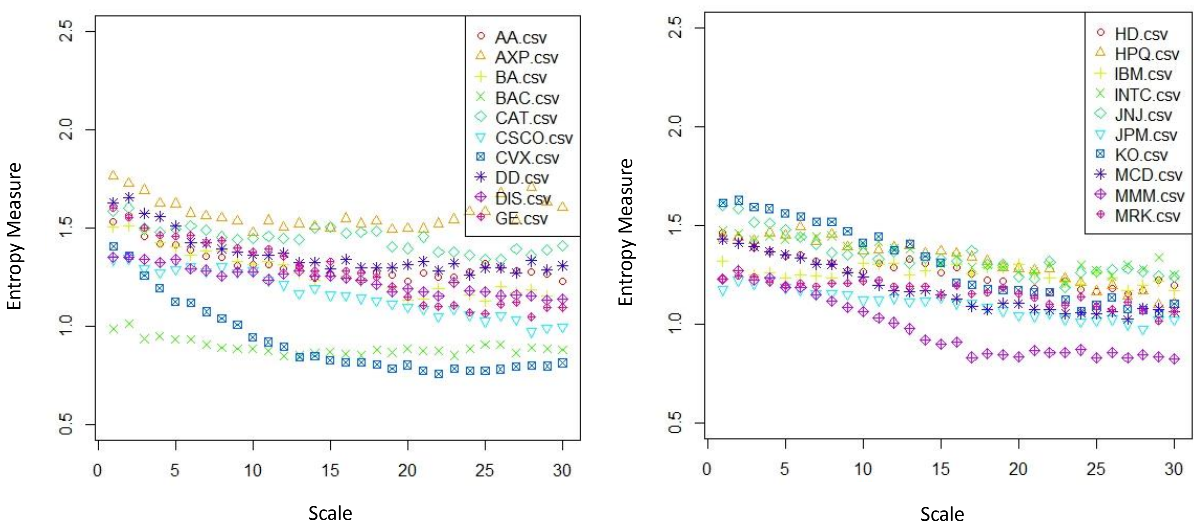
By applying multiscale entropy method, we can identify possible risk and classify different states or categories of institutions in the financial market which provides another detailed market microstructure analysis besides the maximum entropy principle with an emphasis on the time series complexity and the patterns of different scales.

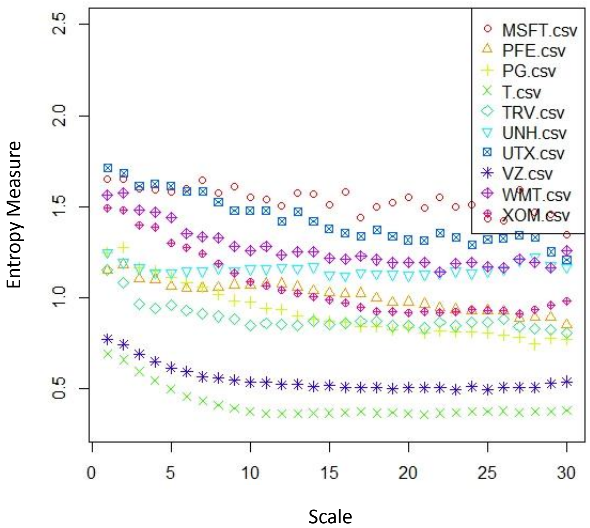
Figure 4: Multiscale entropy of different financial quantities. Black dots: Gaussian simulation (white noise); green dots: volume; red dots: volatility; blue dots: return



From Figure 4, we can find that return is quite similar to white noise while volume is similar to pink noise, and for volatility, it is more like between which indicates the correlation and fractal in the data series. This is possibly the reason why researchers and financial practitioners are very interested in the volatility analysis and prediction since return doesn't have rich information or patterns while both volatility and volume both have.

Figure 5: Multiscale entropy of volatilities of 30 stocks comprising Dow Jones Industrial Average





When looking at the entropy measure of volatilities of different stocks, an interesting find is that AT&T and Verizon have very low MSE which indicates that the stocks of Telecommunication industry is very stable with low complexity. On the other hand, American Express and Microsoft have relatively high MSE which indicates that the stock of Consumer finance and Software industries are very volatile with higher complexity.

Compared with the above maximum entropy principle approach, the multiscale entropy approach also looks closely to explain the market dynamics and tries to understand how the market pattern change over the time. Both derived from the classical definition of entropy, maximum entropy principle approach uses various market factors and the self-organization & competing nature to demonstrate the market patterns evolution and has an important indication of the market as a whole. On the other hand, the multiscale entropy approach focuses on the data series itself and identifies the complex patterns over time and different scales, which is more based on the analysis of the market key quantities like volatility, volume, etc. By combining these two approaches together, we will have a better understanding of the market pattern formation on both macro and micro levels.

Conclusion

In this paper, I look at the stock market as an open complex system and define the "generalized entropy" to describe market complexity and pattern evolution. I explored two different approaches of entropy analysis, i.e. maximum entropy principle and multiscale entropy using data from both China and US stock markets. Through comparison and illustration, I show that both entropy approaches have a promising application in financial market analysis which offers a better understanding of the market dynamics as well as pattern evolution. It is always good to combine these two approaches together which will give a fully picture of the market on different scales across the time and various levels of market. However, further detailed prediction mechanisms still need to be developed and the accuracy of prediction will be verified.

Reference

ARTHUR, W. B. (1999) Complexity and the Economy. *Science*. 2 April 1999, 284, 107-109.

ARTHUR, W. B. (2014). Complexity Economics: A New Framework for Economic Thought. *Complexity and the Economy*. W. B. Arthur, ed., Oxford Univ Press 2014.

CHAI, LH; LI, HB.(2007) A new theoretical analysis on organizing principles of water supply networks. *Journal of Water Supply*. 2007, 56(4): pp. 233-244

COSTA, M.; GOLDBERGER, A. L.; PENG, C.-K. (2002) Multiscale Entropy Analysis of Complex Physiologic Time Series. *Phys. Rev. Lett*. 89, 068102.

COSTA, M., GOLDBERGER, A. L., & PENG, C. K. (2005). Multiscale entropy analysis of biological signals. *Physical review E*. 71(2), 021906.

FARMER, J. D.; F. LILLO. (2004) On the Origin of Power Law Tails in Price Fuctuations, *Quantitative Finance*. 4,1:7–11, 2004.

GRASSBERGER, P. (1991). *Information Dynamics*, edited by H. Atmanspacher and H. Scheingraber (Plenum, New York, 1991), p. 15.

HAKEN, H. (2006). *Information and Self-Organization: A Macroscopic Approach to Complex Systems* (Springer-Verlag, New York, 2006).

PINCUS, S. M. (1991). Approximate entropy as a measure of system complexity. *Proceedings of the National Academy of Sciences U.S.A.* 88, 2297, 1991.

RICHMAN, J. S.; MOORMAN, J. R. (2000) Physiological time-series analysis using approximate entropy and sample entropy. *American Journal of Physiology.* 278, H2039, 2000.

ZHANG, Y.-C. (1991) Complexity and 1/f noise: A phase space approach. *Journal of Physics*. I 1, 971, 1991.