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INCOME INEQUALITY, MEDICAL CONDITIONS, AND HOUSEHOLD BANKRUPTCY

Abstract:

I study disparities in emergency and non-emergency medical conditions between high and low income individuals and their implications on consumption, savings, and bankruptcy. In the Medical Expenditure Panel Survey (MEPS), two patterns emerge. First, low income individuals are more likely to visit emergency rooms than high income individuals, and this gap is disproportionately larger for working age individuals. Second, although the differences between high and low income individuals in non-emergency medical conditions are little in early life, the gap in non-emergency medical conditions is substantial in middle and late life. To explain these facts, I build an overlapping generations general equilibrium model that features (i) endogenous decisions on default and health insurance, (ii) endogenous health that determines labor productivity, (iii) the existence of emergency (non-discretionary) medical expenditures and non-emergency (discretionary) medical expenditures, and (iv) the endogenous distribution of emergency and non-emergency health shocks. I find that low income individuals spend less on their health in early life, leading to their contacting more severe and more frequent health conditions (emergency and non-emergency) following their middle life onwards. This enforces low income individuals to be sicker and to visit emergency rooms more often, while spending more on health cares from their middle life. Moreover, this model shows that this endogenous distribution of health shocks causes low income individuals to have more precautionary savings and less consumption due to their highly volatile earnings from severe health shocks. The poor default more often due to their lower earnings and more frequent emergency (non-discretionary) medical treatments, which arises from their bad health status.

Keywords:

Income Inequality, Household Bankruptcy, Health

JEL Classification: E21, K35, I13

1 Introduction

Based on empirical evidence that medical expenditures are one of the main causes of household bankruptcy, recent studies have emphasized the positive effects of health insurance reform on household finance. This paper reevaluates this hypothesis from a macroeconomic perspective. I ask two questions about the role of overall health insurance on household bankruptcy and the overall macroeconomy. First, can expansions in health insurance decrease the average household bankruptcy rate? Second, what are the macroeconomic and welfare consequences of such policies?

In order to answer these questions I build an overlapping generations general equilibrium model. In the model, households are subject to idiosyncratic shocks on their earnings and health. This general equilibrium model allows me to examine the interaction between changes in relative prices and household decisions. I find that this interaction is important in studying the effects of health insurance policy and assessing their welfare implications, as such policies can have significant effect on individual decisions through price channels. My structural model tries to encompass key variables from empirical studies and quantitatively examine the main insights suggested in the literature. Each period, households choose their consumption, saving or borrowing, along with medical expenditure and decide whether to purchase insurance. Households with debt can also default. Thus, the model features endogenous decisions on medical expenditure, health insurance and default.

My framework is consistent not only with the targeted moments but also with untargeted facts on (1) the aggregate moments in bankruptcy and credit, (2) the lifecycle profile of default, (3) the lifecycle profile of medical expenditures, and (4) the lifecycle profile of insurance take-up rate. To investigate the implications of health insurance reform, I conduct policy analysis comparing three different health insurance scenarios: employer-sponsored health insurance that is the existing scenario before 2014, health insurance with the Affordable Care Act (ACA) subsidies, and universal health insurance.

Findings from the literature generally imply a sizable effect of health insurance reform on household bankruptcy.¹ One type of study focuses on the proportion of household bankruptcies from medical problems. Indeed, medical problems are the biggest reason for bankruptcy found in many studies.² However, these studies do not quantitatively show the effect of health insurance on household bankruptcy. Recent studies have started to quantitatively show the effects of health insurance policies on household bankruptcy. Using the CPS, [Gross and Notowidigdo \(2011\)](#) find that an expansion of Medicaid decreases household bankruptcy filings. [Mazumder and Miller \(2014\)](#) use the Federal Reserve Bank of New York Consumer Credit Panel data set to investigate the effect of the Massachusetts health reform. They find that it had a positive impact on the financial condition of households in the state. The reform improved credit scores and reduced the total amount of unpaid debt as well as the rate of household bankruptcies.

The predictions of my model are different from previous studies. In particular, I show that when the economy changes its health insurance policy and expands coverage, the change in the average default rate is not as large as predicted in earlier work. Moreover, the overall direction of change is somewhat ambiguous, as there is a compositional change in the default rate over age. As health insurance expands, younger generations have a rise in their default rates, while the old experience a fall. Given that medical problems are still a major driving force behind the decision to default, this result may seem to be surprising.

Qualitatively different changes in the default rate occur over age because of a general equilibrium effect from increases in discretionary taxes required to fund the expansions in health insurance. As a result, the aggregate capital stock falls. There are two driving forces behind this result. First, low income households reduce precautionary savings,

¹An exceptional study is [Kuklik \(2010\)](#)'s work. He shows that the effect of expanding health insurance is not large on the average default rate, similar to my result. I will explain the difference in the next section, related literature.

²The range of estimates related to the portion of the bankruptcies from medical reasons is large. For example, [Himmelstein et al. \(2009\)](#) conclude that 62.1% of the household bankruptcies have medical reasons, while [Austin \(2014\)](#) finds that the portion is between 16% and 25%.

driven by the possibility of health shocks, as their health insurance coverage rises. Second, increases in progressive rates of income tax raise taxes on higher income households and reduce their return on capital. The reduction in the aggregate capital stock increases the risk-free interest rate, which raises cost of borrowing through equilibrium loan rate schedules consistent with default risk. This has a disproportionate impact on younger generations. For the young, shocks on earnings are more burdensome than health shocks. Since they are relatively poor, they are more likely to access financial markets to borrow against future earnings. However, they pay higher interest, resulting in higher default rates for young households. Older generations suffer less from this increase in the cost of borrowing, as they have accumulated assets over time. In addition, health is relatively more important for older generations than for younger ones. The prevalence of health insurance lessens the financial burden for poor and old households. It causes a reduction in the default rate for old households. As these intergenerational forces offset each other, the average default rate does not change considerably.

It is important to note that the above result is not the analysis of welfare. Welfare analysis requires the evaluation of changes in health, as improvements in health enhance welfare. In terms of health, younger and poorer individuals are better off. Expansions in health insurance lead to better insurance for younger and poorer individuals against health shocks. Since earnings are also partially determined by health status, expansions in health insurance also raise the income of low income individuals. Thus, expanding health insurance improves welfare at the cost of a decrease in aggregate output and capital. The source of welfare improvement is enhancements in health and consumption for low and middle income households. Expansions in health insurance improve young workers' labor productivity over time through increase in their health. In middle age, their income levels rise, and the improvements in health cause an increase both in income and consumption. Although younger generations face higher interest rates on their debt, the benefits from better health are larger than the losses from the rising cost of borrowing

driven by an increase in the risk free interest rate.

Comparing the economy with ACA subsidies to an economy with universal health insurance, I find several interesting results. First, the gaps in health are very small between the ACA economy and the universal health insurance economy. One of the key components in the ACA is the expansion of the Medicaid. When poor households are fully subsidized for their health insurance, health status in the economy is very similar to the universal health insurance economy. Second, although the welfare in the universal health insurance economy is higher than that with the ACA, the default rate is also higher. To implement universal health insurance, more subsidies are needed, which increases taxes, and thus the risk-free interest rate. This raises the cost of borrowing driven by equilibrium loan rate schedules consistent with default risk.

My results suggest that bankruptcy rates alone are not a good measure for evaluating health insurance policies, because the average default rate does not vary considerably and it may rise with improvements in health and welfare.

Another contribution of my work is the computational method I use to solve my model economy. This induces a potentially large computational burden: the individual state is large, there are multiple discrete choices, and loan prices are functions of individual states. To handle these computational problems, I develop an endogenous grid method for default problems with multiple discrete choices. In particular, my method is the extension of the endogenous grid method in [Fella \(2014\)](#). His algorithm solves multiple discrete choice models with exogenous borrowing constraints. I solve an environment with endogenous individual specific borrowing constraints, arising through default risk, with multiple discrete choices.

The organization of the paper is as follows. Section 2 briefly reviews the literature related to household bankruptcy and health insurance policy. Section 3 and 4 explain the institutional background, economic environments, calibration and model solution. Section 5 describes the result and section 6 concludes this paper.

2 Related Literature

My work is a part of the literature that endogenizes individual health status. The seminal work is [Grossman \(1972\)](#), which provides a theoretical framework for investment in health. Following this approach, [Hall and Jones \(2007\)](#) show why medical expenditures in the U.S. have increased over time. Turning to the effect of health shocks and income, [Prados \(2012\)](#) studies the relationship between the payment of medical expenditures and the inequality of earnings over the life cycle. She finds that a large portion of earnings inequality can be accounted for by feedback from health to income. [Ozkan \(2014\)](#) studies differences in preventative health care usage and the evolution of health between high and low income individuals. He finds that the rich spend more than the poor on medical treatment early in their lives, while the poor spend more than the rich later in life, and constructs a model with preventive health capital that explains these stylized facts.³

Several empirical studies address the relationship between health insurance policies and household bankruptcy. [Mahoney \(2015\)](#) studies the effects of the financial cost of bankruptcy on health insurance choices. He shows that those who have a higher cost of bankruptcy are more likely to buy health insurance. [Gross and Notowidigdo \(2011\)](#) investigate the effect of the expansion of Medicaid on consumer bankruptcy, and find that it has decreased the probability of bankruptcy. [Mazumder and Miller \(2014\)](#) find that the Massachusetts health reform improved financial conditions across households.

The work most closely related to mine is [Kuklik \(2010\)](#). He studies the effect of the Affordable Care Act (ACA) on household bankruptcy using a structural model without health on a state variable. Therefore, health insurance only insures against shocks to medical expenditures. His model shows that, following reform, the reduction in the

³ My paper is also related to the literature on household bankruptcy. In household studies, [Fay et al. \(2002\)](#) use the Panel Study of Income Dynamics (PSID) and show that households strategically file for bankruptcy. [Chatterjee et al. \(2007\)](#) conduct a policy experiment to investigate the effect of the change in the Chapter 7 bankruptcy law. [Livshits et al. \(2007\)](#) investigate how differences in bankruptcy law affect consumption smoothing behavior between the U.S. and European countries. [Athreya \(2008\)](#) investigates the relationship between social insurance and default policies over the life cycle.

bankruptcy is very small, only 0.06 percentage points. However, the reform causes an improvement in welfare of 5.2 percent, largely driven by the redistributive effects of the reform on consumption.

The mechanism of my model is significantly different from that of Kuklik, although results for bankruptcy rate and welfare are similar. First, in Kuklik (2010), expansions in health do not cause changes in aggregate variables such as capital stock, the risk-free interest rate, and output. This difference is driven by my allowing for endogenous health. In my model, health capital substitutes for physical capital and there are redistributive effects following an expansion in health insurance. Second, in Kuklik (2010), expansions in health insurance increase both the fraction of borrowers and the amount of debt. However, in my model, the opposite occurs: the fraction of borrowers and the aggregate level of debt decrease. This difference arises from the change in the aggregate capital stock in my work. In his model, the risk-free interest rate does not change, as capital does not vary. Thus, the expansion of health insurance lowers the cost of borrowing driven by following health shocks becomes less likely. In my model, the risk-free rate rises due to a reduction in the aggregate capital. This raises the cost of borrowing, which leads to a decrease in the fraction of borrowers and the aggregate level of debt.

Furthermore, the sources of improvement in welfare are different. In Kuklik (2010), a major part of welfare improvement is caused by additional income from larger subsidies for low income households. Health care redistributes income from high-income to low-and middle income households through these subsidies and Medicaid. As health is exogenous in his model, welfare is determined by consumption alone. Expansions in health make it possible for low income households to consume more as their medical costs from health shocks are subsidized, which is the main driving force behind welfare improvements in his work. In contrast, in my model, welfare improvements are largely driven by enhancements in health for low-middle income households. These welfare improvements are achieved through the following lifecycle dynamics. Expanding health

insurance improves health for some poor and young households. One resulting effect is that the levels of incomes in middle age increase, as their labor productivity improves, and this raises consumption. The other effect is to improve health itself, which improves welfare in my model. Although both models have welfare improvements with expansions in health insurance, the mechanism is very different.

In contrast to the literature above, my model features both household bankruptcy and endogenous health. Although medical expenditures are an important cause of household bankruptcy, previous studies have usually focused on either household bankruptcy or endogenous decisions on medical expenditures, but not both. My work analyzes the interaction between medical spending and household bankruptcy by endogenizing both decisions.

My work is also related to the literature that examines the role of medical expenditures in increasing precautionary savings by the old. [Kotlikoff \(1986\)](#) shows that the precautionary savings motive arising from possible medical expenditures accounts for a large part of aggregate savings. [Hubbard et al. \(1995\)](#) investigate the interaction between social insurance and precautionary savings. [De Nardi et al. \(2010\)](#) shows that the length and cost of medical treatment explain why higher income elderly have large levels of savings. My work also explores changes in savings incentives of the elderly following expansions in health insurance. Importantly, I explore how changes in the savings of the old affect younger generations' behavior.

My work also contributes to the literature on the endogenous grid method, originally developed by [Carroll \(2006\)](#). [Barillas and Fernández-Villaverde \(2007\)](#) extends it to models with endogenous labor supply. [Fella \(2014\)](#) extends the endogenous grid method for multiple discrete choices problems with exogenous borrowing constraints. The endogenous grid method I used to solve my model extends his work by allowing for default, nonlinear loan rate schedules as well as multiple discrete choices.

3 The Model Economy

I start with an explanation of institutional background for health care. Then I present the model environments and equilibrium.

3.1 Legal Environment

The model economy is based on two institutional features: Chapter 7 bankruptcy filing and the Emergency Medical Treatment and Labor Act (EMTALA).

I model default based on Chapter 7 bankruptcy.⁴ The modeling strategy for default hugely rests on the work of [Chatterjee et al. \(2007\)](#). Suppose that households have a level of unsecured debt and declare bankruptcy. They face the following situation.

- Households have to pay the cost of filing for bankruptcy as much as χ portion of their earnings. Once they file for bankruptcy, all of their debt is discharged.⁵ In the period of filing for bankruptcy, households can neither save nor dissave.
- Once a household declares bankruptcy, the record of bankruptcy remains on their credit record. In the model, "good credit status" means that there is no bankruptcy on the credit record. "Bad credit status" implies that a bankruptcy is recorded for the household on its the credit record.
- Households with bad credit status cannot obtain new loans.
- With an exogenous probability λ , this bad credit status changes to good credit status at the start of any period.⁶

⁴Chapter 7 covers 70% of household bankruptcies. The other type of household bankruptcy is Chapter 13, which I do not address Chapter 13 here.

⁵Chapter 7 has an exemption levels for assets, and this level differs across states. For simplicity, I abstract from exemption.

⁶The value $1/\lambda$ is the average duration of bad credit status. In the US, bankruptcy is reported on a consumer's credit report for ten years.

I model household bankruptcies for medical reasons based on the EMTALA. The EMTALA was enacted by Congress in 1986 in order to prevent hospitals from dumping out patients with emergency conditions. The EMTALA requires hospitals with emergency departments to screen and treat an emergency medical condition, regardless patients' ability to pay or their insurance status. The EMTALA forces hospitals to provide emergency medical treatment to patients on credit. Patients can default on the debt incurred.

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The model has two types of debt. The first type of debt comes through financial intermediaries. This type of debt can be used not only for consumption but also for medical costs. The other type of debt arises from the relationship between hospitals and households, without any financial intermediary. This type of debt exists due to the EMTALA that enforces hospitals to provide emergency medical treatments without monitoring the ability to repay medical bills before the treatment. This debt is incurred only for medical treatments. These two types of debts are unsecured and they are subject to Chapter 7 bankruptcy.

3.2 Model

The economy is populated by a continuum of households in J overlapping generations. Each period, these households make decisions on consumption, saving or borrowing, the purchase of health insurance, medical expenditure, and default. Inelastically supplying labor over their working periods, all retire at age J_r and live until age J .

⁷Holmes and Madans (2013) show that unpaid debts in emergency departments are composed of 6% of the total cost of hospitals. In addition 55% of US emergency care is uncompensated.

3.2.1 Preferences

Preferences are represented by an isoelastic utility function over an aggregate that is itself a Cobb-Douglas utility function over consumption c and health status $(1 - \epsilon_{h,j})h$,

$$u(c, (1 - \epsilon_{h,j})h) = \frac{(c^\alpha ((1 - \epsilon_{h,j})h)^{1-\alpha})^{1-\sigma}}{1 - \sigma} \quad (1)$$

where α is the weight on consumption and σ is the coefficient of relative risk aversion.

3.2.2 Labor Income

Labor income y is comprised to the four terms: the aggregate market wage w , a deterministic age term ω_j ,⁸ a current health status term $(1 - \epsilon_{h,j})h$ ⁹ and an idiosyncratic productivity shock η . The idiosyncratic productivity shock is composed of two part, a persistent shock e , and a transitory shock v .

$$\log(y) = \log(w) + \log(\omega_j) + \log((1 - \epsilon_{h,j})h) + \log(\eta) \quad (2)$$

$$\log \eta = e + v$$

$$e' = \rho_e e + \epsilon, \epsilon \sim N(0, \sigma_\epsilon^2)$$

$$v \sim N(0, \sigma_v^2)$$

Note that labor income is partially endogenous, because the level of health capital, h , is endogenously determined.

3.2.3 Financial Market

There are competitive financial intermediaries and loans are defined by each household state. It implies that with the law of large numbers, ex post realized profits of lenders

⁸I include age, the square of age and the cubic of age to control age

⁹I assume that h and $(1 - \epsilon_{h,j})$ are not separately observable in the data, but the product of both terms $(1 - \epsilon_{h,j})h$ is observed as health status in data sets.

are zero for each type of loans. The lenders can observe the state of each borrower and the price of loans is determined, using individual default probabilities and the risk-free interest rate. Denote $d(a'_1, i', h'; j, e)$ as the default probability of households with a total debt, a'_1 , insurance purchase status, i' , health capital for next period, h' , current age, j and current earnings shock, η . Then, the loan price is

$$q(a'_1, i', h'; j, \eta) = \frac{1 - d(a'_1, i', h'; j, \eta)}{1 + r_f} \quad (3)$$

where r_f is the risk-free saving equilibrium interest rate.

3.2.4 Health Technology

In the spirit of [Grossman \(1972\)](#), health capital evolves as follows:

$$h' = (1 - \epsilon_{h,j})h + \psi m^\varphi \quad (4)$$

where h , $\epsilon_{h,j}$, ψ , and m are the stock of health, health shocks at age j , the efficiency of medical technology and total medical expenditures, respectively. For households at age j , the health capital h is predetermined and a health shock $\epsilon_{h,j}$ is realized at the beginning of the period. The health shock $\epsilon_{h,j}$ causes a depreciation of the stock of health by $\epsilon_{h,j}h$, and the undepreciated part of health capital $(1 - \epsilon_{h,j})h$ represents the households' current health status. Households can choose the level of health capital for next period, h' , using total medical expenditure, m .

Given the endogenous health choice in the model, the timing assumption of the health capital is different from that used in other models to prevent rich individuals from maintaining the best health levels. If health status were fully deterministic, the model would generate unrealistic health profiles that keep the implied health level highest for the very wealthy over the life cycle. The timing assumption used here avoids such unrealistic health profiles.

3.2.5 Health Insurance Plans

Households have access to different types of health insurance over their life cycle. During working ages, households choose whether to purchase private health insurance. In the baseline, I model this health insurance policy as employer-based health insurance. Employer-based health insurance is more likely to be provided in high salary jobs. Thus, I assume that the offer of employer-based health insurance is random and the probability of an offer increases with worker's persistent idiosyncratic productivity, e . Specifically, the probability of an offer of employer-based health insurance es is given by $p(es|e)$ where e is the persistent component of the idiosyncratic shock on earnings. In addition, $p(es|e)$ increases with e . Households after retirement use Medicare which is available to all retirees.

The contract period of health insurance is annual. Households need to make decisions on the purchase of health insurance before health shocks are realized. After the realization of health shocks, households determine the total level of medical spending, m . For the insured, insurance provides coverage qm and $(1 - q)m$ becomes an out-of-pocket medical expenditure of an insured household. Households without health insurances have to pay all of their medical expenditure m as an out-of-pocket medical expenditure.

Health insurance premiums, $p(j_g)$ are determined by age group : 25 – 34, 35 – 44, 45 – 54, 55 – 64. There are many private health insurance providers and these premiums are competitively determined. Since insurance providers can freely enter health insurance markets, their total revenue is equal to the cost in each age j sub-market.

3.2.6 Tax System and Government Budget

Taxes are levied from two sources: payroll and income. On the one hand, social security Ψ , and Medicare q_{med} are financed from payroll tax. τ_{ss} is the payroll tax rate for Social security, τ_{med} is that for Medicare. On the other hand, income tax covers the

government expenditures G , and finances the subsidy for health insurance, $\Xi(\cdot)$, that is a function of income, y .

I choose the progressive tax function from [Gouveia and Strauss \(1994\)](#), as this has been widely used in the macroeconomic policy literature. The income tax function $T(y)$ is given by

$$T(y) = a_0 \{y - (y^{-a_1} + a_2)^{-1/a_1}\} + \tau_y y \quad (5)$$

where y is taxable income. a_0 means the upper bound of the progressive tax as income y goes infinite. a_1 determines the curvature of the progressive tax function, and a_2 is a scale parameter. To use [Gouveia and Strauss \(1994\)](#)'s estimation result, I take their estimates in a_0 and a_1 . a_2 is calibrated to match a target that is the fraction of total revenues financed by progressive income tax is 65% (OECD Revenue Statistics 2002). τ_y is chosen to balance the total government budget.

3.2.7 Firm

The economy has a representative firm. The firm maximizes its profit by solving the following problem,

$$\max_{K,N} F(K, N) - wN - rK \quad (6)$$

where K is the aggregate capital stock and N is aggregate labor.

3.2.8 Overview of Household's Decisions

Each period consists of three sub-periods. At the beginning of sub-period 1, health shocks, $\epsilon_{h,j}$, are realized. Then, households choose their total medical expenditures, m , and pay out-of-pocket costs, $(1 - q\mathbb{1}_{i=1})m$ where $\mathbb{1}_{i=1}$ is the indicator function for health

insurance. The stock of health capital next period, h' , is determined by the law of motion, $h' = (1 - \epsilon_{h,j})h + \psi m^\varphi$. After paying the out-of-pocket medical costs, remaining assets are $a_2 = a_1 - (1 - q\mathbb{1}_{i=1})m$, where a_1 was assets at the start of period.

At the beginning of sub-period 2, idiosyncratic shocks on labor productivity, η , are realized. Next, households with good credit choose whether or not to default. Those whose credit status is bad have no option to default.

In sub-period 3, households determine consumption, saving or borrowing, and the purchase of health insurance. Their feasible choices differ with credit status and default decision in sub-period 2. Non-defaulters with good credit can choose consumption, c , savings or debt, a'_1 , and the purchase of health insurance for next period, i' . Defaulters can neither save nor dis-save, and they can choose consumption, c , and health insurance, i' . Households with bad credit are not allowed to borrow. They choose their level of consumption, c , savings $a'_1 \geq 0$, and health insurance for next period, i' .

At the end of sub-period 3, the credit status of households is updated. Non-defaulters who repay their debts maintain good credit status, while defaulters have bad credit status. Households with bad credit recover good credit status with probability λ .

3.2.9 Household's Dynamic Problem

The household optimal decision problems can be represented recursively. I begin with the problems of working households. At the beginning of sub-period 1, the state is $(a_1, i, h, \epsilon_{h,j}, \eta_{-1})$ and $v \in \{G, B\}$, where a_1 is their level of assets before paying medical bills, i is health insurance, $\epsilon_{h,j}$ is health shock, η_{-1} is the idiosyncratic shocks on labor productivity, and v is credit status, where G and B mean good credit and bad credit, respectively. Let $V_j^G(a_1, i, h, \epsilon_{h,j}, \eta_{-1})$ ($V_j^B(a_1, i, h, \epsilon_{h,j}, \eta_{-1})$) denote the value function of age $j < Jr$ agent with good (bad) credit in sub-period 1. Then in sub-period 1, households

with good credit solve

$$V_j^G(a_1, i, h, \epsilon_{h,j}, \eta_{-1}) = \max_m \sum_{\eta} \pi_{\eta_{-1}, \eta} v_j^G(a_2, i, h', h, \epsilon_{h,j}, \eta) \quad (7)$$

$$a_2 = a_1 - (1 - q\mathbb{1}_{i=1})m$$

$$h' = (1 - \epsilon_{h,j})h + \psi m^\varphi$$

, and households with bad credit solve

$$V_j^B(a_1, i, h, \epsilon_{h,j}, \eta_{-1}) = \max_m \sum_{\eta} \pi_{\eta_{-1}, \eta} v_j^B(a_2, i, h', h, \epsilon_{h,j}, \eta)$$

$$a_2 = a_1 - (1 - q\mathbb{1}_{i=1})m$$

$$h' = (1 - \epsilon_{h,j})h + \psi m^\varphi$$

where m is total medical expenditure, a_2 is their level of assets after paying out of pocket medical expenditures, h' is the stock of health capital for next period, and q is the insurance coverage rate. $v_j^G(a_2, i, h', h, \epsilon_{h,j}, \eta)$ ($v_j^B(a_2, i, h', h, \epsilon_{h,j}, \eta)$) denotes the value function of age $j < Jr$ households with good (bad) credit in sub-period 2.

At the beginning of sub-period 2, idiosyncratic earnings shocks, η , are realized. Afterwards, households with a good credit status decide whether to default. They solve

$$v_j^G(a_2, i, h', h, \epsilon_{h,j}, \eta) = \max \{v_j^{G,R}(a_2, i, h', h, \epsilon_{h,j}, \eta), v_j^{G,D}(i, h', h, \epsilon_{h,j}, \eta)\} \quad (8)$$

where $v_j^{G,R}(a_2, i, h', h, \epsilon_{h,j}, \eta)$ is the value function if debt is repaid and $v_j^{G,D}(i, h', h, \epsilon_{h,j}, \eta)$ is the value of defaulting.

In sub-period 3, non-defaulters with good credit status solve

$$v_j^{G,R}(a_2, i, h', h, \epsilon_{h,j}, \eta) = \max_{\{c, a_1, i' \in \{0,1\}\}} \frac{(c^\alpha ((1 - \epsilon_{h,j})h)^{1-\alpha})^{1-\sigma}}{1 - \sigma} + \beta \sum_{\epsilon_{h',j+1}} \pi_{\epsilon_{h',j+1}} V_{j+1}^G(a_1', i', h', \epsilon_{h',j+1}, \eta) \quad (9)$$

$$c + q(a_1', i', h'; j, \eta)a_1' + p(i', j_g) = we_j + a_2 - Tax(we_j, y)$$

$$a_2 = a_1 - (1 - q\mathbb{1}_{i=1})m(h', h, \epsilon_{h,j})$$

$$\tilde{p}(i', j_g) = \begin{cases} 0 & \text{if } i' = 0 \\ p_{ins}(j_g) & \text{if } i' = 1 \end{cases}$$

$$Tax(we_j, y) = (\tau_{ss} + \tau_{med})we_j + T(y)$$

$$y = we_j + \left(\frac{1}{q^{rf}} - 1\right)a_1 \cdot \mathbb{1}_{a_1 > 0}$$

$$e_j = \omega_j(1 - \epsilon_{h,j})h\eta$$

where $q(a_1', i', h'; j, \eta)$ is the loan price of households with future endogenous state $(a_1', i', h'; j, \eta)$, $p(i', j_g)$ is the health insurance premium for age group j_g , e_j is households efficiency unit of labor at age j , and $Tax(we_j, y)$ represents total taxes. Non-defaulters with good credit have earnings, we_j . A positive (negative) value of a_2 implies savings (debt). $m(h', h, \epsilon_{h,j})$ is the decision rule for medical expenditures from sub-period 1. Households pay tax based on their payroll income, we_j , and total income y . $(\frac{1}{q^{rf}} - 1)a_1 \cdot \mathbb{1}_{a_1 > 0}$ is capital income where q^{rf} is the risk-free bond price, a_1 is the level of asset before paying medical costs in sub-period 1, and $\mathbb{1}$ is the indicator function for savings.

Households make decisions on whether to purchase health insurance for next period and pay a premium, $\tilde{p}(i', j_g)$. Next, they access financial intermediary to borrow at prices that reflect their the default risk. Afterwards, households determine their level of consumption, c . It is worth noting that medical expenditures, $m(h', h, \epsilon_{h,j})$, are not in the

decision problem here. They were determined in sub-period 1.

Defaulting households who had good credit solve the following problem,

$$v_j^{G,D}(i, h', h, \epsilon_{h,j} \eta) = \max_{\{c, i' \in \{0,1\}\}} \frac{(c^\alpha ((1 - \epsilon_{h,j})h)^{1-\alpha})^{1-\sigma}}{1 - \sigma} + \beta \sum_{\epsilon_{h,j+1}} \pi_{\epsilon_{h,j+1}} V_j^B(0, i', h', \epsilon_{h,j+1}, \eta) \quad (10)$$

$$c + p(i', j_g) = (1 - \chi)we_j - Tax(we_j, y) - (1 - q \cdot \mathbb{1}_{i=1})m(h', h, \epsilon_{h,j}) \cdot \mathbb{1}_{\epsilon_{h,j} \in EMTALA}$$

where χwe_j is a pecuniary cost of filing for bankruptcy. $\mathbb{1}_{\epsilon_{h,j} \in EMTALA}$ is an indicator function representing health shocks that require patients to use emergency medical treatment subject to the EMTALA. I assume that if health shocks are in the top 20% of the distribution, medical treatments are subject to the EMTALA.¹⁰ Defaulters can determine the level of consumption c , and the purchase of health insurance for next period, i' , while they can neither save nor dissave in this period.

In sub-period 2, households with bad existing credit have no default choice, and in sub-period 3, they solve

$$v_j^B(a_2, i, h', h, \epsilon_{h,j}, \eta) = \max_{\{c, a'_1 \geq 0, i' \in \{0,1\}\}} \frac{(c^\alpha ((1 - \epsilon_{h,j})h)^{1-\alpha})^{1-\sigma}}{1 - \sigma} \quad (11)$$

$$+ \beta \sum_{\epsilon_{h,j+1}} \pi_{\epsilon_{h,j+1}} \left(\lambda V_{j+1}^G(a'_1, i', h', \epsilon_{h,j+1}, \eta) + (1 - \lambda) V_{j+1}^B(a'_1, i', h', \epsilon_{h,j+1}, \eta) \right)$$

$$c + q^{rf} a'_1 + p(i', j_g) = we_j + a_2 - Tax(y)$$

$$a_2 = a_1 - (1 - q \mathbb{1}_{i=1})m(h', h, \epsilon_h)$$

$$a'_1 \geq 0$$

where q^{rf} is the price of a risk-free bond. The problem of households with bad credit is similar to that of repaying households with good credit except for two differences. The

¹⁰Holmes and Madans (2013) show that 20% of US residents visited an emergency room in 2011.

first difference is that the households with bad credit are not allowed to borrow. The price of a bond is independent of households state, because the return on saving is always their risk-free return in the economy. The other difference is that households with bad credit recover good credit status in the next period with an exogenous probability of λ . This process reflects the exclusion penalty in Chapter 7 Bankruptcy of 10 years in the US.

Households after retirement solve

$$V_j^{rt}(a, h, \epsilon_{h,j}) = \max_{\{c, m \geq 0, a' \geq 0\}} \frac{(c^\alpha ((1 - \epsilon_{h,j})h^{1-\alpha})^{1-\sigma})^{1-\sigma}}{1 - \sigma} + \beta \sum_{\epsilon_{h,j+1}} \pi_{h, \epsilon_{h,j+1}} V_{j+1}^{rt}(a', h, \epsilon_{h,j+1}) \quad (12)$$

$$c + q^{rf} a' + p_{med} = a - (1 - q_{med})m + ss - Tax(0, y)$$

$$h' = (1 - \epsilon_{h,j})h + \psi m^\varphi$$

where p_{med} is the Medicare premium, and q_{med} is the Medicare coverage rate. I assume that retired workers cannot access private health insurance markets. Retired households do not have labor income but they are allowed to have capital income. In addition, they can save, but they are not allowed to borrow.

3.2.10 Hospital sector

For a households with state $(a_2, i, h, h', \epsilon_{h,j}, \eta)$ in sub-period 3, a hospital earns the following revenue.¹¹

$$(1 - g_{df,j}(a_2, i, h, h', \epsilon_{h,j}, \eta))m(h', h, \epsilon_{h,j}) + g_{df,j}(a_2, i, h, h', \epsilon_{h,j}, \eta) \max(a_2, 0) \quad (13)$$

where $g_{df,j}(a_2, i, h, h', \epsilon_{h,j}, \eta)$ is the binary decision rule of default with household's state $(a_2, i, h, h', \epsilon_{h,j}, \eta)$ in sub-period 2 and $m(h', h, \epsilon_{h,j})$ is the decision rule of medical expenditures with their state (h', h, ϵ) from sub-period 1. Non-defaulters repay all the medical

¹¹Since households pay the medical bills in sub-period1 and make a decision on default in sub-period 2, the revenue of the hospital is determined in sub-period 3.

expenditures to the hospital, but defaulters provide their assets instead of paying medical bills. If the asset level of the household is less than 0, the hospital receives no payment. As Chatterjee et al. (2007) assume, the mark-up is adjusted to have zero profits in the equilibrium.

3.2.11 Equilibrium

I define a measure space to describe equilibrium. The state of households in sub period 1 consists of assets, $a_1 \in \mathbf{A}_1$, health insurance status, $i \in \mathbf{I}$, health stock, $h \in \mathbf{H} = [0, 1]$, health shock $\epsilon_{h,j} \in \mathbf{HS}$, and idiosyncratic shock on earnings from the previous period, $\eta_{-1} \in \mathbf{ES}_{-1}$. Denote $\mathbf{S}_1 = \mathbf{A}_1 \times \mathbf{I} \times \mathbf{H} \times \mathbf{HS} \times \mathbf{ES}_{-1}$ as the state space of households in sub-period 1. In addition, let $\mathbb{B}(\mathbf{S}_1)$ denote the Borel σ -algebra on \mathbf{S}_1 . Then, for each age $j \in J$, a probability measure μ_j is defined on the Borel σ -algebra $\mathbb{B}(\mathbf{S})$ such that $\mu_j : \mathbb{B}(\mathbf{S}_1) \rightarrow [0, 1]$. $\mu_j(B)$ represents the measure of age j households whose state lies in $B \in \mathbb{B}(\mathbf{S})$ as a proportion of all age j agents in sub-period 1. At the beginning of sub-period2, idiosyncratic shocks on earnings, $\eta \in \mathbf{ES}$ are realized. Then, household in sub-period 2 is defined by $\mathbf{S}_2 = \mathbf{A}_2 \times \mathbf{I} \times \mathbf{H} \times \mathbf{H}' \times \mathbf{HS} \times \mathbf{ES}$. The distribution of households' evolves as follows:

$$\mu_{j+1}(B_1) = \int_{\{\mathbf{s}_1, \mathbf{s}_2 | (g_{a_1}(\mathbf{s}_2), g_i(\mathbf{s}_2), g_h(\mathbf{s}_1), \epsilon_{h,j+1}, \eta) \in B_1\}} Pr(\epsilon_{h,j+1}, \eta | \eta_{-1}) \mu_j(ds_1) \text{ for all } B_1 \in \mathbb{B}(\mathbf{S}_1) \quad (14)$$

where $\mathbf{s}_1 = (a_1, i, h, \epsilon_{h,j}, \eta_{-1}) \in \mathbf{S}_1$ is the household state in sub-period 1 and $\mathbf{s}_2 = (a_1, i, h, g_h(\mathbf{s}_1), \epsilon_{h,j}, \eta) \in \mathbf{S}_2$ is household state in sub-period2. $g_{a_1}(\cdot)$ is the policy function for assets in sub-period3, $g_i(\cdot)$ is the policy function for insurance decision in sub-period 3, and $g_h(\cdot)$ is the policy function for health investment in sub-period 1. In addition, $Pr(\epsilon_{h,j+1}, \eta | \eta_{-1})$ is the transition probability for $\epsilon_{h,j+1}$ and η conditional on η_{-1} .

Definition 1 Given a health insurance policy $(q, q_{med}, \Xi(\cdot))$, a tax policy $(\text{Tax}(\cdot, \cdot), \tau_{ss}, \tau_{med})$, and a social security policy Ψ , a recursive stationary competitive equilibrium is a set of prices $(w, q^{rf}, r, \{q(\cdot, \cdot, \cdot, j, \cdot)\}_{j=1}^{Jr}, \{p(\cdot, j)\}_{j=1}^{Jr}, p_{med})$, a mark-up of the hospital Θ , a set of decision rules $(\{g_{df,j}(\cdot), g_{a_1,j}(\cdot), g_{i,j}(\cdot), g_{h,j}(\cdot)\}_{j=1}^J)$, a default probability function $\{d(\cdot, \cdot, \cdot; j, \cdot)\}_{j=1}^{Jr}$, and values $(\{V_j^G(\cdot), V_j^B(\cdot), v_j^G(\cdot), v_j^B(\cdot), v_j^{G,R}(\cdot), v_j^{G,D}(\cdot)\}_{j=1}^{Jr}, \{V_j^{rt}(\cdot)\}_{j=Jr+1}^J)$, and distributions of household $\{\mu_j\}_{j=1}^J$ such that

- i. Households solve the values, and attain the decision rules.
- ii. Firm is competitive pricing.
- iii. Loan prices and default probabilities are consistent, whereby lenders earn zero expected profits on each loan of size a'_1 .

$$q(a'_1, i', h'; j, e) = \frac{1 - d(a'_1, i', h'; j + 1, e)}{1 + r_f}$$

$$d(a'_1, i', h'; j, e) = \sum_{e'} \pi_{e',e} \sum_{v'} \pi_{v'} g_{df,j+1}(s'_2)$$

where $s'_2 = (a'_1, i', h', g_{h,j+1}(s'_1), \epsilon_{h,j+1}, \eta') \in \mathbf{S}_2$

and $g_{df,j+1}(\cdot)$ is the policy function of default.

- iv. The hospital has zero profit.

$$\sum_{j=1}^J \left(\int \sum_{\eta} \left[(1 - g_{df}(s_2))m(s_1) + g_{df}(s_2) \max(a_2, 0) - \frac{m(s_1)}{\Theta} \right] \pi_{\eta, \eta_1} \mu_j(ds_1) \right) = 0$$

where $\mathbf{s}_1 = (a_1, i, h, \epsilon_{h,j}, \eta_{-1}) \in \mathbf{S}_1$, $\mathbf{s}_2 = (a_1, i, h, g_h(\mathbf{s}_1), \epsilon_{h,j}, \eta) \in \mathbf{S}_2$,
 $m(\mathbf{s}_1) = g_h(\mathbf{s}_1) - (1 - \epsilon_{h,j})h$, and $a_2 = a_1 - (1 - q\mathbb{1}_{i=1})m(\mathbf{s}_1)$.

v. Markets for bond, capital and labor clear.¹²

vi. The government budget constraint satisfies

$$\begin{aligned} \sum_{j=Jr+1}^J \int \Psi \mu_j(d\mathbf{s}_1) &= \sum_{j=1}^{Jr} \int \tau_{ss} \sum_{\eta} w e_j \pi_{\eta, \eta-1} \mu_j(d\mathbf{s}_1) \\ \sum_{j=Jr}^J \int (q_{med} m(\mathbf{s}_1) - p_{med}) \mu_j(d\mathbf{s}_1) &= \sum_{j=1}^{Jr} \int \tau_{med} \sum_{\eta} w e_j \pi_{\eta, \eta-1} \mu_j(d\mathbf{s}_1) \\ G + \sum_{j=1}^{Jr} \int \Xi(\mathbf{y}) \mu_j(d\mathbf{s}_1) &= \sum_{j=1}^J \int \sum_{\eta} T(\mathbf{y}) \pi_{\eta, \eta-1} \mu_j(d\mathbf{s}_1) \end{aligned}$$

vii. The insurance market for working households households clears. For each age group J_g ,

$$\sum_{j \in J_g} \int q \cdot \mathbb{1}_{i=1} m(\mathbf{s}_1) \mu_j(d\mathbf{s}_1) = \sum_{j \in J_g} \int p(g_i(\mathbf{s}_2), j) g_i(\mathbf{s}_2) \mu_{j-1}(d\mathbf{s}_1)$$

where $\mathbf{s}_1 = (a_1, i, h, \epsilon_{h,j}, \eta_{-1}) \in \mathbf{S}_1$ and $\mathbf{s}_2 = (a_1, i, h, g_h(\mathbf{s}_1), \epsilon_{h,j}, \eta) \in \mathbf{S}_2$

viii. Distributions are consistent with individual behavior. For all $j \leq J - 1$ and
for all $B_1 \in \mathbb{B}(\mathbf{S}_1)$

$$\mu_{j+1}(B_1) = \int_{\{\mathbf{s}_1, \mathbf{s}_2 | (g_{a1}(\mathbf{s}_2), g_i(\mathbf{s}_2), g_h(\mathbf{s}_1), \epsilon_{h,j+1}, \eta) \in B_1\}} Pr(\epsilon_{h,j+1}, \eta | \eta_{-1}) \mu_j(d\mathbf{s}_1) \quad (15)$$

where $\mathbf{s}_1 = (a_1, i, h, \epsilon_{h,j}, \eta_{-1}) \in \mathbf{S}_1$ and $\mathbf{s}_2 = (a_1, i, h, g_h(\mathbf{s}_1), \epsilon_{h,j}, \eta) \in \mathbf{S}_2$

¹²The market clearing conditions are the same as Chatterjee et al. (2007)

4 Calibration and Solution

4.1 Calibration

Table 1: Baseline Parameter Values

Parameter	Value	Description	Calibration Strategy (Model Result)
A. Parameters Determined Ex-Ante			
θ	0.36	Curvature of production function	Capital Share is 0.36.
δ	0.076	Depreciation rate	I/K=0.076
σ	2	Curvature of utility function	Coefficient of RRA is 2.
λ	0.1	Probability of recovering credit status	Average duration of exclusion is 10 years.
q	0.75	Insurance coverage rate	Average coverage rate in the ACA
τ_{ss}	0.106	Social security payroll tax rate	Social security tax rate is 10.6%
τ_{med}	0.029	Medicare payroll tax	Medicare tax rate is 2.9%
a_0	0.258	$a_0\{y - (y^{-a_1} + a_2)^{-1/a_1}\}$	Gouveia and Strauss (1994)
a_1	0.768	$a_0\{y - (y^{-a_1} + a_2)^{-1/a_1}\}$	Gouveia and Strauss (1994)
ρ_η	0.972	Persistence of productivity shock	PSID (1983-2011)
σ_η	0.385	Std of persistent shock	PSID (1983-2011)
σ_ϵ	0.273	Std of transitory shock	PSID (1983-2011)
$\{\epsilon_{j,h}\}_{j=1}^J$		Health shocks	MEPS(2000-2010)
B. Parameters that Require Solving the Model			
β	0.966	Discount factor	Risk free interest rate is 4% (4%)
χ	0.853	Cost of filing for bankruptcy	The ratio of debt to output is 0.67% (0.67%)
ψ	0.700	Efficiency of health technology for the old	$\frac{\text{Old medical expenditure}}{\text{Young medical expenditure}} = 2.12$ (1.93)
φ	0.870	Curvature of health technology	$Var\left(\frac{\text{Medical expenditure}}{\text{Output}}\right) = 0.054$. (0.049)
$1 - \mu$	0.36	Weight on health in utility	$\frac{\text{Total medical expenditure}}{\text{Output}} = 0.156$ (0.156)
a_2	3.323	$a_0\{y - (y^{-a_1} + a_2)^{-1/a_1}\}$	$\frac{\text{Revenue from progressive tax}}{\text{Total tax revenue}} = 0.65$ (0.65)
τ_y	0.083	Proportional tax	Government budget is Balanced.

My calibration strategy induces separating parameters into two groups. The first set of parameters is determined outside the model. The other set of parameters requires solving the model to match moments generated by model to their empirical counterparts.

The first set has 13 parameters. I choose the values of these parameters from standard macroeconomic studies and policies. The capital share θ is chosen to reproduce that the share of capital income is 0.36. The depreciate rate δ is targeted to match the ratio of investment to capital that is 0.076. The coefficient of relative risk aversion is fixed at 2. The probability of recovering credit status λ is chosen to match the average duration of exclusion that is 10 years on Chapter 7 bankruptcy filing. The insurance coverage rate q_{us}

is 75%, which is the average of insurance coverage rates in the Affordable Care Act and close to the average coverage rate of employer-sponsored insurance, 72%. The values of parameters related to tax are chosen from the corresponding policies. The rate of social security payroll tax τ_{ss} is 10.6%, and that of Medicare payroll tax τ_{med} is 2.9%. The values of two progressive parameter is taken from [Gouveia and Strauss \(1994\)](#). The upper bound parameter a_0 is 0.258, and the curvature parameter a_1 is 0.768.

The parameters of earnings components and its shocks are estimated from two sets of data. I use the Physical Component of Survey (PCS) in the Medical Expenditure Panel Survey (MEPS) as the measure of health status, but the MEPS is an improper data set to estimate the process of earnings because of the structure of data.¹³ I estimate the deterministic parts of earnings from the MEPS, while I obtain the residuals from the PSID to estimate the parameters for shocks on earnings. I control the health status through the variable of self-reported health status in the PSID, and estimate the parameters of shocks on earnings. I use the sample from 1983 to 2011, because the PSID provides the health information from 1983. The persistence of the shocks ρ_e is 0.972, a standard deviation of the persistent shocks is 0.385, and that of transitory shocks is 0.273.

I calculate health shocks based on the procedure of [Prados \(2012\)](#). She keeps track of individual disease records in the MEPS, and estimates health shocks by age with the disability weight form the World Health Organization (WHO).¹⁴ I follow the same procedure for the estimation of health shocks.

7 parameters require solving the model for the calibration. Since the model is based on the setting of incomplete markets, parameters are not independently determined. The 5 parameters—including the discount rate β , the cost of filing for bankruptcy χ , the efficiency of medical treatment for the old ψ , the curvature of medical expenditure function ψ , the weight of health in the utility function $1 - \mu$ —are determined jointly by minimizing the square of the log differences between moments from data and those generated by

¹³The MEPS does not keep track of an individual more than 3 years.

¹⁴The details are covered in the appendix.

the model. The targets I choose are the risk-free interest rate, debt to output ratio, the average ratio of medical expenditure, and the ratio of the average medical expenditures over working age households to that of medical expenditures over retired households. The other two parameters – the scale parameter of the progressive tax function a_2 and the parameter of proportional income tax τ_y – are targeted to match the portion of the revenue from progressive income tax in the total tax revenue and the balanced government budget constraint, respectively.

4.2 Numerical Solution

There are substantial computational burdens in solving the model. The model has a large number of household state variables. Moreover, loan prices depend on the state of households due to the endogenous default setting.¹⁵

To solve the model, I extend the endogenous grid method.¹⁶ The algorithm is as follows. Let's assume that a household repays its debt at age j . First, I apply the algorithm to the variable of assets, a'_1 , and take all of the endogenous state as given states. Second, I numerically set up bounds for the solution of asset holdings, a'_1 , which is necessary in this model with endogenous borrowing constraints. Third, using the endogenous grid method, I calculate the First Order Condition for asset holdings (Euler equation), which is a necessary condition for an optimal choice of assets, a'_1 . Fourth, I use the Fella (2014)'s algorithm to handle non-concavities, which are caused by multiple discrete choices, and find the global solution of asset holdings, a'_1 , among the candidates provided by the FOC in the previous step.

In the first step, it is worth noting that this endogenous grid method is for solving one dimensional problems. I apply my algorithm to a continuous endogenous state vari-

¹⁵Here, I demonstrate the solution for the value function if debt is repaid with existing good credit. The value function of defaulting is solved by optimizing a few discrete choices. The value function with bad credit is solved by Fella (2014)'s endogenous grid method.

¹⁶More details are in the appendix.

able, while discretizing the other endogenous state variables. The continuous variable I choose is assets, a' for the algorithm, and discretize the states of health capital, h' , and insurance choice, i' . It is because the variation in assets, a' , is the largest.

In the second step, I set up the lower bound of feasible sets for the solution of asset holdings, a'_1 . This process is unnecessary in other heterogeneous household models with an exogenous borrowing constraint or a borrowing constraint depending on collateral. It is because these types of borrowing constraints are not from these models' equilibrium conditions but from their technology. However, my model has borrowing constraints that endogenously arises from its equilibrium. Moreover, these borrowing constraints depend on households' state.

To set up the bounds for the solution of assets, a'_1 , I find a numerical method, motivated by the work of Clausen and Strub (2013) and Arellano (2008). They show that for every optimal debt contract, the size of the loan $q(a'_1)a'_1$ increases in a'_1 . In addition, they define the risky borrowing limit (credit limit) to be the lower bound of the set for these optimal debt contracts. I calculate the risky borrowing limits for each households state and fix them as the lower bound of the feasible set for the solution of asset holdings, a'_1 .¹⁷

Definition 2 For each $(a'_1, i', h'; j, \eta)$, $a'_{rbl}(a'_1, i', h'; j, \eta)$ is the risky borrowing limit if

$$\forall a'_1 \geq a'_{rbl}(a'_1, i', h'; j, \eta),$$

$$\frac{\partial q(a'_1, i', h'; j, \eta)a'_1}{\partial a'_1} = \frac{\partial q(a'_1, i', h'; j, \eta)}{\partial a'_1} a'_1 + q(a'_1, i', h'; j, \eta) > 0$$

The FOC is a necessary condition for an optimal choice of asset holdings, a'_1 .¹⁸ Let $V_{j+1}^G(a'_1, i', h', \eta)$ denote $\beta \sum_{\epsilon_{h,j+1}} \pi_{\epsilon_h} V_{j+1}^G(a'_1, i', h', \epsilon_{h,j+1}, \eta)$. Moreover, for each state (i', h', j, η) , let $a'_{rbl}(i', h', j, \eta)$ denote the risky borrowing limit. I formally describe the FOC.

¹⁷I describe how to numerically find them in the appendix.

¹⁸I show that the FOC for asset holdings exists and it is a necessary condition for an optimal choice for asset holding, a'_1 . Clausen and Strub (2013) proves that the FOC exists and is a necessary condition for an optimal choice for assets, a'_1 in the case with iid idiosyncratic shocks on earnings. I extend their proposition to default models with persistent shocks on earnings.

Proposition 1

For any (i', h', j, η) and for any $a'_1 \geq a'_{rbl}(i', h', j, \eta)$,

$$D_1 u(c, (1 - \epsilon_h)h) = \frac{1}{D_1 q(a'_1, i', h'; j, \eta) a' + q(a'_1, i', h'; j, \eta)} D_1 V_{j+1}(a'_1, i', h', \eta)$$

is a necessary condition for an optimal choice of asset holdings, a'_1 .

Proof. See the appendix. ■

In the third step, I apply the endogenous grid method to obtain the above FOC for asset holdings (Euler equation), and save this necessary condition for an optimal choices for each household state.

It is worth noting that the FOC is not sufficient but necessary. I find the global optimum for asset holdings, a'_1 , by using information provided by the FOC. To do this, I use the algorithm of [Fella \(2014\)](#). He suggests a numerical algorithm dividing the set of solutions into two regions: a concave region and a non-concave region. In the concave region, the FOC is both necessary and sufficient condition, so the FOC directly provides a solution of an optimal choice of assets, a'_1 . In the non-concave region, the FOC gives good candidates for a global solution of assets holdings, a'_1 . [Fella \(2014\)](#) checks whether a candidate from the FOC is the global solution or not in the non-concave region. Although this requires searching grid points for the solution, it is less computationally burdensome. One reason is that it does not search over the whole feasible space of solution sets. The range of grid search is restricted to the non-concave set. The other reason is that, even on the non-concave region, the processes of searching can stop once it faces a point generating a higher value than the value provided by the FOC.¹⁹ I follow his algorithm to

¹⁹[Fella \(2014\)](#) introduces a way to decrease grid searching times by using the monotonicity on value functions. He checks a few points closest to the candidate of the solutions. Despite efficiency gains from this step, I do not use it, because it can miss the solution if too few points are searched. Rather, I search all grid points on the non-concave region for the precision.

find a global solution for asset holdings, a'_1 . Using the implied decision rule for assets, a'_1 , from these step, I retrieve households' value functions given other discretized endogenous state variables: health capital, h' , and insurance, i' .

Note that the endogenous grid method is used only for the asset state variable, a'_1 . After the above step, I find the optimal choices for other discrete endogenous states of health capital, h' , and insurance, i' , by searching over their choice grid. This procedure is not computationally burdensome due to a small numbers of grid points on the discrete choice state variables: 2 grid points for health insurance and 20 grid points for health. By solving the value functions at age j with this method, I update the loan prices of household at age $j - 1$. These steps are repeated until the initial age.

5 Result

In this section, first I check whether the benchmark matches the features of US data. In particular, I test the validation of the model by examining the performance in fitting the untargeted moments. Then, I compare the result from the baseline model to other two economies differing in health insurance policies: health insurance with the ACA subsidies, universal health insurance. The former exercise is motivated by the current health reform, the Affordable Care Act, and the latter is for examining the necessity of national wide universal health insurance in the future.

5.1 Performance of the benchmark

The values of moments are from previous studies and the MEPS. The target for the fraction of debt holders is from [Nakajima and Ríos-Rull \(2014\)](#). I use the values for the default rate and the average borrowing interest rate from [Livshits et al. \(2007\)](#) and [Athreya et al. \(2009\)](#), and I calculate the employer-sponsored insurance take-up rate from the MEPS.

Table 2: Aggregate Result of the Benchmark

	Fraction of debt holders	Default rate	Average r^b	Insurance take-up ratio*
Data	8.40%	0.85%-1.2%	11.2-12.8%	52.90%
Benchmark	9.20%	0.94%	15.56%	49.05%

* The insurance only indicates the holding rate of employer-sponsored health insurance between ages 25 and 64.

Table 2 shows the moments that are untargeted in the calibration. At the aggregate level, the model closely matches key features of the data. Regarding the moments related to debt and default, it is worth noting that only debt to output ratio was targeted moment. The benchmark model the fraction of debt holders and default rate. The fraction of debt holders is 9.2% in the model, which is close to the value in data, 8.4%. The model has a default rate of 0.94%, which is in the range of default rate reported in previous studies, 0.85%-1.2%. However, the average borrowing interest rate in the benchmark is 15.56%, which is a little bit higher than the range in the data.²⁰ The model generates 92.7% of the employer-sponsored take-up rate observed in the data (49.05% compared to 52.9%).

The benchmark generates interesting patterns in life-cycle implications. Figure 1 shows that the pattern of default in the benchmark broadly matches with US data. The model illustrates qualitatively similar bankruptcy filing patterns over the life cycle: younger households suffer more bankruptcy, and the default rate decreases with age. However, the benchmark generates a default profile that is quantitatively different from that in US data. The model shows an overshooting of the default rate for young households compared to the data, while the rate of old households in the model is lower than that in the data.

Figure 2 illustrates medical expenditures over the life cycle. The model matches the life cycle profile of medical expenditures in data well. Note that this result is en-

²⁰This target is from Livshits et al. (2007) in which the interest target is the average interest rate on credit cards. If unsecured loans contain other type of financial services such as pay day loans, the target interest rate is higher than the target borrowing interest rate proposed by Livshits et al. (2007).

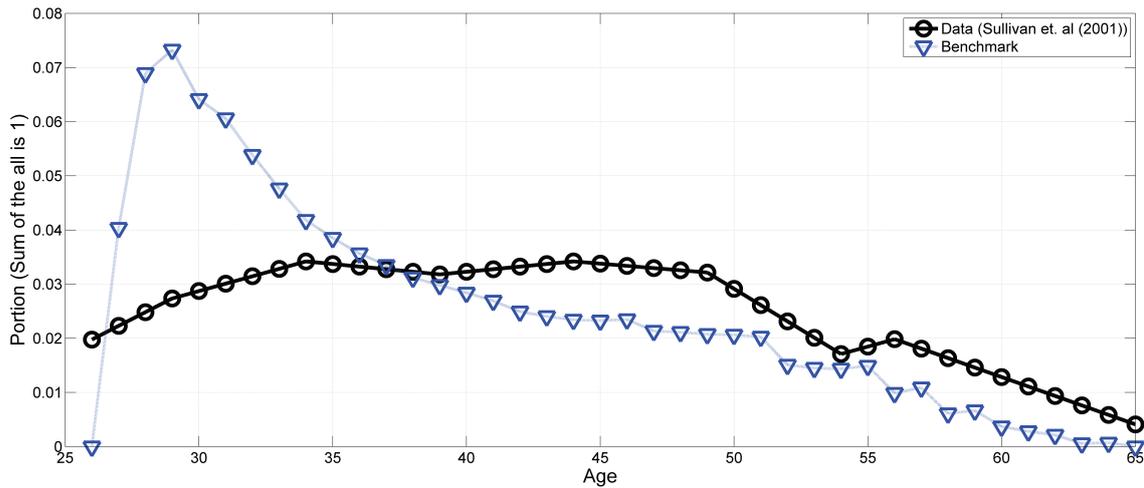


Figure 1: Default Model and Data

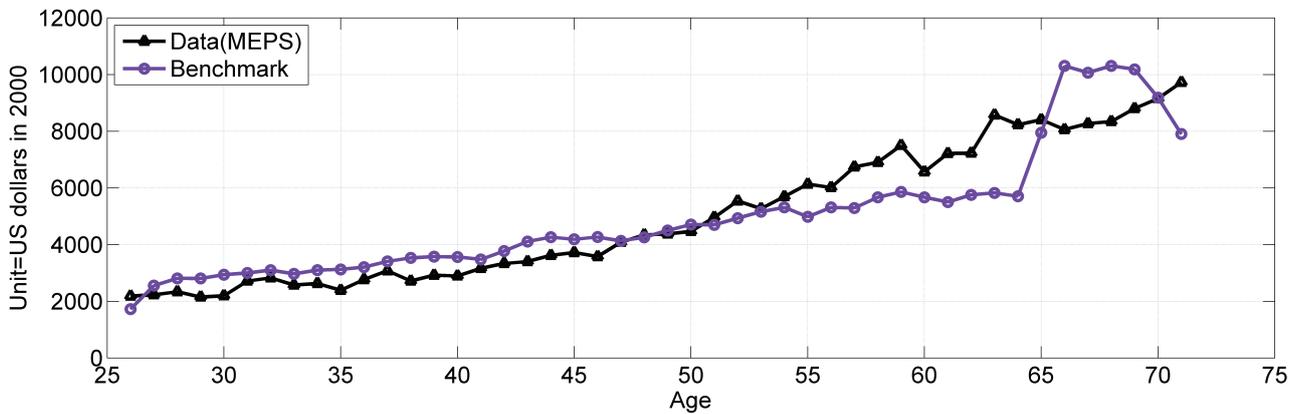


Figure 2: Medical Expenditure by Age

dogenously driven by the model mechanism, as the age profile is not targeted by the calibration. The model shows similar patterns in medical expenditures up to age 55 as in the data, while a gap appears from age 60 to age 64. A sudden jump at age 65 takes place, because of the assumption that the efficiency of medical treatment falls after retirement. This assumption helps to match the profile of medical expenditures for retired workers.

Figure 3 shows the take-up ratio of employer-sponsored health insurance. The model matches well the take-up ratio up to age 55, while the gap between the model

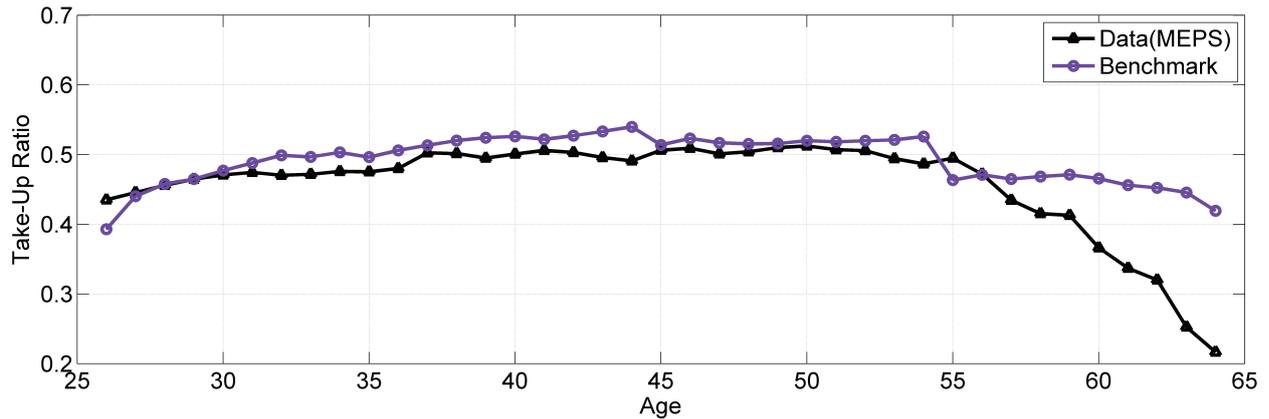


Figure 3: Insurance Take-Up Rate by Age

and data gets larger after age 55. This gap appears due to the assumption of exogenous retirement. In the data, those who are older than 55 start to retire and they give up the employer-sponsored health insurance. Since workers in the model exogenously retire at age 65, the model does not perform well in fitting the take-up ratio of households around retirement age.

5.2 Policy Analysis

The benchmark model well replicates macroeconomic moments targeted by the calibration. In addition, the model performs well in fitting the untargeted moments relevant to bankruptcy, medical expenditure, and health insurance. This suggests it is a useful framework for investigating the effects of expansions in health insurance on household bankruptcy, macroeconomy, and welfare.

I implement two types of expansion in health insurance: health insurance with the Affordable Care Act (ACA) subsidies and universal health insurance with the same coverage rate as the benchmark policy. I choose these health insurance policies, as the former is close to the current health care form in US, and the latter has been frequently mentioned in debates of health care reforms. For each policy, the proportional income tax rate, τ_y , is

employer-sponsored health insurances under the benchmark.

The second health insurance policy alternative is based on the universal health insurance. Under this policy, everyone has health insurance provided by the government. Thus, there is no choice problem in the health insurance²². This health insurance policy is more expansionary than the first type, in the sense that this extends coverage to every. For the whole cost of universal health insurance is covered by income tax. Since income tax is progressive, rich households cover more costs than poor households.

Table 4 shows the result of these policies analysis at the aggregate level. Firstly, ag-

Table 4: Aggregate Policy Analysis

	Benchmark	ACA	Universal HI
A. Output, Capital, and Proportional Income Tax			
Output	1*	0.983	0.986
Consumption	0.360	0.358	0.360
Risk-free interest rate	1.04	1.045	1.047
Capital to output ratio	3	2.87	2.836
Proportional income tax rate	8.3%	8.9%	11.98%
B. Default Rate and Debt			
Default rate	0.95%	0.84%	0.89%
Debt to output ratio	0.67%	0.29%	0.17%
Average borrowing interest rate	1.153	1.183	1.186
C. Health Related Variables			
Insurance take-up ratio	49.05%	80.8%	100%
Medical expenditure to output ratio	15.6%	16.6%	16.9%
Average health**	0.594	0.636	0.642

* I normalize the output value in the benchmark model as 1.

** Health is measured by the normalized PCS in the section 4. I divide the PCS of all the individuals by the PCS of the healthiest. Thus, the normalized PCS is between 0 and 1.

gregate variables such as consumption, capital, and the income tax rate react to changes in health insurance policies. The level of output decreases as health insurance expands. The fall in output is driven by a decrease in the aggregate capital level. There are two reasons why expanding health insurance causes a decrease in capital. The first reason is that greater insurance coverage reduces the precautionary saving motive of poor households over different ages. Health shocks are one of the important reasons of accumulating assets

²²Although countries that have universal health insurance also have private market of health insurance, the portion of the private market in these countries is relatively small. For example, the WHO shows only 7.6% of the total medical expenditures in the UK is covered by private health insurance.

over age in the absence of health insurance. For this reason, poor households have lower consumption and higher savings. However, an expansion in health insurance that extends coverage to such households curtails this precautionary saving motive. As a result, poor households increase consumption and decrease their savings. The second reason for a reduction in capital is the distortion from increase in income tax rate that decrease the return on savings. To provide more health insurance, income taxes need to rise. An increase in income tax incurs the lower return on saving for rich households due to the progressive tax. This brings about a reduction of saving. Since the aggregate level of capital is largely determined by wealthy households, the distortion of tax caused by higher taxes is more important in reducing the aggregate capital stock. As we move from the benchmark model, which represents existing health insurance policies to universal health insurance, only 15% of reduction in capital is explained by a fall in precautionary savings motive, while the remaining 85% is from the reduction in the net return to saving as a result of higher taxes.

Regarding the average default rate, the effects of expansion in health insurance are not sizable. In addition, the direction of change in the default rate is somewhat ambiguous between the ACA and universal health insurance. This is related to the reduction in capital. In the model, the default probability is determined by the risk-free interest rate as well as an individual state. Since expanding health insurance decreases the aggregate capital level, it increases the risk-free interest rate, and this raises the overall cost of borrowing driven by equilibrium loan rate schedules consistent with default risk. On the other hands, greater health insurance coverage reduces the probability of default for sick and old people overall. Since these two effects offset each other, the effect of health insurance on the average bankruptcy rate is small across economies and the direct of changes is ambiguous between the economy with the ACA and that with universal health insurance. However, the quantity and price of debt shows clear changes. As health insurance expands, the debt to output ratio falls and the average borrowing interest rate rise. These

changes are the result of the rising cost of borrowing driven by equilibrium loan rate schedules consistent with default risk.

The take-up rate for health insurance increases as health insurance is expanded. Since it is difficult for poor households to buy health insurance, progressive subsidies are helpful in increasing the take-up rate. In addition, medical expenditures and average health rise. In particular, the effect of expanding health insurance policies on health is sizable. From the benchmark to the economy with universal health insurance, health improves by 8.1% with the normalized PCS unit.

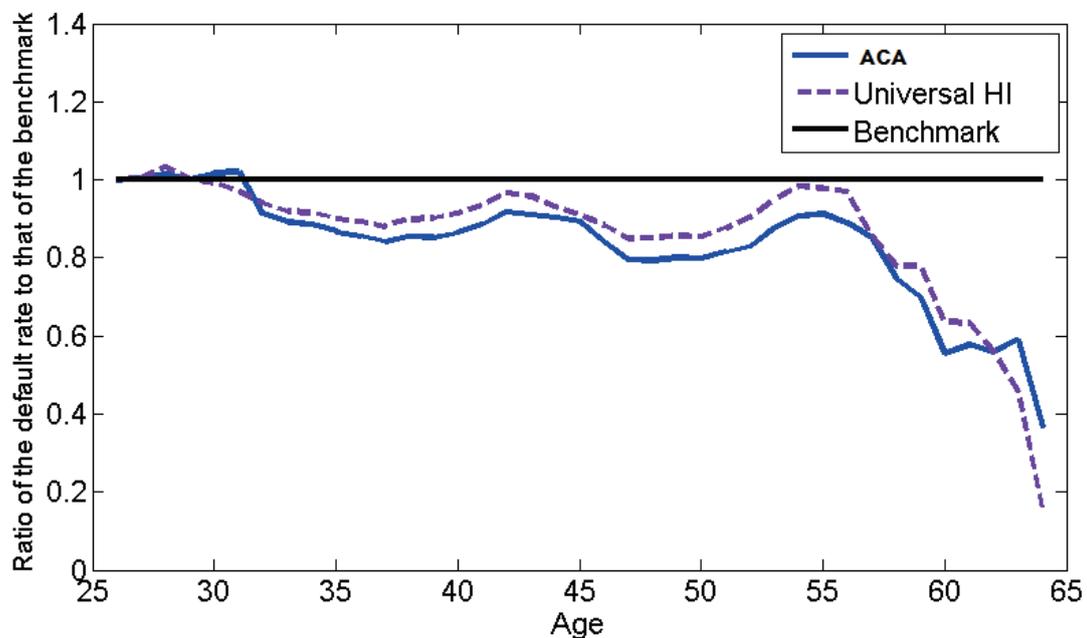


Figure 4: Ratio of the Default Rate to that in the Benchmark

Figure 4 illustrates the ratio of the default rate in an economy to that in the benchmark over age. It shows that the effect of broader health insurance differs over generations. The default rate rises for younger generations, while that falls for older generations. The default rate in economies with the ACA subsidies or universal health insurance is higher than that in the benchmark up to age 31, while older households suffer from less default rate. This change takes place for the following reason. Expanding health insurance

tends to decrease the aggregate capital stock, because the required increase in income tax reduces the return on savings, and low income households reduce the precautionary savings motive of. The reduced aggregate capital stock implies the rising cost of borrowing driven by equilibrium loan rate schedules consistent with default risk, and this has a larger impact on younger generations, while older households suffer less from these. As a result, default rate rises for young, who pay higher interest rates and suffer earnings shocks. It falls for the old who are more sensitive to health shocks.

To further explore the reasons for changes in default rate over age, note that expanding health insurance mitigates the burden of medical spending for older households. There are two sources of idiosyncratic shocks: earnings and health. For young generations, on average, idiosyncratic shocks on earnings are more important than health. To smooth their consumption, some young households need access to financial markets. However, with the rising cost of borrowing driven by equilibrium loan rate schedules consistent with default risk, under a more expansion in health insurance, the young needs to pay higher interest rates, and it is harder for them to repay their debts. This change increases the default rate of young households. In contrast, idiosyncratic shocks on health are more important for old households. When health insurance expands, health shocks have a less impact on medical spending. This decreases the default rate of older generations. As a result, default rate rises for young who pay higher interest rates and suffer earnings shocks, while it falls for the old who are more sensitive to health shocks

Figure 5 illustrates the evolution of health over the life cycle across different health insurance policies. Expanding health insurance policies leads to substantially better health over the life cycle. In particular, more generous health insurance amplifies the improvement of health over age. For example, at age 30, when the economy changes its health insurance from that in benchmark model to universal health insurance, health improves around by 8.3% overall. However, health capital in households rises around to 18.5% by

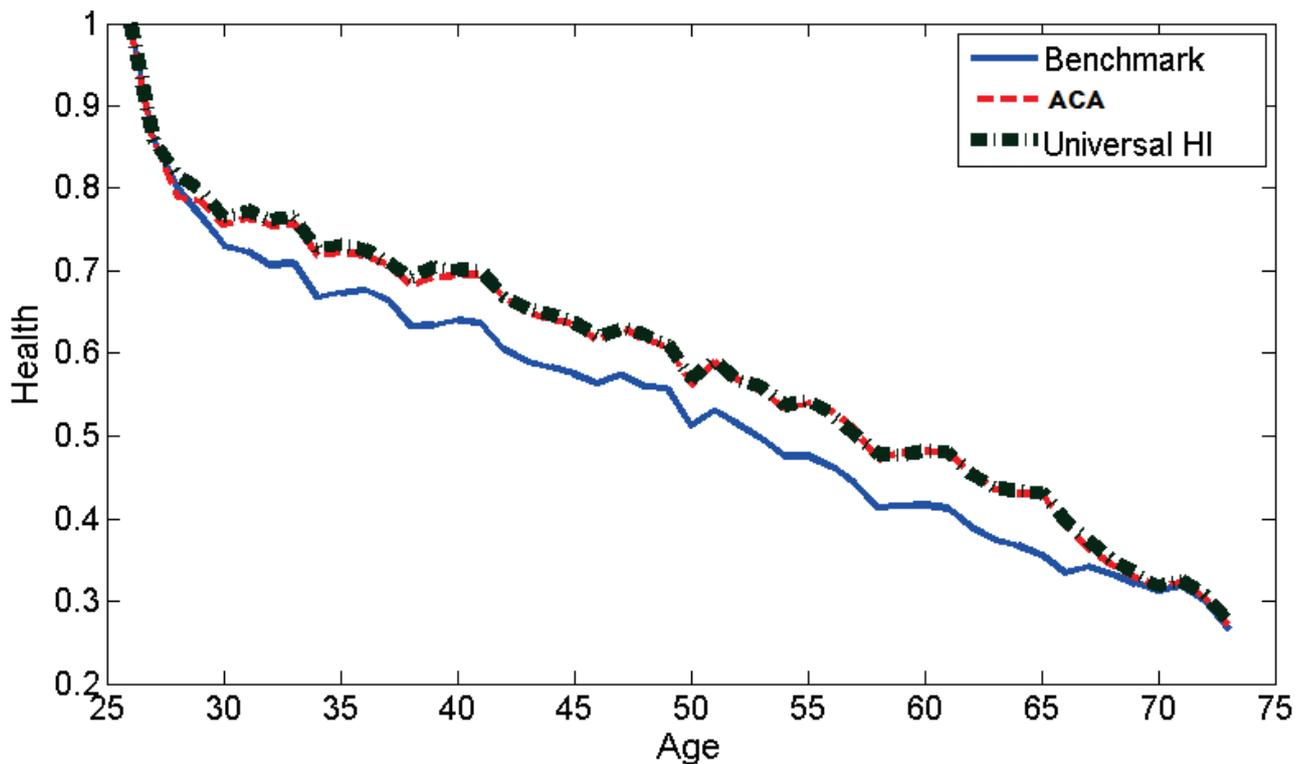


Figure 5: Health over the Lifecycle

age 65. The gap between the economy with the ACA subsidies and the universal health insurance economy is very small. This implies that low income households face borrowing that affects their cost of borrowing for their medical expenditures. Low income households do not pay insurance premiums, when health insurance is characterized by the ACA subsidies, as the government provides Medicaid. In terms of health, providing health insurance for low income households is not too different from supplying it to everyone.

Table 5 shows working households' distribution of health by income. Expanding health insurance improves health overall, and the improvements are largely for the bottom and next low income households. For example, from the baseline economy to the economy with universal health insurance, the bottom 20 percentile income households

Table 5: Health for Working Households by Income Percentile

Income	Income Percentile				
	0-20pct	20-40pct	40-60pct	60-80pct	80-100pct
Baseline (Health)	0.292	0.446	0.645	0.732	0.780
ACA (Health)	0.357	0.572	0.705	0.737	0.775
Universal HI (Health)	0.350	0.566	0.716	0.759	0.782
Changes from baseline to Universal HI	19.9%	26.91%	11%	3.7%	-0.2%

* Health is measured by the normalized PCS in the section 4. I divide the PCS of all the individuals by the PCS of the healthiest. Thus, the normalized PCS is between 0 and 1.

experience health improvements of 19.9%. For those whose incomes are between 20 and 40 percentile, health capital rises by 26.9%. However, the improvement in health for high income households is not as big as that of health for low income households. Households with income between 60 and 80 percentile have only 3.7% of the improvement in their health stock. Households in the top 20 percentile of their income have no improvement at all.

Table 6: Consumption for Working Households by Income Percentile

Income	Income Percentile				
	0-20pct	20-40pct	40-60pct	60-80pct	80-100pct
Baseline (Cons)	0.018	0.050	0.124	0.319	1.06
ACA (Cons)	0.017	0.053	0.138	0.323	1.04
Universal HI (Cons)	0.017	0.050	0.136	0.337	1.04
Changes from baseline to Universal HI	-0.5%	0%	9.6%	5.6%	-1.9%

* The unit measure is the value of output in the benchmark model.

Table 6 illustrate working households' distribution of consumption by income level. A changes in consumption differs from those in health. First, the magnitude of changes in consumption is not as large as that health. The largest change is less than 10%. The other difference from health is that improvements in consumption are concentrated among middle income groups. Low income households have almost no changes in income, and the top 20 percentile of income households see a small loss in their consumption.

Table 5 and 6 show that expanding health insurance has a different impact over dif-

ferent income groups. Young households are likely to be low income households because of a lack of savings and the presences of age-dependent hump-shape earnings profiles. When young workers face health shocks and they have health insurance, they spend more on medical treatment, and this improves their labor productivity over time through increases in their health. In middle age, their income levels rises, and the improvements in health cause an increase both income and consumption. However, high income households pay higher tax without improvement in health. The progressive tax system implies that they pay more tax than before to finance these expansions in health insurance. Moreover, their health is not improved by these, as they were previously able to cover medical expenditures. Therefore, expanding health insurance decreases disposables income for high income households.

Table 7: Welfare % Changes from the Benchmark

ACA	Universal HI
6.53%	6.62%

Table 7 shows the welfare changes from the benchmark. The welfare is measured by the percentage increase in consumption in all dates and states that leaves a newborn household indifferent between the benchmark economy and another economy. It should be very cautious in interpreting the above result. First, it is important to note that the benchmark model omits many institutional features involving social insurance for low income households. Before the ACA, Medicaid was not available to all poor working people. Its provision was based not only on income but also family structure, which makes it difficult to model. This absence of social insurance may be the reason for the large welfare improvement, as the welfare is sensitive to low income households. Table 5 and 6 indicates the source of welfare improvement is an enhancement in health for low income households and consumption for middle income.

Recall that the bankruptcy rate in the economy with universal health insurance is

higher than that in the economy with the ACA subsidy. Table 7 shows that welfare may improve with an increase in the average bankruptcy rate. Therefore, my results suggest that just focusing on the bankruptcy rate alone is not a good measure with which to evaluate health insurance policies. When a health insurance reform is implemented, the bankruptcy rate does not fall substantially. However, this does not mean expanding health insurance does not contribute to an improvement in welfare. Welfare can improve through the distributive changes of consumption and health.

6 Conclusion

I have studied how health insurance policies affect household bankruptcy. I have found that the economy's average default rate does not exhibit the large changes produced by the previous empirical work, and the direction of change is ambiguous, following an expansion in health insurance coverage. Moreover, I find a compositional change in default rate over age. Expanding health insurance increases the default rate of younger generations, while that of older generations falls. I show that these are driven by an interaction between the distortion from taxes and general equilibrium effects of a falling stock.

Another issue this paper addresses is the macroeconomic and welfare consequences of health insurance policies. I find that increases in health insurance coverage improve health for low and middle income households at the cost of decreases in aggregate output and capital. The improvements in health is the key channel for improvements in welfare.

In terms of future research, endogenizing labor supply decisions seems important. Reforms in health insurance are closely related to the demand or supply behavior of labor. While I abstracted from labor supply decisions, endogenizing labor supply decisions will generate other interesting predictions from health insurance reform.

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Appendices

A Data Details

A.1 MEPS

I choose the MEPS waves from 2000 to 2011. Data series before 2000 are not used, because they do not include the identifier of the Health Insurance Eligibility Units (HIEU), which is the definition of a household in this paper. A HIEU includes spouses, unmarried natural or adoptive children of age 18 or under and children under 24 who are full-time students. I use the MEPS longitudinal weight for each individual when computing empirical targets.

Since each survey of the MEPS covers 2 consequent years, I stack individuals in the 10 different panels into one data set. I convert all nominal values into the value of dollars in a base year 2000 with the CPI . To handle the weight for this data set, I use the way in [Jeske and Kitao \(2009\)](#). Since the number of sample is different in each year, I rescale the longitudinal weight in each survey in the way that the sum of the weight is equal to the number of HIEUs. In this way, I address the issues of different size of samples across surveys in the MEPS. The number of observations in each panel is as follows.

Year	2000-2001	2001-2002	2002-2003	2003-2004	2004-2005	2005-2006
Numbers	5218	10187	7484	7577	7548	7294
Year	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	
Numbers	7721	5835	8611	7988	7020	

Table 8: MEPS Panel Sample Size

In the MEPS, the variable of total medical expenditure includes medical expenditures paid for by Veteran's Affairs, Workman's Compensation and other sources. Since my model does not cover these variables, I discharge three variables for the value of total

medical expenditures.

B Measuring Health Shocks

I follow the work of [Prados \(2012\)](#) to compute health shocks from the data. I use the medical condition files in the MEPS that provide individuals' medical conditions. I quantify each of these medical conditions, using the disability weight computed by the World Health Organization (WHO). Let's assume that a household has D medical conditions. Denote d_i as the WHO index for medical condition i , where $i = 1, \dots, D$. For this household, its health shock ϵ_h is represented by

$$(1 - \epsilon_h) = \prod_{i=1}^D (1 - d_i) \quad (16)$$

Next, I rearrange households by age and obtain age-specific empirical distributions on health shocks. I use the values of disability weight for medical conditions from [Prados \(2012\)](#).²³

²³[Prados \(2012\)](#) addresses more details of this measure.

C Applying Clausen and Strub (2013)'s Envelope Theorem to the model

Clausen and Strub (2013) introduce an envelope theorem to prove that First Order Conditions are necessary conditions for the global solution. They show that the envelope theorem is applicable to default models with iid idiosyncratic shocks. I extend this to solve my model with persistent idiosyncratic shock. To use their envelope theorem, it is necessary to introduce the following definition.

Definition 3 We say that $F : C \rightarrow \mathbb{R}$ is *differentiably sandwiched* between the lower and upper support functions $L, U : C \rightarrow \mathbb{R}$ at $\bar{c} \in C$ if

1. L is a differentiable lower support function of F at \bar{c} , i.e. L is differentiable, $L(c) \leq F(c)$ for all $c \in C$, and $L(\bar{c}) = F(\bar{c})$.
2. U is a differentiable upper support function of F at \bar{c} , i.e. U is differentiable, $U(c) \geq F(c)$ for all $c \in C$, and $U(\bar{c}) = F(\bar{c})$.

Let's begin with the First Order Condition. Given a state $(i', h'; j, \eta)$, let $a'_{rbl}(i', h'; j, \eta)$ denote the risky borrowing constraints (credit limits). Then, the First Order Condition (FOC) is :

$$\text{For each state } (i', h', j, \eta) \text{ and } \forall a' \geq a'_{rbl}(i', h'; j, \eta),$$

$$D_1 u(c, (1 - \epsilon_h)h) = \frac{1}{D_1 q(a'_1, i', h'; j, \eta) a'_1 + q(a'_1, i', h'; j, \eta)} D_1 V_{j+1}(a'_1, i', h', \eta)$$

where $D_1 V_{j+1}(a'_1, i', h', \eta) = \beta D_1 \sum_{\epsilon_{h,j+1}} \pi_{\epsilon'_h} V_{j+1}^G(a'_1, i', h', \epsilon_{h,j+1}, \eta)$. Clausen and Strub (2013) prove that if each constituent function of the FOC has a differentiable lower support function at a point, the constituent function are differentiable and the First Order

Condition is a necessary condition for the global solution. For example, given a state $(\bar{i}', \bar{h}', j, \eta)$ and $(1 - \epsilon_h)h$, if there are differentiable lower support function of the utility function $u(\cdot, (1 - \epsilon_h)h)$, price function $q(\cdot, \bar{i}', \bar{h}', \bar{j}, \bar{\eta})$, and value function $V_{j+1}(\cdot, \bar{i}', \bar{h}', \bar{\eta})$ with respect to a'_1 , the FOC holds and is a necessary condition for the global solution. Formally,

Corollary 0.1

For each state (i', h', j, η) and $\forall a' \geq a'_{rbl}(i', h'; j, \eta)$,

$$D_1 u(c, (1 - \epsilon_h)h) = \frac{1}{D_1 q(a'_1, i', h'; j, \eta) a'_1 + q(a'_1, i', h'; j, \eta)} D_1 V_{j+1}(a'_1, i', h', \eta)$$

holds and this is a necessary condition for the global solution.

Proof.

Claim: $u(\cdot, (1 - \epsilon_h)h)$, price function $q(\cdot, \bar{i}', \bar{h}', \bar{j}, \bar{\eta})$, and value function $V_{j+1}(\cdot, \bar{i}', \bar{h}', \bar{\eta})$ have differentiable lower support functions.

Since the utility function is differentiable, it the utility function itself is the differentiable lower support function.

By Lemma 0.1 and Lemma 0.2, price function $q(\cdot, \bar{i}', \bar{h}', \bar{j}, \bar{\eta})$, and value function $V_{j+1}(\cdot, \bar{i}', \bar{h}', \bar{\eta})$ have differentiable lower support functions.

By theorem 1 (Envelope theorem) in Clausen and Strub (2013), the FOC holds and it is a necessary condition for the global solution. ■

Lemma 0.1 Let a state in sub-period 2 $(\bar{a}_2, \bar{i}, \bar{h}', \bar{h}, \epsilon_{\bar{h}, j}, \bar{\eta}; \bar{j})$ and insurance status for the next period \bar{i}' be given. Let a^* be the risky borrowing limit (risk-free credit limit) of $q(\cdot, \bar{i}', \bar{h}', ; j, \bar{\eta})$. For all a'_1 with $a'_1 > a_{rbl}$, the bond price function, $q(a'_1, \bar{i}', \bar{h}', \bar{j}, \bar{\eta})$ has a differentiable lower support function.

Proof. For simplicity, denote $\bar{y}_1 = (\bar{i}', \bar{h}'; \bar{j}, \bar{\eta})$ and $\bar{y}_2 = (\bar{a}_2, \bar{i}, \bar{h}', \bar{h}, \epsilon_{\bar{h}, j}, \bar{\eta}; \bar{j})$.

For any $a'_1 \geq 0$, $q(a'_1, \bar{y}_1) = \frac{1}{1+r_f}$ and $D_1 q(a'_1, \bar{y}_1) = 0$. Thus, $q(a'_1, \bar{y}_1)$ itself is a differential lower support if $a'_1 > 0$.

Take any $a'_1 \in (a_{rbl}, 0)$. Note that $q(a'_1, \bar{y}_1) = \frac{1-d(a'_1, \bar{y}_1)}{1+r_f}$. It implies that finding a lower differential support function of $q(a'_1, \bar{y}_1)$ is equivalent to doing an upper differential support function of $d(a'_1, \bar{y}_1)$.

■

Lemma 0.2 Let a state in sub-period 2 $(\bar{a}_2, \bar{i}, \bar{h}', \bar{h}, \epsilon_{h,j}^-, \bar{\eta}; \bar{j})$ and insurance status for next period \bar{i}' be given. Let a_{rbl} be a risk-free credit limit of the price function. For any a'_1 with $a'_1 \geq a_{rbl}$, there is a differentiable lower support function of $D_1 V_{j+1}(a'_1, i', h', \eta) = \beta D_1 \sum_{\epsilon_{h,j+1}} \pi_{\epsilon_{h,j+1}} V_{j+1}^G(a'_1, i', h', \epsilon_{h,j+1}, \eta)$.

D Computation Method

It is not a trivial task to solve the model, because not only the dimension of individual state is large, but also the value functions of the model are involved with many non-concave and non-smooth factors: the choice of default, health insurance, medical cost, and progressive subsidy or tax policies.

To solve the model with these complexities, I develop an endogenous grid method for default models with discrete choices. This method is an extended version of Fella (2014)'s endogenous grid method. Fella provides an algorithm to handle non-concavities on the value functions with an exogenous borrowing constraint. I generalize the method for default problems in which borrowing constraints differ among individuals.

Whereas there are a few types of value functions in the model, the computational issues are mainly related to two types of value functions: the value function of repaying debt $v_j^{G,R}(a_2, i, h', h, \epsilon_{h,j}, \eta)$, and the value function with bad credit status $v_j^B(a_2, i, h', h, \epsilon_{h,j}, \eta)$.

²⁴ The value function with bad credit status is solved with the algorithm of Fella (2014), because in this case the economy has an exogenous borrowing constraint with discrete choice, consistent with the setting of Fella (2014). My computation method is for solving the value function of repaying debts $v_j^{G,R}(a_2, i, h', h, \epsilon_{h,j}, \eta)$ in which borrowing constraints differ across individuals states.

Before introducing details of the solution method, I briefly repeat the value function of repaying debts in sub period 2 and describe the whole picture of the solution method.

²⁴The value function of filing for default is not involved with any continuous choice variable. Also, the other value functions are restricted to a few discrete choices.

Those who decide to repay their debts with good credit solve

$$v_j^{G,R}(a_2, i, h', h, \epsilon_{h,j}, \eta) = \max_{\{c, a_1', i' \in \{0,1\}\}} \frac{(c^\alpha ((1 - \epsilon_{h,j})h)^{1-\alpha})^{1-\sigma}}{1 - \sigma} + \beta \sum_{\epsilon_{h,j+1}} \pi_{\epsilon_h'} V_{j+1}^G(a_1', i', h', \epsilon_{h,j+1}, \eta) \quad (17)$$

$$c + q(a_1', i', h'; j, \eta)a_1' + p(i', j) = we(j, h, \epsilon_{h,j}, \eta) + a_2 - Tax(we, y)$$

$$a_2 = a_1 - (1 - q\mathbb{1}_{i=1})m(h', h, \epsilon_{h,j})$$

The whole picture of computation is the following.

1. Except for the asset variable for the next period, take all the other states as given states, which means the discretization of the other states.
2. For each individual state, calculate the risky borrowing limit by using the price function.
3. Obtain the FOC, retrieve the value function, and store the cash on hands.
4. With [Fella \(2014\)](#)os algorithm, divide the domain of the expected future value function into concave area and non-concave area.
5. Find the global solution and obtain the value function on the exogenous grid.
6. Optimize the discrete choices that are given from the first step.
7. Update the price.

In the following sub sections, the details of each step in the above are covered.

D.1 Discretization of states

In the model, households need to make choices on three individual state variables: asset, health insurance, and health status (health expenditure). I discretize the state of

health insurance and health status as well as other exogenous state variables. It is to use the endogenous grid method with respect to the state of asset. This way is efficient because the range of asset in the equilibrium is wide and the heterogeneity of asset is the largest comparing to the state of health insurance and health. By doing this, the problem becomes a one dimensional optimization problem given the other states. In the equation 17, the endogenous individual state variable is only a'_1 in this state.

D.2 Calculating risky borrowing limits (credit limits)

In order to solve default models with the endogenous grid method, it is necessary to set up feasible sets for the solution. Finding the feasible set is not a trivial work because of two reasons. The first reason is that, in default models, borrowing limits are not given, as they are endogenously determined in the equilibrium. Therefore, this step is not required for other types of models in which the borrowing constraint are given before solving the models. The second reason is that the range of the feasible sets depends on the state of individuals, because the borrowing limits are affected by individual default risks that are the functions of individual state.

I set up the feasible sets of the solution based on the work in [Arellano \(2008\)](#); [Clausen and Strub \(2013\)](#). They investigate the property of the risky borrowing limits (credit limits) in their work. [Arellano \(2008\)](#); [Clausen and Strub \(2013\)](#) show that the size of loan $q(a^n)a^n$ is increases in a' under every optimal debt contract. If the size of loan $q(a^n)a^n$ decreases in a^n , households can increase their consumption by increasing debts, which is unstable debt contracts. [Arellano \(2008\)](#) ([Clausen and Strub \(2013\)](#)) defines the risky borrowing limit (credit limit) to be the lower bound of the set for optimal contract. Figure D.2 illustrate the risky borrowing limit.

For each state $(\bar{i}', \bar{h}', \bar{j}, \bar{\eta})$, I calculate the risky borrowing limit $a'_{rbl}(\bar{i}', \bar{h}', \bar{j}, \bar{\eta})$ such that

$$\forall a'_1 \geq a'_{rbl}(\bar{i}', \bar{h}', \bar{j}, \bar{\eta}), \frac{\partial q(a'_1, \bar{i}', \bar{h}'; \bar{j}, \bar{\eta})a'_1}{\partial a'_1} = \frac{\partial q(a'_1, \bar{i}', \bar{h}'; \bar{j}, \bar{\eta})}{\partial a'_1} a'_1 + q(a'_1, \bar{i}', \bar{h}'; \bar{j}, \bar{\eta}) > 0 \quad (18)$$

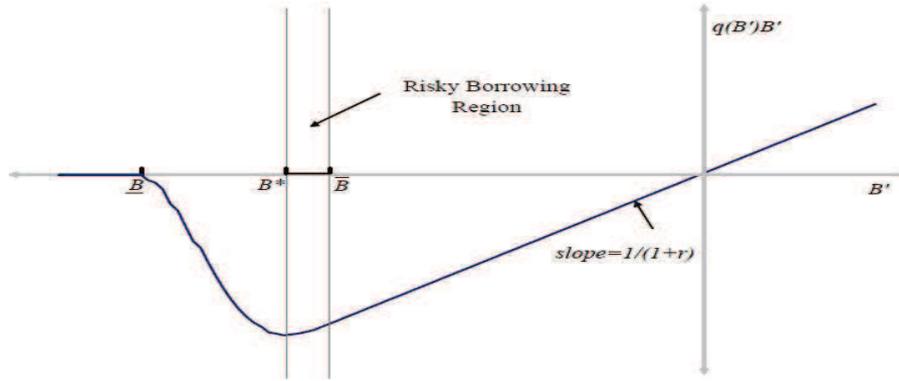


Figure 6: Risky borrowing limit, Arellano(2007)

and save them. I use the risky borrowing limits as the lower bound of feasible sets. These sets include every global solution and differ across the state of individuals. To obtain the value of derivative, $\frac{\partial q(a'_1, \bar{i}', \bar{h}', \bar{j}, \bar{\eta})}{\partial a'_1}$, I use the slope of the price function $q(a'_1, \bar{i}', \bar{h}', \bar{j}, \bar{\eta})$ between a'_1 and its nearest right point on the grid.

D.3 Obtaining the FOC as the candidate of the global solution

By corollary 0.1, the FOC is a necessary condition for the global solution and exists. With the risky borrowing limits in the previous step, the FOC is defined as follows.

$$\text{For each state } (\bar{i}', \bar{h}', \bar{j}, \bar{\eta}) \text{ and } \forall a'_1 \geq a'_{rbl}(\bar{i}', \bar{h}', \bar{j}, \bar{\eta}), \quad (19)$$

$$D_1 u(c, (1 - \bar{\epsilon}_h) \bar{h}) = \frac{1}{D_1 q(a'_1, \bar{i}', \bar{h}', \bar{j}, \bar{\eta}) a'_1 + q(a'_1, \bar{i}', \bar{h}', \bar{j}, \bar{\eta})} D_1 V_{j+1}(a'_1, \bar{i}', \bar{h}', \bar{\eta})$$

On the grid $G_{a'_1}$ of a'_1 , calculate the FOC (19). For the derivative, I use the slope of the functions between a'_1 and its nearest right point on the grid.

Using the FOC, compute the endogenously-determined level of consumption $c(a'_1, \bar{i}', \bar{h}', \bar{\epsilon}_{h,j}; \bar{j}, \bar{\eta})$. Next, substitute the consumption $c(a'_1, \bar{i}', \bar{h}', \bar{\epsilon}_{h,j}; \bar{j}, \bar{\eta})$ into the utility function, and retrieve the value function of repaying debts:

$$\tilde{\vartheta}_j^{G,R}(a'_1, \vec{i}, \vec{i}, \vec{h}', \vec{h}, \epsilon_{\vec{h},j}, \vec{\eta}) = \frac{\left(c(a'_1, \vec{i}, \vec{h}', \epsilon_{\vec{h},j}, \vec{\eta})^\alpha ((1 - \epsilon_{\vec{h},j})\vec{h})^{1-\alpha}\right)^{1-\sigma}}{1 - \sigma} + V_{j+1}(a'_1, \vec{i}, \vec{h}', \vec{\eta}) \quad (20)$$

In addition, for each state $(\vec{i}, \vec{h}', \epsilon_{\vec{h},j}, \vec{\eta})$ and a'_1 on the grid $G_{a'_1}$, store cash on hands, $coh_j(a'_1, \vec{i}, \vec{h}', \epsilon_{\vec{h},j}, \vec{\eta})$:

$$coh_j(a'_1, \vec{i}, \vec{h}', \epsilon_{\vec{h},j}, \vec{\eta}) = c(a'_1, \vec{i}, \vec{h}', \epsilon_{\vec{h},j}, \vec{\eta}) - q(a'_1, \vec{i}, \vec{h}'; \vec{j}, \vec{\eta})a'_1 \quad (21)$$

D.4 Identifying the concave and non-concave regions

It is worth noting that the FOC is a necessary condition because of the non-concavities on the expected value function $V_{j+1}(a'_1, \vec{i}, \vec{h}', \vec{\eta})$. If the concave regions can be identified, the FOC is a sufficient and necessary condition on the concave region, which decrease the burden of computations. I use Fella (2014)'s algorithm to divide the domain of the expected value functions $V_{j+1}(a'_1, \vec{i}, \vec{h}', \vec{\eta})$ into the concave and non-concave regions.

With the cash on hands, the FOC (19) can be represented in the following way.

$$\text{For each state } (\vec{i}, \vec{h}', \vec{j}, \vec{\eta}; \vec{h}, \epsilon_{\vec{h},j}) \text{ and } \forall a'_1 \geq a'_{rbl}(\vec{i}, \vec{h}'; \vec{j}, \vec{\eta}), \quad (22)$$

$$D_1 u(coh_j(\vec{a}'_1, \vec{i}, \vec{h}', \epsilon_{\vec{h},j}, \vec{\eta}) + q(a'_1, \vec{i}, \vec{h}'; \vec{j}, \vec{\eta})a'_1, (1 - \epsilon_{\vec{h}})\vec{h}) = \frac{D_1 V_{j+1}(a'_1, \vec{i}, \vec{h}', \vec{\eta})}{D_1 q(a'_1, \vec{i}, \vec{h}'; \vec{j}, \vec{\eta})a'_1 + q(a'_1, \vec{i}, \vec{h}'; \vec{j}, \vec{\eta})}$$

where \vec{a}'_1 in $coh_j(\vec{a}'_1, \vec{i}, \vec{h}', \epsilon_{\vec{h},j}, \vec{\eta}) + q(a'_1, \vec{i}, \vec{h}'; \vec{j}, \vec{\eta})a'_1$ is constant. The LHS of (22) is monotone increasing in a'_1 , because the value of a'_1 is larger than the risky borrowing limit, $a'_{rbl}(\vec{i}, \vec{h}'; \vec{j}, \vec{\eta})$ ²⁵ and the utility function is strictly concave.

The issues of non-concavities are involved with the RHS. If the expected value function is concave, the RHS is monotonic decreasing. Therefore, an algorithm of identifying

²⁵Note that under every optimal contract, $q(a'_1, \vec{i}, \vec{h}'; \vec{j}, \vec{\eta})a'_1$ is weakly increasing in a'_1 .

the non-concave regions needs to be used on the RHS. I take Fella (2014) algorithm that is as follows.

1. For each state $(\bar{i}', \bar{h}', \bar{j}, \bar{\eta}; \bar{h}, \bar{\epsilon}_{h,j})$, calculate the value of derivative on the RHS of (22) at each grid point on G_{a_1}' .
2. Find grid points in which the value of derivative jumps in a_1' , and save the jumped points with logical clause. (ex. jumping points=1, the others=0)
 - To find the jumped points, check the derivative values at two adjunct grid points, $a_{1,i}' < a_{1,i+1}'$ on the grid G_{a_1}' . If the value of the derivative at the greater point, $a_{1,i+1}'$ is greater than the value of the derivative at the lower point $a_{1,i}'$, $a_{1,i}'$ is one of the jumped points.
3. For each state $(\bar{i}', \bar{h}', \bar{j}, \bar{\eta}; \bar{h}, \bar{\epsilon}_{h,j})$, find the maximum value of the derivative $v_{max}(\bar{i}', \bar{h}', \bar{j}, \bar{\eta}; \bar{h}, \bar{\epsilon}_{h,j})$, and minimum value of the derivative $v_{min}(\bar{i}', \bar{h}', \bar{j}, \bar{\eta}; \bar{h}, \bar{\epsilon}_{h,j})$ among the jumped grid points on the grid G_{a_1}' .
4. For each state $(\bar{i}', \bar{h}', \bar{j}, \bar{\eta}; \bar{h}, \bar{\epsilon}_{h,j})$, search for $a_{\bar{i}}'(a_{\bar{i}}')$ such that for all $a' < a_{\bar{i}}'(a' > a_{\bar{i}}')$, the value of derivative on the RHS of (22) is greater (smaller) than $v_{max}(v_{min})$.²⁶
5. For each state $(\bar{i}', \bar{h}', \bar{j}, \bar{\eta}; \bar{h}, \bar{\epsilon}_{h,j})$, the non-concave region is $G_{a_1}'^{nc} = \{a_{\bar{i}}' = a_{min}, \dots, a_{\bar{i}}' = a_{max}\}$, while the rest, $G_{a_1}'^{cc} = G_{a_1}' \setminus G_{a_1}'^{nc}$ constitutes the concave region.

D.5 Collecting the global solutions and updating the value function

Since the concave regions are identified by the previous procedures, the FOCs are sufficient and necessary conditions for the global solutions on the concave regions $G_{a_1}'^{cc}$. For each state $(\bar{i}', \bar{h}', \bar{j}, \bar{\eta}; \bar{h}, \bar{\epsilon}_{h,j})$, save the pair of cash on hands and asset grids, $\{coh_j(a_1', \bar{i}', \bar{h}', \bar{\epsilon}_{h,j}, \bar{\eta}), a_1'\}$ if $a_1' \in G_{a_1}'^{cc}$.

²⁶For example, in FORTRAN, to find $a_{\bar{i}}'$, we can use **minloc** function for the derivative values, conditional on these values are greater than v_{max} .

If $a'_1 \in G_{a'_1}^{nc}$, the FOCs are not sufficient but necessary. However, the FOCs provide good candidates of the global solutions. For each state $(\bar{i}', \bar{h}', \bar{j}, \bar{\eta}; \bar{h}, \bar{\epsilon}_{h,j})$, it is required to check whether $a'_1 \in G_{a'_1}^{nc}$ is a global solution by searching grids on the non-concave region, $G_{a'_1}^{nc}$. If the grid $a'_1 \in G_{a'_1}^{nc}$ is consistent with the solution from the grid search, save the pair $\{coh_j(a'_1, \bar{i}', \bar{h}', \bar{\epsilon}_{h,j}, \bar{\eta}), a'_1\}$.

With the saved pairs, retrieve the value functions, $v_j^{G,R}(a'_1, \bar{i}', \bar{i}, \bar{h}', \bar{h}, \bar{\epsilon}_{h,j}, \bar{\eta})$. It is important to note that these value function is on endogenously determined grids in a_1 . Since the value functions preserve the monotonicity, it is possible to retrieve the value functions on the exogenous grid I set up by using linear interpolations.

The above steps are summarized in the following.

1. If $a'_1 \in G_{a'_1}^{cc}$, save $\{coh_j(a'_1, \bar{i}', \bar{h}', \bar{\epsilon}_{h,j}, \bar{\eta}), a'_1\}$
2. If $a'_1 \in G_{a'_1}^{nc}$, solve the following problem

$$a'_{sol} = \underset{a'' \in G_{a''}^{nc}(\bar{i}', \bar{h}', \bar{j}, \bar{\eta}; \bar{h}, \bar{\epsilon}_{h,j})}{\operatorname{argmax}} u(coh_j(\bar{a}', \bar{i}', \bar{h}', \bar{\epsilon}_{h,j}, \bar{\eta})) + q(\mathbf{a}'', \bar{i}', \bar{h}', \bar{j}, \bar{\eta}) \mathbf{a}'', (1 - \bar{\epsilon}_{h,j})\bar{h}) + V_{j+1}(\mathbf{a}'', \bar{i}', \bar{h}', \bar{\eta})$$

- If $a'_{sol} = \bar{a}'$, save the pair $\{coh_j(a'_1, \bar{i}', \bar{h}', \bar{\epsilon}_{h,j}, \bar{\eta}), a'_1\}$.

If $a'_{sol} \neq \bar{a}'$, discard the pair $\{coh_j(a'_1, \bar{i}', \bar{h}', \bar{\epsilon}_{h,j}, \bar{\eta}), a'_1\}$.

3. With the saved pairs $\{coh_j(a'_1, \bar{i}', \bar{h}', \bar{\epsilon}_{h,j}, \bar{\eta}), a'_1\}$, obtain the value functions $v_j^{G,R}(a'_1, \bar{i}', \bar{i}, \bar{h}', \bar{h}, \bar{\epsilon}_{h,j}, \bar{\eta})$ that are on the endogenously determined grids of a_1 .
4. Use linear interpolations to $v_j^{G,R}(a'_1, \bar{i}', \bar{i}, \bar{h}', \bar{h}, \bar{\epsilon}_{h,j}, \bar{\eta})$ and calculate the values on the exogenous grid of a_1 .

D.6 Optimize the discrete choices

Until this step, the choice of health insurance i' and the status of invested health (medical expenditure) $h'(m)$ are given statuses. Optimize these two choices by searching the grid for each variable. The number of grid points for these variables is relatively smaller than that of grid points on asset a_1 . Therefore, the computation is not so costly in this procedure. Formally, solve the following problems:

$$v_j^{G,R}(a'_1, \bar{i}, \bar{h}', \bar{h}, \epsilon_{\bar{h},j}, \bar{\eta}) = \max_{\{i' \in \{0,1\}\}} \tilde{v}_j^{G,R}(a'_1, i', \bar{i}, \bar{h}', \bar{h}, \epsilon_{\bar{h},j}, \bar{\eta}) \quad (23)$$

$$V_j^G(a_1, i, h, \epsilon_{h,j}, \eta_{-1}) = \max_m \sum_{\eta} \pi_{\eta_{-1}, \eta} v_j^G(a_2, i, h', h, \epsilon_{h,j}, \eta),$$

where $h' = (1 - \epsilon_j)h + Am^\psi$

D.7 Updating the prices

I have explained how to obtain $v_j^{G,R}(a'_1, \bar{i}, \bar{h}', \bar{h}, \epsilon_{\bar{h},j}, \bar{\eta})$. The other value functions, $v_j^{G,D}$ and v_j^B , are solved by simple grid search for a few discrete choices and using the algorithm of Fella (2014).²⁷ Then, the default probability is

²⁷The value function of defaulter is solved by optimize the discrete choice on health insurance. In addition, the value function with bad credit is solved by the algorithm of Fella (2014), because it has the discrete choices with an exogenous borrowing constraint, consistent with the circumstance of his algorithm.

$$dp(a'_1, i', h'; j-1, \eta_{-1}) = \sum_{\epsilon_{h,j}} \sum_{\eta} \pi_{\eta, \eta_{-1}} g_{df,j}(s'_2) \quad (24)$$

$$g_{df,j}(s'_2) = \begin{cases} 0 & \text{if } v_j^{G,R}(a'_2, i', g_h(s'_1), h', \epsilon_{h,j}, \eta) > v_j^{G,D}(i', g_h(s'_1), h', \epsilon_{h,j}, \eta) \\ 1 & \text{if } v_j^{G,R}(a'_2, i', g_h(s'_1), h', \epsilon_{h,j}, \eta) \leq v_j^{G,D}(i', g_h(s'_1), h', \epsilon_{h,j}, \eta) \end{cases}$$

$$s'_2 = (a'_2, i', g_h(s'_1), h', \epsilon_{h,j}, \eta)$$

$$s'_1 = (a'_1, i', h', \epsilon_{h,j}, \eta_{-1})$$

$$a'_2 = a'_1 - (1 - q\mathbb{1}_{i'=1})m(g_h(s'_1), h', \epsilon_{h,j})$$

With the default probability $dp(a'_1, i', h'; j-1, \eta_{-1})$, the loan price at age $j-1$ is where r_f is the equilibrium risk-free interest rate. Repeat the procedures until the beginning age.