A WAVELET TRANSFORMATION APPROACH TO CRUDE OIL PRICE AND CZK/USD EXCHANGE RATE DEPENDENCE

Abstract:
In this paper, we contribute to the literature on the dependency between oil price returns and CZK/USD exchange rate returns. Oil price is one of the most important determinants for an explanation of the long-term behaviour of exchange rates, especially for USD dollar. The oil shock transmission mechanism is through the exchange rate channel and so the deeply understanding of this process is essential not only for investors but also for monetary authority. We utilised wavelet transform analysis so that we can analyse the dependency in the time-frequency domain. Our analysis finds that the connection between returns time series changes in time and scales. The major implications of our findings are important for effective monetary policies aimed at controlling inflationary pressures.

Keywords:
wavelet, wavelet coherence, oil price, dependence

JEL Classification: C22, F40, C58
Introduction

The analysis of dependence between oil price and exchange rates is very important not only for investors and risk management but also for monetary policy. The negative oil shocks have important impact on inflationary pressures and macroeconomic conditions. On the other hand, there is a question about impact of oil price on the exchange rates. Because the settlement currency in oil markets is US dollar, the main channel of oil shock transmission is through the exchange rates. Following Reboredo and Rivera-Castro (2013) the oil price in foreign currency is given by:

\[ p_f = e_{USD} + p_{USD} \]  \hspace{1cm} (1)

where \( p_f \) is logarithmic oil price in foreign currency, \( e_{USD} \) is logarithmic foreign currency per unit of USD and \( p_{USD} \) is logarithmic price of oil in USD. The relationship between oil price and exchange rate was described by Krugman in 1983. Krugman assumed that oil is essential commodity for oil importing countries, but the effect of a risen oil price on exchange rate is not clear. The effect depends through the balance of payment on the economics structure. Amano (1995) worked with Krugman’s idea and found that the changes in oil price influence terms of trade.

Following Reboredo and Rivera-Castro (2013) the home consumer price index \( p \) and foreign country consumer price index \( p^* \) is given as:

\[ p = (1 - \alpha)p^T + \alpha p^N \] \hspace{1cm} (2)

\[ p^* = (1 - \alpha^*)p^{T*} + \alpha^* p^{N*} \] \hspace{1cm} (3)

Where \( p^T(p^{T*}) \) is logarithmic price of traded goods in the home (foreign) country and \( p^N(p^{N*}) \) is logarithmic price of nontraded goods in the home (foreign) country. \( \alpha(\alpha^*) \) is weight for home (foreign) country of expenditure on nontraded goods. From the law of one price we can express from the equation 1, 2, 3 the nominal exchange rate \( e \) as:

\[ e = (p^N - p^{N*}) + (1 - \alpha)(p^{T*} - p^{N*}) - (1 - \alpha)(p^T - p^N) \] \hspace{1cm} (4)

For simplicity assuming that \( \alpha \equiv \alpha^* \). Then for oil-importing countries the effect of negative oil shock on the nominal exchange rate \( e \) depends on the relative price of traded goods in the home country with respect to the relative price of traded goods in the foreign country. So, if the inequality \( \frac{p^T}{p^{T*}} > 1 \) is hold then after negative oil shock we could expect the depreciation in the home currency and Vice versa. The traditional approach to analyze the dependency between these two time series comes from econometric approach. This approach ignores the heterogeneity of markets participants and distinguishes only short term and long term. But on the markets operate different agents with different “behavior” horizon. In the short run the connection is from contagion reason, on the other hand the long term is associated with the co-movement or independency (Gallegati, 2012). It depends on whether the country is oil-importer or exporter and also whether the country’s industry is oil intensive or not. The solution is in using wavelet analysis of the dependency. Wavelet
transformation is very popular in finance and macroeconomics. For example Hanus and Vacha (2016) describe the business cycle synchronization of the Visegrad Four and the European Union via wavelet coherence, or Vacha and Barunik (2012) analysed the co-movement of energy commodities. The dependency among oil prices and economic activity analysed Naccache (2011).

In this article, we apply wavelet coherence on the WTI oil price data and CZK/USD exchange rate. The wavelet coherence method gives us detailed insight into the behavior of dependency between both time series. Thanks to this we will understand better to the oil shocks transition.

The paper is organized as follows. We begin with a short literature review and methodology description presenting wavelet analysis. A data description and an empirical analysis of oil return and CZK/USD dependency follow. The last part is devoted to the discussion and conclusion.

**Literature review**

In 1995 Zhou analyzed the main sources of real shocks on the real exchange rate movements. He found that the oil price fluctuation is one of the most important factors. Camarero and Tamarit (2002) analyzed the relationship between real oil price and Spanish peseta real exchange rate. They found that the real oil price is one of the most significant factors for long-term real exchange rate. Lizardo and Mollick (2010) demonstrated the similar conclusion. They found that between oil price and USD rate exists negative relationship and that oil price is one of the most important factor in long-term USD rate determination. The relatively small impact of changes in real oil price on the exchange rate demonstrate Huang and Guo (2007). They applied VAR methodology to the China’s real exchange rate. Cifarelli and Paladino (2010) utilized multivariate generalized autoregressive conditional heteroskedasticity model on the oil and exchange rates return and found that for the most of the time the signs of correlation were negative.

With similar conclusion came Yousefi and Wirjanto (2004). They reported the negative correlation between USD and OPEC oil return and Reboredo (2012) measured the dependency between oil return and range of currencies with copula function. He found that the dependence is relatively weak with some substantially rises. One of the most significant increases was after the global financial crisis. The detailed insight to the problem of dependency bring Reboredo and Rivera-Castro (2013). They analyzed the dependency behavior before and after global financial crisis 2008. The analysis was built on the wavelet transformation when they estimated the wavelet correlation between oil return and seven currency returns for different time scale. They found that for pre-crisis period doesn’t exist significant correlation on any scales. However, they found it after global crisis period for all scales. The most of currencies had negative correlation value.

**Methodology**

Following Percival (2000) the wavelet function satisfied these two conditions:

\[
\int_{-\infty}^{\infty} \psi(t) dt = 0
\]  

(5)
Then the wavelet is

$$\int_{-\infty}^{\infty} \psi^2(t) dt = 1$$

Then the wavelet is

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t - u}{s}\right)$$ \hspace{1cm} (6)

Where $s$ is the scale (or dilatation) and $u$ the translation (or shift) and $\frac{1}{\sqrt{s}}$ is a normalization factor. The coefficients from continuous wavelet transformation is obtained by projecting the wavelet $\psi(.)$ onto time series $x(t)$. The continuous wavelet transform is defined as:

$$W_x(u, s) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{s}} \psi\left(\frac{t - u}{s}\right) dt$$ \hspace{1cm} (7)

where $W_x(u, s)$ are wavelet coefficients for different time position $u$ and different time scale $s$. Then $|W_x(u, s)|^2$ is called as wavelet spectra. The spectra measures the local variation of time series $x(t)$ on the scale $s$. Because we want to study the behavior of two time series, we have to use cross-wavelet spectra. Let's have the equation:

$$W_{xy}(u, s) = W_x(u, s)W_y^*(u, s)$$ \hspace{1cm} (8)

where $W_x(u, s)$ and $W_y(u, s)$ are wavelet coefficients from the continuous wavelet transformation of time series $x(t)$, $y(t)$ and * denotes a complex conjugate. Then the absolute value of $|W_{xy}(u, s)|$ is cross-wavelet spectra. This spectra is estimation of local covariation of time series $x(t)$ and $y(t)$ at different time scales. Because we want to compare different scales, we have to use the wavelet coherence $R^2(u, s)$ from of Torrence and Webster (1999), Which is defined as:

$$R^2(u, s) = \frac{|S\left(s^{-1}W_{xy}(u, s)\right)|^2}{S(s^{-1}|W_x(u, s)|^2)S(s^{-1}|W_y(u, s)|^2)}$$ \hspace{1cm} (9)

Where $S$ is smooth operator Percival (2000) and the value of $R^2(u, s)$ is bounded on $0 \leq R^2(u, s) \leq 1$. The wavelet coherence measures the local quadratic correlation between $x(t)$ a $y(t)$. The main disadvantage is in the positive number of $R^2(u, s)$. We
can not measure positive/negative correlation between time series. From this purpose, we use Morlet’s wavelet which has real and imaginary part. Then we can estimate the phase difference $\phi_{xy}(u, s)$, which is defined as:

$$\phi_{xy}(u, s) = \tan^{-1} \left[ \frac{\Im \left\{ S \left( s^{-1}W_{xy}(u, s) \right) \right\}}{\Re \left\{ S \left( s^{-1}W_{xy}(u, s) \right) \right\}} \right]$$

(10)

where $\Im \left\{ S \left( s^{-1}W_{xy}(u, s) \right) \right\}$ is imaginary part and $\Re \left\{ S \left( s^{-1}W_{xy}(u, s) \right) \right\}$ is real part of wavelet coherence. We can use this information to measure the sigh of correlation. The phase difference will be depicted as arrow in wavelet coherence scaleogram. The zero phase difference means that the examined time series move together at a particular scale $s$. The arrows pointing to the right (left) means that the time series are in-phase (anti-phase), i.e. positively (negatively) correlated Vacha and Barunik (2012).

Another look at the structure of the dependency is from the wavelet correlation. Wavelet correlation is the estimator for correlation at different scale $s$.

$$\rho_{xy}(u, s) = \frac{\text{cov} \left( W_x(u, s), W_y(u, s) \right)}{\left[ \text{var}(W_x(u, s)) \text{var}(W_y(u, s)) \right]^{1/2}}$$

(11)

Where $W_x(u, s)$ and $W_y(u, s)$ are vectors of wavelet coefficients for time series $x(t)$ and $y(t)$ at scale $s$ from continuous wavelet transformation and cov (var) is estimated covariance (variance) at scale $s$.

**Data**

For our analysis, we use WTI daily price and CZK/USD daily price. The sample period spans from 07/01/1992 until 30/12/2016. From the reason of data nonstationary we computed crude oil price and exchange rate returns on a continuous compounding basis as the difference between the log of the current price and that of the one-period lagged price. The return series were stationary processes. The summary statistics of the returns are in the table 1. We can see that the unconditional mean value is zero for both processes. Volatility of WTI is higher than CZK/USD rate. The more interesting are higher moments. The estimated skewness suggests negatively skew returns and kurtosis suggests fat tails when the CZK/USD rate has fatter tails than WTI. The last column in the table 1 with name JB is P-value of Jarque-Bera test. The value suggest that we can reject the null hypothesis about normal distribution of returns.

**Table 1: Descriptive statistics for daily oil price and exchange rate returns for the period from 07/01/1992 until 30/12/2016.**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std.dev</th>
<th>Skew</th>
<th>Kurt.</th>
<th>JB</th>
</tr>
</thead>
<tbody>
<tr>
<td>WTI</td>
<td>0.0000</td>
<td>0.0102</td>
<td>-0.1802</td>
<td>4.6760</td>
<td>0.0000</td>
</tr>
<tr>
<td>CZK/USD</td>
<td>0.0000</td>
<td>0.0021</td>
<td>-0.0693</td>
<td>7.0877</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
Empirical part
At the figure 1 we can see the scaleogram, where on the x axis is time and on the y axis are scales. The color spectra represent the value of wavelet coherence from equation. The co-movement is stressed by yellowish color. The stronger co-movement would be the more yellowish color is. The yellow islands with black contour are the statistically significant areas of co-movement and the arrow measure the correlation sign. We can see that on the highest frequencies does not exist significant co-movement of time series. The first statistically positive correlation is viewed during the period of 32 to the 64-th days. But the most interesting periods are from 128-th days. Especially after 2008 (value 3000 on x axis) we can see negative correlation between oil return and CZK return. The different pattern we can see on the scale of 4 years (1024). There exists positive correlation between time series and CZK returns before global financial crisis.

Figure 1: Scaleogram for wavelet coherence between oil return and CZK/USD return series

Another look on the structure of correlation is viewed on the figure 2. On the x axis are time scales and on the y axis is correlation coefficient. The estimation from the equation 11 depicted by black line. The lines indicated U and L denote the upper and lower limits for the confidence interval of 95%. We can see the similar pattern as in the figure 1. On the low scale, we cannot speak about correlation. But from the figure 2 the correlation is the function of scales. During the period from the first half year (128) to one year (256) the correlation is negative, but for higher scales up to 4 years (1024) we can see positive correlation.
Figure 2: Wavelet correlation estimates between oil return and CZK/USD return series for different scale.

Discussion
The empirical results from oil price and CZK/USD exchange rate interdependence through wavelet transformation has three major conclusions. On the highest frequency we can agree with works from Huang and Guo (2007) and Reboredo and Rivera-Castro (2013). We didn’t find statistically significant correlation between time series on this scale. The statistically significant local correlation was found for scale from 128 days. This result is in connection with Lizardo and Mollick (2010) and Reboredo and Rivera-Castro (2013) too. We can see different behavior before and after global financial crisis. Before financial crisis we couldn’t see statistical significant correlation around scale between half year and year. After financial crisis, we were able to see the statistically significant correlation. The correlation had the negative sign which correspond with works from the literature review. The new findings in our analysis comes from scale represents 4 years period. On this scale, we can see statistically significant correlation. This pattern was observed before global financial crisis but not after. In contrast with lower scales the correlation has the positive sign with value greater than 0.5.

Conclusion
In this paper, we contribute to the literature on the dependency between oil price returns and exchange rate returns. We applied the wavelet coherence method and wavelet correlation estimation on the WTI oil and CZK/USD return series which is a novelty for this currency. Our analysis finds that the interconnection between returns time series changes significantly in time and scales. We found that on the low scales there does not exist statistically significant correlation. The first statistically significant correlation was found for scales between 128 and 512 days but only for post-global
crisis period. The sign of correlation was negative. These conclusions are in connection with previously cited journals. The difference we found on scales up 1024. The wavelet coherence reveals positive and statistically significant correlation before the global financial crisis.

Acknowledgements

The work on this paper was supported by the grant IGS F4/73/2016 of the University of Economics, Prague.

References


