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INTERNATIONAL REAL BUSINESS CYCLE MODELS WITH INCOMPLETE INFORMATION

Abstract:

Standard international real business cycle (IRBC) models formulated by Backus, Kehoe, and Kydland (BKK, 1992) have been considered a natural starting point to assess the quantitative implications of dynamic stochastic general equilibrium (DSGE) models in an open economy environment. Since the standard IRBC model under assumptions of flexible prices and perfect competition cannot replicate all the observed characteristics of international business cycles, a number of extended models with more realistic features have been developed in the past two decades. We introduce a noisy information structure into an otherwise standard international real business cycle model with two countries. When domestic firms observe current foreign technology with some noise, predictions of the model on international correlation can be very different from those of a standard perfect information model. We show that the model can explain: (i) positive output correlation both in complete and incomplete market models; (ii) consumption correlation smaller than output correlation with an introduction of information-constrained consumers; and (iii) observation of both positive and negative productivity-hours correlation in two countries.

Keywords:

Cross-country correlations; Imperfect information; Incomplete markets

JEL Classification: E32, F41, G15

Introduction

Standard international real business cycle (IRBC) models formulated by Backus, Kehoe, and Kydland (BKK, 1992, 1995) have been considered a natural starting point to assess the quantitative implications of dynamic stochastic general equilibrium (DSGE) models in an open economy environment. Since the standard IRBC model under assumptions of flexible prices and perfect competition cannot replicate all the observed characteristics of international business cycles, a number of extended models with more realistic features have been developed in the past two decades. Most importantly, incorporating monopolistic competition and sticky prices, along with the monetary sector in open economy DSGE models has been proven to be very successful in matching the data. In contrast to a large interest in the role of nominal rigidities, however, few studies have attempted to formally assess the quantitative implications of introducing informational frictions in the model. In this paper, we introduce a noisy information structure in an otherwise standard IRBC model and show that an extension in this direction is also useful in understanding some key features of international comovements of output, consumption, and labor.

We consider an imperfect information variant of a standard two-country bond-economy IRBC model similar to the one used in Baxter and Crucini (1995) and Heathcote and Perri (2002), except that we exclude capital accumulation from the model. While we believe that an open economy DSGE model with nominal rigidities is more realistic, we maintain the assumptions of perfect competition and flexible prices in this paper simply because they provide a reasonable benchmark in evaluating the pure effect of imperfect information on the international business cycle properties. In terms of explaining the international comovement, the original BKK model predicts negative (or near-zero) output correlation, near-perfect consumption correlation, and negative correlation of factors of production, all of which contradict the data. To improve the performance of the model, Baxter and Crucini (1995) and Kollman (1996) replaced the complete market assumption of the BKK model with the incomplete market assumption, so that consumers only have access to a real bond market. A convenient approach to ensure a unique stationary solution to an open economy model of incomplete market is to impose a (small) real cost of bond holding (see Heathcote and Perri, 2002, and Schmitt-Grohe and Uribe, 2003). According to the simulation results reported by Boileau and Normandin (2008, Table 1), under the stationary technology process with positive international spillovers, an incomplete market model with a tiny bond holding cost can yield positive international output correlation, but its magnitude is still less than the data[1]. As in the original BKK model, we focus on stationary technology shocks with international spillovers as a source

[1] Baxter and Crucini (1995) emphasized the better performance of the bond economy model when the technology is highly persistent and there is no international spillover.

of aggregate fluctuations. However, domestic firms are assumed to observe the current foreign technology with noise. We first show that when the information noise is sufficiently large, the model can match the positive output comovement in the data not only for the case of incomplete market but also for the case of perfect international risk sharing.

Even in the case of incomplete market where international consumption risk sharing is restricted, the standard IRBC models with stationary technology shocks are known to predict international consumption correlation higher than the international output correlation (see Boileau and Normandin, 2008, Table 1). The data, however, typically suggest that the former is lower than the latter (see Ambler, Cardia, and Zimmermann, 2004). To narrow the gap between output correlation and consumption correlation predicted by the model, several different channels have been emphasized in the literature. For example, the proposed channels include non-traded goods (Stockman and Tesar, 1995), endogenous incomplete market with limited enforcement (Kehoe and Perri, 2002), sticky prices (Chari, Kehoe, and McGrattan, 2002) and variable capital utilization (Baxter and Farr, 2005). In this paper, we highlight the information channel and show that the presence of a noisy information structure in the household sector helps to fill the gap between the cross country output correlation and consumption correlation.

In the recent global financial crisis of 2007-2009, employment and hours worked declined both in the US and Euro area. Such a positive comovement is not predicted by the standard IRBC models but can be generated in our imperfect information variant of the model. Furthermore, since the labor declined more in the US than in the Euro area, observed labor productivity increased in the US, which contrasts to the Euro area where near-zero or negative productivity growth was observed. The empirical observation of near-zero (or negative) correlation between productivity and hours worked has been viewed as a productivity-hours anomaly in the macroeconomic literature, since the standard real business cycle models predict positive response of hours worked to positive technology shocks, provided an upward sloping labor supply curve (see for example, Galí, 1999, and Christiano, Eichenbaum, and Vigfusson, 2003). To explain the negative productivity-hours correlation, Galí (1999) emphasizes the role of monetary policy shocks and sticky prices. In this paper, we show that negative productivity-hours correlation can also be predicted from the noisy information structure even if prices are flexible and that heterogeneous observations in two regions can be obtained if the fraction of information-constrained consumers differs across regions.

We note that there are other studies that emphasize the role of imperfect information structures in open economy macroeconomic models. For example, Gourinchas and Tornell (2004) discuss distorted beliefs of investors, while Bacchetta and van Wincoop (2006), Crucini, Shintani, and Tsuruga (2010) and Guo (2016), respectively, introduce the heterogeneous information and sticky information structures in open economy monetary models. However, these studies mainly focus on explaining nominal and real exchange rate dynamics rather than the international comovement of real variables. Luo, Nie, and Young (2010) introduce the rational inattention to an intertemporal current account model. However, since the intertemporal current account model is a small open partial equilibrium model, it is not suitable for understanding cross-country correlations. In our paper, we introduce a noisy information structure in a two-country economy general equilibrium model with direct implications on cross-country comovements. Our approach is similar in spirit to Angeletos and La'O (2009), who introduce an imperfect common knowledge structure in a close-economy real business cycle model and show that the model can induce a negative short-run response of employment to productivity shocks. Unlike their model where heterogeneous information across firms plays an important role, we assume homogeneous information across firms but only allow heterogeneous information between countries. Even with such a simple information structure, the model still has rich implications on international business cycle features.

The remainder of the paper is organized as follows. Our two-country model with noisy information is introduced in Section 2. Section 3 discusses the implications of our model for output, consumption, and labor. Section 4 presents conclusion. All the proofs are provided in Appendix.

Model

Our baseline international real business model is a simplified version of the two-country bond-economy model of Baxter and Crucini (1995) from which we have eliminated capital accumulation. We introduce information noise to both firms and households in the baseline model and compare it with the case of perfect information. Foreign country variables are denoted by stars.

Firms

Firms in the domestic country produce the same final good as firms in the foreign country. Labor is internationally immobile but the labor market is competitive. Firms produce the output using a diminishing-returns-to-scale technology

$$Y_t = A_t N_t^{\theta} \tag{1}$$

$$Y_t^* = A_t^* N_t^{*\theta}$$

where $Y_t(Y_t^*)$ is the output in the home (foreign) country, $A_t(A_t^*)$ is the technology level in the home (foreign) country, $N_t(N_t^*)$ is labor employed in the home (foreign) country, and θ∈(0,1). Domestic and foreign firms maximize expected value of their profits, $\pi_t = P_t Y_t - W_t N_t$ and $\pi_t^* = P_t Y_t^* - W_t^* N_t^*$, respectively, where common price P_t of the final goods in two countries is normalized to one and $W_t(W_t^*)$ is the wage rate in the domestic (foreign) country. We assume firms in a country are owned by the residents of the same country so that the profits π_t and π_t^* are given to consumers in corresponding countries[2]. Technology follows the VAR(1) model given by

$$\begin{bmatrix} \log A_t \\ \log A_t^* \end{bmatrix} = \begin{bmatrix} \rho & \nu \\ \nu & \rho \end{bmatrix} \begin{bmatrix} \log A_{t-1} \\ \log A_{t-1}^* \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \epsilon_t^* \end{bmatrix} \qquad (2)$$

Where v(>0) represents technology spillovers, and $\varepsilon_t, \varepsilon_t^* \sim N(1, 1/k_a)$ and $corr(\varepsilon_t, \varepsilon_t^*) \equiv \eta$. Domestic firms know their own level of technology, but receive a signal (with noise) for the technology level of firms in the foreign country. The signals received by home and foreign firms at the beginning of each period t are respectively given by

$$x_t = \log A_t^* + \upsilon_t \quad \text{and} \quad x_t^* = \log A_t + \upsilon_t^*$$

where $\upsilon_t, \upsilon_t^* \sim N(1, 1/k_x)$.

Households

Each country consists of two types of consumers. The first type (type 1) decides the consumption level based on the same information set as the firms located in the same country. The fraction of the type 1 consumers in the home (foreign) country is represented by $\kappa(\kappa^*)$. The remaining consumers choose their consumption level after the information on the foreign technology level is revealed. Households consume the final products and supply labor to firms located in the same country. Each type of consumer in the home country maximizes the expected value of the discounted sum of utility given by

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{C_{it}^{1-\gamma}}{1-\gamma} - \frac{N_{it}^{1+\epsilon}}{1+\epsilon}\right]$$

conditional on the information available at the decision timing, where C_{it} and N_{it} are consumption and labor supply of type *i* (*i*=1,2) consumers, γ(≥0) is the reciprocal of the intertemporal elasticity of substitution or relative risk aversion, ε(≥0) is the reciprocal of the Frisch elasticity of labor supply, and *β* is the discount factor. The international asset

[2] Here we exclude the possibility of cross-border ownerships of firms.

market is restricted to trade only non-contingent bonds. The household budget constraint is given by

$$C_{it} + Q_t B_{it+1} + \frac{\pi}{2} B_{it+1}^2 \leq \Pi_t + W_t N_{it} + B_{it}$$

where B_{it} is bonds held by the type i consumers, Q_t (=(1+r_t)$^{-1}$) is the price of bonds in units of good, r_t is the world interest rate, and $(\pi/2)B_{it+1}^2$ is a quadratic holding cost of bonds with π being a small positive value. The household maximization problem is similarly defined for foreign consumers with preferences identical to domestic consumers.

For the timing of decisions made by firms and households, we follow the setting of Angeletos and La'O (2009) and consider each period in two stages. Let Ω_t be the set of full information revealed at the end of period *t*, defined as $\Omega_t = \{\log A_t, \log A_t^*, x_t, x_t^*\} \cup \Omega_{t-1}$. At the beginning of each time period (stage 1), firms and labor representatives of households meet and decide the production level based on the information set $\{\log A_t, x_t\} \cup \Omega_{t-1}$. All households make labor supply decisions at this stage. The type 1 consumers, κ fraction of households, also determine their consumption level (which cannot be adjusted in the next stage). Firms produce final goods. Then, at the end of each time period (stage 2), information on foreign productivity is revealed. The type 2 consumers, the remaining 1-κ fraction of households, make their consumption-saving decisions based on the updated information set Ω_t. The interest rate level and the real wage rate level are determined where the bond market and the labor market clear. Countries export or import goods in the world market.

Reis (2006) built a microfoundation of inattentive consumers, who update their information sporadically. Mankiw and Reis (2006) further considered the role of inattentive consumers in a general equilibrium framework. In our model, type 1 consumers play a role similar to that of inattentive consumers (planner) considered in Mankiw and Reis (2006), except that we allow our consumers to observe a signal. The presence of type 2 consumers, who make their consumption decision after all the information is revealed, is essential in closing our model so that κ=1 case is excluded in the analysis. The timing of the decision made by type 2 consumers is important in avoiding strategic responses by firms and type 1 consumers. In the beginning of each period, neither firms nor type 1 consumers can observe prices to extract the information about the state of the economy. Since there is no strategic response by firms, they make their production decisions based on their expected value of the price, conditional on their restricted information set. Likewise, type 1 consumers make their saving-borrowing decisions based on their conditional expectation of the interest rate.

Equilibrium

Labor is internationally immobile so that the labor market clearing condition for each country is respectively given by

$$N_t = \kappa N_{1t} + (1-\kappa) N_{2t}$$

and

$$N_t^* = \kappa^* N_{1t}^* + (1-\kappa^*) N_{2t}^*$$

Trade across countries is allowed so that the world goods-market clearing condition (resource constraint) is given by

$$Y_t - C_t + Y_t^* - C_t^* - \frac{\kappa\pi}{2} B_{1t+1}^2 - \frac{\kappa^*\pi}{2} B_{1t+1}^{*2} - \frac{(1-\kappa)\pi}{2} B_{2t+1}^2 - \frac{(1-\kappa^*)\pi}{2} B_{2t+1}^{*2} = 0$$

where

$$C_t = \kappa C_{1t} + (1-\kappa) C_{2t}$$

and

$$C_t^* = \kappa^* C_{1t}^* + (1-\kappa^*) C_{2t}^*.$$

Finally, the Walras' Law implies that the remaining bond market clears as

$$\kappa B_{1t} + \kappa^* B_{1t}^* + (1-\kappa) B_{2t} + (1-\kappa^*) B_{2t}^* = 0$$

so that bonds are in zero net supply at the world level.

Implications for International Business Cycles

International Output Correlation (κ=κ^{∗}=0)

We first solve the model and investigate its implication on the international output correlation when $\kappa=\kappa^*=0$ so that all the consumers can decide their consumption levels after the information about foreign technology is revealed. This setting is convenient for comparing the implication of the model under incomplete market assumption and that of the model under complete market assumption. To solve the model, we log-linearize all the first-order conditions and then use the guess-verification approach. That is, we assume a policy function to take a linear form and plug it into the model to match the coefficients of the same state variables in the two sides of the equations.

Let $y_t = \log Y_t - \log Y (y_t^* = \log Y_t^* - \log Y^*)$ and $b_t = \log B_t - \log B (b_t^* = \log B_t^* - \log B^*)$ where variables with no subscript imply steady state values. We then have the following results on the level of output.

Proposition 1: Suppose κ=κ*=0 under the incomplete market assumption. Then, (i) the equilibrium level of output in the home country and in the foreign country is given by

$$\begin{aligned} y_t &= m_{-1} \log A_{t-1} + m_{-1}^* \log A_{t-1}^* + m \log A_t + m_x x_t + m_b b_t \\ y_t^* &= m_{-1} \log A_{t-1}^* + m_{-1}^* \log A_{t-1} + m \log A_t^* + m_x x_t^* + m_b b_t^* \end{aligned} \quad (3)$$

for some coefficients $(m_{-1}, m_{-1}^*, m, m_x, m_b)$; and

(ii) the equilibrium value of the coefficients $(m_{-1}, m_{-1}^*, m, m_x, m_b)$ satisfies the following properties: m_{-1} and m_{-1}^* approach zero as $k_a/k_x \to 0$; m approaches a positive value as $k_a/k_x \to 0$; m_x approaches a negative value as $k_a/k_x \to 0$ and approaches zero as $k_z/k_x \to \infty$; and m_b is invariant to k_a/k_x.

Proof: See the Appendix.

To illustrate the reason why the model with noisy information provides a quantitatively different result on international output correlation from the full information model, it is helpful to first consider the case of the complete market which has a closed form solution. For the complete market case, firms' problems are the same as before but the households maximize expected value of

$$\sum_{t=0}^{\infty} \beta^t \{ \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{N_t^{1+\epsilon}}{1+\epsilon} + \frac{C_t^{*1-\gamma}}{1-\gamma} - \frac{N_t^{*1+\epsilon}}{1+\epsilon} \} \quad (4)$$

which is common across countries subject to the world resource constraint

$$Y_t - C_t + Y_t^* - C_t^* = 0.$$

For the full information case, the solution is given by

$$\begin{aligned} y_t &= m \log A_t + m^* \log A_t^* \\ y_t^* &= m \log A_t^* + m^* \log A_t \end{aligned} \tag{5}$$

with m=((1-α/2)/(1-α))ζ>0 and m^*=((α/2)/(1-α))ζ<0 where α=-θγ/(1+ε-θ)<0, ζ=(1+ε)/(1+ε-θ)>0. Note that the combination of m>0 and m^*<0 explain the reason why the domestic output responds negatively to foreign technology shocks. When the empirical performance of the model is evaluated, both data series and the simulated series are typically filtered either by using the Hodorick-Prescott filter or the first difference filter. In this paper, we employ the latter and focus on the international correlations in terms of the log growth rates $\Delta y_t = y_t - y_{t-1}$ and $\Delta y_t^* = y_t^* - y_{t-1}^*$. Our choice of filter here is convenient for computing the predicted correlation directly when a closed form solution is provided, as in the case of (4). Given the technology process (2) with a typical choice of parameters, it is straightforward to show that (4) yields negative correlation of Δy_t and Δy_t^*.

If the noisy information structure is introduced in this complete market model, we have the following result.

Proposition 2: κ=κ*=0 under the complete market assumption. Then, (i) the equilibrium level of outputs in the home country and in the foreign country is given by

$$\begin{aligned} y_t &= m_{-1} \log A_{t-1} + m_{-1}^* \log A_{t-1}^* + m \log A_t + m_x x_t \\ y_t^* &= m_{-1} \log A_{t-1}^* + m_{-1}^* \log A_{t-1} + m \log A_t^* + m_x x_t^* \end{aligned} \tag{6}$$

where

$$\begin{cases} m_{-1} = \frac{[\frac{\alpha}{2}\rho + (1-\frac{\alpha}{2})\nu](1-\frac{\alpha}{2})\zeta\alpha}{2(1-\alpha)[(1-\frac{\alpha}{2})^2 + (1-\alpha)(k_a/k_x)^{-1}]} \\ m_{-1}^* = \frac{[\rho - \frac{\alpha}{2}(\rho-\nu)](1-\frac{\alpha}{2})\zeta\alpha}{2(1-\alpha)[(1-\frac{\alpha}{2})^2 + (1-\alpha)(k_a/k_x)^{-1}]} \\ m = \frac{(1-\frac{\alpha}{2})\zeta(1+(k_a/k_x)^{-1})}{(1-\frac{\alpha}{2})^2 + (1-\alpha)(k_a/k_x)^{-1}} \\ m_x = \frac{\frac{\alpha}{2}\zeta(k_a/k_x)^{-1}}{(1-\frac{\alpha}{2})^2 + (1-\alpha)(k_a/k_x)^{-1}} \end{cases}$$

(ii) the equilibrium value of the coefficients $(m_{-1}, m_{-1}^{*}, m, m_x)$ approaches (0, 0, ((1-α/2)/(1-α))ζ, ((α/2)/(1-α))ζ) as $k_a/k_x \rightarrow 0$.

Proof: See the Appendix.

If we compare the coefficients in (4) and (5), the output responds less to currently observed variables and more to old information. When the relative precision of information becomes worse, firms rely less on the signal x_t, and more on old information $\log A_{t-1}^{*}$ so that m_{-1}^{*} becomes more negative as k_a/k_x increases. Even if there is a positive technology shock in the home country, since foreign firms cannot directly observe it, they do not reduce their production level as much as the full information case. Consequently, the home firms do not increase their production as much as the full information case and m becomes smaller as k_a/k_x increases. Again, we can easily compute the correlation of Δy_t and Δy_t^{*} explicitly based on (5).

To better understand the difference of the impact of imperfect information on complete and incomplete markets, we conduct a simple calibration exercise using the results from Propositions 1 and 2. We set parameters at θ=0.64, ε=0.5, γ=2 and β=0.99, values that are commonly used in the literature. We set π=.0001 for the quadratic cost of bond holding to assure a unique steady state. For the parameters appear in technology process (2), we use our own estimated values based on the quarterly series of output and hours worked from the US and Euro area. For the hours worked series in the Euro area, we obtain quarterly average weekly or monthly hours of work in manufacturing from 1989Q1 to 2009Q4 for Austria, France, Germany and Spain from LABORSTA. We then convert these series to quarterly hours worked series in all sectors, by using the ratio of annual hours of worker in manufacturing sector to that in all sectors, for each country, obtained from OECD Main Economic Indicator. The hours worked series for the US is obtained from the BLS. Quarterly real GDP series, obtained from OECD Quarterly National Accounts is used to construct output series for the US and Euro area. We then transform the hours worked series and output series to $\log A_t$ using (1) combined with θ=0.64. Using the estimation procedure employed by BKK, we obtain ρ=0.931, v=0.046 and η=0.040, values that are very close to the ones used by BKK. The output (growth) correlation of the US and Euro area from 1989Q1 to 2009Q4 is 0.54 when the Euro area is based on the four countries we used to construct $\log A_t$. When we expand the output series of the Euro area to those from 15 European countries (Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Norway, Netherlands, Portugal, Spain, Sweden and the United Kingdom), the output correlation from the same period becomes 0.32.

Figure 1 shows how the predicted correlation of Δy Δy_t and Δy_t^* changes in response to changes in the relative precision of information k_a/k_x under two different asset market assumptions. The left panel shows the complete market case based on (5) and the right panel shows the incomplete market case based on (3). When the information is perfect (k_a/k_x=0), the output correlation is negative for the complete market. As k_a/k_x increases, the correlation monotonically increases and becomes positive. In case of the incomplete market, the output correlation is positive but is much smaller than what the data suggests. Again, the correlation increases as k_a/k_x increases. For both cases, the model with a sufficiently large noise matches the observed output correlation from the data (0.54 and 0.32).

An intuitive explanation on the role of restricted information in increasing output correlation is as follows. The main reason why standard IRBC models generate negative or near zero correlation of output is that the domestic and foreign firms respond to technology shocks in the opposite direction. For example, with a positive productivity shock in the home country, domestic firms increase their production, while foreign firms decrease their production. In contrast, if foreign firms do not directly observe a positive shock at home country, they do not reduce their production. Furthermore, as a result of excess supply caused by uninformed foreign firms, home firms do not increase production as much as the fully informed case. Combining the effect of weaker responses with positively correlated technologies across two countries can yield positive output correlation.

International Consumption Correlation ($\kappa=\kappa^*>0$)

We now focus on the bond-economy IRBC model when there are two types of consumers. We show that introducing type 2 consumers in the economy will make the international correlation of consumption lower compared to the benchmark model with full information. To simplify the argument, we here maintain that the fraction of type 1 consumers is common across the countries. As in the previous subsection, we use the first difference filter to investigate the international consumption correlation. Typically, the data suggests that international consumption growth correlation is less than the international output growth correlation. For example, Obstfeld and Rogoff (2000) use the annual Penn World Table data over 1973 to 1992 and find that the average international correlation in real GDP growth rates is 0.53, while the average consumption growth correlation is 0.40. We also compute the consumption growth rate correlation based on the data from OECD Quarterly National Accounts. If we use four countries for the Euro area, consumption correlation is 0.46 compared to the outputs correlation of 0.54 during the period from 1989Q1 to 2009Q4. When we use 15 European countries to construct Euro aggregates, the consumption correlation is 0.26, but the outputs correlation is 0.32. In either case, consumption correlation is lower than the output correlation, which cannot be predicted by the standard full information model[3].

To solve the model with $\kappa=\kappa^*>0$, we need to combine an extended version of Sims' (2001) approach and the guess-verification approach. We decompose heterogeneous expectations into homogeneous expectation component and expectation error component. We then solve the model by treating as if the latter is an exogenous shock in the first step. In the second step, we use the method of undetermined coefficients to assure the endogenous expectation errors consistent with the solution from the first step (see technical appendix for details). All the parameter values are the same as before except that we set κ=0.6. The solutions are obtained for both (y_t, y_t^*) and (c_t, c_t^*) where $c_t = \log C_t - \log C$ and $c_t^* = \log C_t^* - \log C^*$.

To understand the characteristics of the model, we compute the impulse response of consumption to one standard positive deviation of domestic technology shocks with three different choice of relative precision of information, k_a/k_x=0, 1, and 25, which is shown in Figure 2[4]. Since the information is revealed at the end of each period, the effects of information precision become almost negligible after one period. In the perfect information case (k_a/k_x=0), households in the home country increase their consumptions as their income increases. Households in the foreign country also increase consumption, since the spillover effects of the positive technology shocks make foreign households to borrow from the home country. When the information noise becomes large (k_a/k_x= 1, and 25), foreign households cannot predict the increase in income in the future and do not borrow as much as they should from the international asset market. Therefore, even if foreign firms produce relatively more than the perfect information case, foreign households still decrease their consumption. This asymmetric responses of c_t and c_t^* is even more amplified by taking the first difference Δc_t and Δc_t^*. This makes consumption growth correlation decreasing with respect to the magnitude of information noise.

Figure 3 demonstrates the dynamics of consumptions growth correlation and outputs growth correlation in response to different degrees of information frictions. As in the case of $\kappa=\kappa^*=0$, we can see information noise increases outputs growth correlation, and at the same time it reduces consumptions-growth correlation. When the relative precision of information (k_a/k_x) reaches 5, the consumption growth correlation becomes less than the

[3] Pakko (2004) uses 10 country data from 1973:Q1 to 2002:Q4 and show that for all countries, the correlations of output growth rates is higher than that of consumption growth rates.

[4] Since both information-constrained and unconstrained consumers have rational expectations, as long as κ is not extremely large the calibration of our exercise shows the response of interest rates to technology shocks is not unrealistic.

output growth correlation which dramatically reduces the gap between the prediction of the model and the data.

International Productivity-Hours Dynamics (κ≠κ*)

In the recent global financial crisis of 2007-2009, employment and hours worked declined both in the US and Euro area. Such a positive comovement is not predicted by the standard IRBC models. Furthermore, since the labor declined more in the US than in Euro area, observed labor productivity increased in the US which contrast to the Euro area where near-zero or negative productivity growth was observed. This fact was first investigated by Ohanian (2010). The empirical observation of near-zero (or negative) correlation between productivity and hours worked has been viewed as a productivity-hours anomaly in the macroeconomic literature since the standard real business cycle model predicts a positive response of hours worked to positive technology shocks, provided an upward sloping labor supply curve (see Galí, 1999; Christiano, Eichenbaum and Vigfusson, 2003).

Let us first show that given a certain range of parameter values, our model can predict the positive comovement of labor input, which cannot be obtained in the full information model. In our data, the hours worked (growth) correlation between the US and Euro area based on four European countries is positive at 0.20. Using the same solution technique as before, we can obtain the solution for (n_t, n_t^*) where $n_t = \log N_t - \log N$ and (c_t, c_t^*) where $n_t^* = \log N_t^* - \log N^*$. Figure 4 shows the predicted international correlation of hours worked using the same set of parameter values as before. For the perfect information case with k_a/k_x=0, the correlation is negative. The correlation is not monotonically increasing in k_a/k_x. However, it predicts the positive correlation when k_a/k_x lies between the values of 0.1 and 0.5.

We also solve the model when the fraction of information constrained consumers differs across the country. Figure 5 shows the predicted correlation of hour worked growth, $\Delta n_t(\Delta n_t^*)$, and measured productivity growth, $\Delta y_t - \Delta n_t(\Delta y_t^* - \Delta n_t^*)$, when κ=0.1 and κ*=0.7. It shows that when k_a/k_x increases, the model can predict negative productivity-hours correlation in one region and positive productivity-hours correlation in the other region, where the former represents the Euro area and the latter represents the US.

Conclusion

We introduced a noisy information structure into an otherwise standard international real business cycle model with two countries. When domestic firms observe current foreign technology with some noise, the prediction of the model on international correlation turned out to be very different from that of a standard perfect information model. First, we found that the imperfect information model can explain positive output correlation both in complete and incomplete market models. Second, consumption correlation became smaller than output correlation when the precision of the information becomes worse in the presence of information constrained consumers. Third, the model can explain the observation of positive productivity-hours correlation in one country and negative correlation in the other country.

There are several directions in which our model can be extended. First, we can allow for information heterogeneity not only across the countries but also within a country. When firms in the same country face different signals about the foreign technology, the lagged foreign technology will have a role of public information, in addition to its role as the predictor of the current foreign technology. This may amplify the effect of noisy information and increase the predicted international output comovement. Second, we can introduce nominal shocks into the model and consider the possibility of confusion between nominal and real shocks. Third, we can investigate the role of the possible correlation of noise shocks across countries for the output correlations. These extensions are left for future research.

Appendix

Proof of Proposition 1:

The bond economy can be fully characterized by the following first-order conditions (we normalize P_t=1):

$$E_{ht}\omega_t - E_{ht}(A_t\theta N_t^{\theta-1}) = 0 \tag{A.1}$$

$$E_{ft}\omega_t^* - E_{ft}(A_t^*\theta N_t^{*\theta-1}) = 0 \tag{A.2}$$

$$Y_t - A_t N_t^{\theta} = 0 \tag{A.3}$$

$$Y_t^* - A_t^* N_t^{*\theta} = 0 \tag{A.4}$$

$$\lambda_t - C_t^{-\gamma} = 0 \tag{A.5}$$

$$\lambda_t\omega_t - N_t^{\epsilon} = 0 \tag{A.6}$$

$$\lambda_t(1+\pi B_{t+1}) - E_t(\beta\lambda_{t+1}R_{t+1}) = 0 \tag{A.7}$$

$$B_{t+1} + \frac{\pi}{2}B_{t+1}^2 + C_t - Y_t - R_t B_t = 0 \tag{A.8}$$

$$\lambda_t^* - C_t^{*-\gamma} = 0 \tag{A.9}$$

$$\lambda_t^*\omega_t^* - N_t^{*\epsilon} = 0 \tag{A.10}$$

$$\lambda_t^*(1+\pi B_{t+1}^*) - E_t(\beta\lambda_{t+1}^* R_{t+1}) = 0 \tag{A.11}$$

$$B_{t+1}^* + \frac{\pi}{2}B_{t+1}^{*2} + C_t^* - Y_t^* - R_t B_t^* = 0 \tag{A.12}$$

$$B_t + B_t^* = 0 \tag{A.13}$$

and boundary conditions:

$$\lim_{t\to\infty}\beta^t\lambda_t B_t = 0 \tag{A.14}$$

$$\lim_{t\to\infty}\beta^t\lambda_t^* B_t^* = 0, \tag{A.15}$$

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where $\lambda_t(\lambda_t^*)$ is the Lagrange multiplier associated with the budget constraint of domestic (foreign) households. Log-linearize equations around the steady state and jointly solve equations (A.1), (A.3), (A.5) and (A.6), and we have

$$y_t = c_0 + \zeta \log A_t + \alpha E_{ht} c_t. \tag{A.16}$$

Then jointly solve equations (A.2), (A.4), (A.9) and (A.10),

$$y_t^* = c_0^* + \zeta \log A_t^* + \alpha E_{ft} c_t^*, \tag{A.17}$$

where α=((-θγ)/(1+ε-θ)) and ζ=((1+ε)/(1+ε-θ)), $y_t = \log Y_t - \log Y (y_t^* = \log Y_t^* - \log Y^*)$, and $c_t = \log C_t - \log C(c_t^* = \log C_t^* - \log C^*)$. Likewise, jointly solve equations (A.5) and (A.7),

$$c_t - E_t c_{t+1} = \frac{1}{\gamma} E_t(-r_{t+1} + \pi b_{t+1}),$$

and equations (A.9) and (A.11),

$$c_t^* - E_t c_{t+1}^* = \frac{1}{\gamma} E_t(-r_{t+1} + \pi b_{t+1}^*),$$

where r_t=logR_t-log$R$ and b_t=B_t/Y. With $c_t + c_t^* = y_t + y_t^*$,

$$c_t - E_t c_{t+1} = \frac{1}{2}[y_t + y_t^* - E_t(y_{t+1} + y_{t+1}^*) + \frac{2\pi}{\gamma} b_{t+1}].$$

Then by assuming

$$c_t = \frac{1}{2}(y_t + y_t^* + d_t) \tag{A.18}$$

and

$$c_t^* = \frac{1}{2}(y_t + y_t^* - d_t),$$

we have

$$d_t = E_t[d_{t+1} + \frac{2\pi}{\gamma} b_{t+1}] = E_t[d_{t+2} + \frac{2\pi}{\gamma} b_{t+1} + \frac{2\pi}{\gamma} b_{t+2}] = \cdots. \tag{A.19}$$

With $B_{t+1} = Y_t - C_t - R_t B_t - (\pi/2) B_{t+1}^2$,

$$\begin{aligned} \beta^k B_{t+k} &= \beta^k [(Y_{t+k-1} - C_{t+k-1} - \frac{\pi}{2} B_{t+k}^2) + (Y_{t+k-2} - C_{t+k-2} - \frac{\pi}{2} B_{t+k-1}^2) R_{t+k-1} \\ &\quad + \cdots + R_{t+k-1} \cdots R_t B_t]. \end{aligned}$$

From the boundary condition (A.14) and λ_t>0, we have $\lim_{k\to\infty} E_t \beta^k B_{t+k} = 0$. Using $\lim_{k\to\infty} E_t \beta^k B_{t+k} = 0$, (A.19), and the results that in the steady state Y=C and βR=1, Y_t-C_t≈$Y(y_t$-$c_t)$,

$$d_t = (1-\beta)\{E_t[\sum_{k=0}^{\infty} \beta^k (y_{t+k} - y_{t+k}^*) + \sum_{k=0}^{\infty} \beta^{k+1} \frac{2\pi}{\gamma(1-\beta)} b_{t+1+k} + 2b_t]\}. \tag{A.20}$$

To solve the model, we use guess-verification approach and assume

$$y_t = m_{-1} \log A_{t-1} + m_{-1}^* \log A_{t-1}^* + m \log A_t + m_x x_t + m_b b_t, \tag{A.21}$$

and

$$y_t^* = m_{-1} \log A_{t-1}^* + m_{-1}^* \log A_{t-1} + m \log A_t^* + m_x x_t^* + m_b b_t^*. \tag{A.22}$$

Plug equations (A.21) and (A.22) into (A.20) and then simplify the expression by using the assumption of vector-autoregressive technology processes

$$\begin{bmatrix} \ln A_t \\ \ln A_t^* \end{bmatrix} = \begin{bmatrix} \rho & \nu \\ \nu & \rho \end{bmatrix} \begin{bmatrix} \ln A_{t-1} \\ \ln A_{t-1}^* \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \epsilon_t^* \end{bmatrix}, \tag{A.23}$$

and then we have

$$\begin{aligned} d_t &= (1-\beta)(m_{-1} - m_{-1}^*)(\log A_{t-1} - \log A_{t-1}^*) + \frac{1-\beta}{1-\beta(\rho-\nu)}[\beta(m_{-1} - m_{-1}^*) + m - \beta m_x(\rho-\nu)] \quad \text{(A.24)} \\ &\quad (\log A_t - \log A_t^*) + (1-\beta)[m_x(x_t - x_t^*) + (2m_b + \frac{2\pi}{\gamma(1-\beta)}) E_t \sum_{k=0}^{\infty} \beta^k b_{t+k} + (2 - \frac{2\pi}{\gamma(1-\beta)}) b_t]. \end{aligned}$$

Plug (A.20) into (A.19), we have

$$\begin{aligned} & E_t[\sum_{k=0}^{\infty}\beta^k(y_{t+k}-y^*_{t+k})+(2m_b+\frac{2\pi}{\gamma(1-\beta)})\sum_{k=0}^{\infty}\beta^k b_{t+k}+(2-\frac{2\pi}{\gamma(1-\beta)})b_t] \qquad (A.25)\\ = & E_t[\sum_{k=0}^{\infty}\beta^k(y_{t+1+k}-y^*_{t+1+k})+(2m_b+\frac{2\pi}{\gamma(1-\beta)})\sum_{k=0}^{\infty}\beta^k b_{t+1+k}+2b_{t+1}]. \end{aligned}$$

With

$$\begin{aligned} & E_t\sum_{k=0}^{\infty}\beta^k(y_{t+k}-y^*_{t+k})\\ = & E_t\sum_{k=0}^{\infty}\beta^k[(m_{-1}-m^*_{-1})(\log A_{t-1+k}-\log A^*_{t-1+k})+m(\log A_{t+k}-\log A^*_{t+k})+m_x(x_{t+k}-x^*_{t+k})]+2m_bE_t\sum_{k=0}^{\infty}\beta^k b_{t+k}\\ \equiv & U_t+2m_bV_t, \end{aligned}$$

equation (A.25) can be rewritten as

$$(m_b+1)V_t-(\beta+\frac{\pi}{\gamma}+m_b+1)E_tV_{t+1}+\beta E_tV_{t+2}=-\frac{1}{2}(U_t-E_tU_{t+1}).$$

By using Lag-operator, and taking $b_t=V_t-\beta E_tV_{t+1}$ as initial condition and U_{t+k} as given, we have

$$V_t=(1-\frac{\beta}{\lambda_2})^{-1}\frac{\beta}{2\lambda_2(1+m_b)}\{U_t-\frac{(1-\lambda_1)E_tU_{t+1}}{1-\lambda_1(\rho-\nu)}\}+\frac{1}{1-\frac{\beta}{\lambda_2}}b_t, \qquad (A.26)$$

where $\lambda_1<1<\lambda_2$ solve the equation $(m_b+1)\lambda^2-(\beta+(\pi/\gamma)+m_b+1)\lambda+\beta=0$. Plug (A.26) into (A.24), then into (A.18), then into (A.16) and finally compare the coefficients with (A.21), and we have

$$\left\{\begin{array}{l} m_{-1}=\frac{\alpha(m_{-1}+m^*_{-1})}{2}+\frac{\alpha(1-\beta)(1+d_1)(m_{-1}-m^*_{-1})}{2}-\frac{\alpha(1-\beta)(1+d_1)(\beta(m_{-1}-m^*_{-1})+m-\beta m_x(\rho-\nu))k_a\nu}{2(1-\beta\rho+\beta\nu)(k_a+k_x)}\\ +\frac{\alpha m k_a\nu}{2(k_a+k_x)}+\frac{\alpha(1-\beta)d_2(m_{-1}-m^*_{-1}+\frac{(\rho-\nu)(\beta(m_{-1}-m^*_{-1})+m-m_x)}{(1-\beta\rho+\beta\nu)})k_a\nu}{2(k_a+k_x)};\\ m^*_{-1}=\frac{\alpha(m_{-1}+m^*_{-1})}{2}-\frac{\alpha(1-\beta)(1+d_1)(m_{-1}-m^*_{-1})}{2}-\frac{\alpha(1-\beta)(1+d_1)(\beta(m_{-1}-m^*_{-1})+m-\beta m_x(\rho-\nu))k_a\rho}{2(1-\beta\rho+\beta\nu)(k_a+k_x)}\\ +\frac{\alpha m k_a\rho}{2(k_a+k_x)}+\frac{\alpha(1-\beta)d_2(m_{-1}-m^*_{-1}+\frac{(\rho-\nu)(\beta(m_{-1}-m^*_{-1})+m-m_x)}{(1-\beta\rho+\beta\nu)})k_a\rho}{2(k_a+k_x)};\\ m=\zeta+\frac{\alpha(m+m_x)}{2}-\frac{\alpha(1-\beta)(1+d_1)m_x}{2}+\frac{\alpha(1-\beta)(1+d_1)(\beta(m_{-1}-m^*_{-1})+m-\beta m_x(\rho-\nu))}{2(1-\beta\rho+\beta\nu)}\\ -\frac{1}{2}\alpha(1-\beta)d_2(m_{-1}-m^*_{-1}+\frac{(\rho-\nu)(\beta(m_{-1}-m^*_{-1})+m-m_x)}{1-\beta\rho+\beta\nu});\\ m_x=\frac{\alpha m k_x}{2(k_x+k_a)}+\frac{\alpha m_x}{2}+\frac{\alpha(1-\beta)(1+d_1)m_x}{2}-\frac{\alpha(1-\beta)(1+d_1)(\beta(m_{-1}-m^*_{-1})+m-\beta m_x(\rho-\nu))k_x}{2(1-\beta\rho+\beta\nu)(k_a+k_x)}\\ +\frac{\alpha(1-\beta)d_2(m_{-1}-m^*_{-1}+\frac{(\rho-\nu)(\beta(m_{-1}-m^*_{-1})+m-m_x)}{(1-\beta\rho+\beta\nu)})k_x}{2(k_a+k_x)};\\ m_b=\frac{\alpha(1-\beta)(1-\frac{\pi}{\gamma(1-\beta)})(1-\frac{\beta}{\lambda_2})+\alpha(1-\beta)\frac{\pi}{\gamma(1-\beta)}}{1-\frac{\beta}{\lambda_2}-\alpha(1-\beta)}, \end{array}\right.$$

where $\alpha=((-\theta\gamma)/(1+\varepsilon-\theta))$, $\zeta=((1+\varepsilon)/(1+\varepsilon-\theta))$, $d_1=(m_b+(\pi/(\gamma(1-\beta))))(1-(\beta/(\lambda_2)))^{-1}(\beta/((\lambda_2(1+m_b)))$, $d_2=(m_b+(\pi/(\gamma(1-\beta))))(1-(\beta/(\lambda_2)))^{-1}((\beta(1-\lambda_1))/((\lambda_2(1+m_b)[1-\lambda_1(\rho-v)]))$. There are five undetermined variables and five equations, so we solve the coefficients $(m_{-1}, m_{-1}^*, m, m_x, m_b)$. We can easily verify that m_b is invariant to k_a/k_x.

For part (ii), when $k_a/k_x \to 0$, the above equations become

$$\begin{cases} m_{-1} = \frac{\alpha(m_{-1}+m^*_{-1})}{2} + \frac{\alpha(1-\beta)(1+d_1)(m_{-1}-m^*_{-1})}{2}; \\ m^*_{-1} = \frac{\alpha(m_{-1}+m^*_{-1})}{2} - \frac{\alpha(1-\beta)(1+d_1)(m_{-1}-m^*_{-1})}{2}; \\ m = \zeta + \frac{\alpha(m+m_x)}{2} - \frac{\alpha(1-\beta)(1+d_1)m_x}{2} + \frac{\alpha(1-\beta)(1+d_1)(\beta(m_{-1}-m^*_{-1})+m-\beta m_x(\rho-\nu))}{2(1-\beta\rho+\beta\nu)} \\ -\frac{1}{2}\alpha(1-\beta)d_2(m_{-1}-m^*_{-1}+\frac{(\rho-\nu)(\beta(m_{-1}-m^*_{-1})+m-m_x)}{1-\beta\rho+\beta\nu}); \\ m_x = \frac{\alpha(m+m_x)}{2} + \frac{\alpha(1-\beta)(1+d_1)m_x}{2} - \frac{\alpha(1-\beta)(1+d_1)(\beta(m_{-1}-m^*_{-1})+m-\beta m_x(\rho-\nu))}{2(1-\beta\rho+\beta\nu)} \\ +\frac{\alpha(1-\beta)d_2(m_{-1}-m^*_{-1}+\frac{(\rho-\nu)(\beta(m_{-1}-m^*_{-1})+m-m_x)}{(1-\beta\rho+\beta\nu)})}{2}; \\ m_b = \frac{\alpha(1-\beta)(1-\frac{\pi}{\gamma(1-\beta)})(1-\frac{\beta}{\lambda_2})+\alpha(1-\beta)\frac{\pi}{\gamma(1-\beta)}}{1-\frac{\beta}{\lambda_2}-\alpha(1-\beta)}. \end{cases}$$

so,

$$\begin{cases} m_{-1} = m^*_{-1} = 0; \\ m = \frac{\zeta}{1-\alpha} - \frac{\alpha[(1-\beta\rho+\beta\nu)-(1-\beta)(1+d_1)+(1-\beta)d_2(\rho-\nu)]\zeta}{2(1-\alpha)[(1-\beta\rho+\beta\nu)-\alpha(1-\beta)(1+d_1)+\alpha(1-\beta)d_2(\rho-\nu)]}; \\ m_x = \frac{\alpha[(1-\beta\rho+\beta\nu)-(1-\beta)(1+d_1)+(1-\beta)d_2(\rho-\nu)]\zeta}{2(1-\alpha)[(1-\beta\rho+\beta\nu)-\alpha(1-\beta)(1+d_1)+\alpha(1-\beta)d_2(\rho-\nu)]}; \\ m_b = \frac{\alpha(1-\beta)(1-\frac{\pi}{\gamma(1-\beta)})(1-\frac{\beta}{\lambda_2})+\alpha(1-\beta)\frac{\pi}{\gamma(1-\beta)}}{1-\frac{\beta}{\lambda_2}-\alpha(1-\beta)}. \end{cases}$$

Then, let us prove the case under the condition $\pi \to 0$, $m>0$ and $m_x<0$. At first, we prove $-1<m_b<0$ as a preparation. Since $0<\lambda_1<1<\lambda_2$, $0<\beta<1$, and $\alpha<0$, $(\beta/\lambda_2)<1$, if $\pi \to 0$, we have $m_b<0$ and,

$$m_b + 1 = \frac{(1-\frac{\beta}{\lambda_2})(1-\alpha(1-\beta)) + \frac{\alpha\beta\pi}{\gamma\lambda_2}}{1-\frac{\beta}{\lambda_2}-\alpha(1-\beta)} > 0.$$

Therefore, we have $-1<m_b<0$. For d_1, we have

$$\begin{aligned} d_1 &= (m_b + \frac{\pi}{\gamma(1-\beta)})(1-\frac{\beta}{\lambda_2})^{-1}\frac{\beta}{\lambda_2(1+m_b)} \\ &= \frac{\alpha(1-\beta)-\frac{\alpha\pi}{\gamma}+\frac{\pi}{\gamma(1-\beta)}}{1-\frac{\beta}{\lambda_2}-\alpha(1-\beta)}\frac{\beta}{\lambda_2(1+m_b)} = \frac{\frac{\alpha\beta(1-\beta)}{\lambda_2}-\frac{\alpha\beta\pi}{\gamma\lambda_2}+\frac{\pi\beta}{\gamma\lambda_2(1-\beta)}}{-\frac{\alpha\beta(1-\beta)}{\lambda_2}+\frac{\alpha\beta\pi}{\gamma\lambda_2}+1-\frac{\beta}{\lambda_2}} < 0. \end{aligned}$$

Similarly, we can also have d_2<0. Because

$$-d_2(\rho - \nu) < -d_2 = -d_1 \frac{1-\lambda_1}{1-\lambda_1(\rho-\nu)} < -d_1,$$

we have $-d_1+d_2(\rho-\nu)>0$. Thus, m_x's numerator:

$$\begin{aligned} & \alpha[(1-\beta\rho+\beta\nu)-(1-\beta)(1+d_1)+(1-\beta)d_2(\rho-\nu)]\zeta \\ = \quad & \alpha\{\beta-\beta\rho+\beta\nu+(1-\beta)[-d_1+d_2(\rho-\nu)]\} < 0. \end{aligned}$$

Because

$$d_1 + 1 = \frac{1-\frac{\beta}{\lambda_2}+\frac{\pi\beta}{\gamma\lambda_2(1-\beta)}}{-\frac{\alpha\beta(1-\beta)}{\lambda_2}+\frac{\alpha\beta\pi}{\gamma\lambda_2}+1-\frac{\beta}{\lambda_2}} > 0,$$

m_x's denominator

$$2(1-\alpha)[(1-\beta\rho+\beta\nu)-\alpha(1-\beta)(1+d_1)+\alpha(1-\beta)d_2(\rho-\nu)] > 0.$$

Overall,

$$m_x = \frac{\alpha[(1-\beta\rho+\beta\nu)-(1-\beta)(1+d_1)+(1-\beta)d_2(\rho-\nu)]\zeta}{2(1-\alpha)[(1-\beta\rho+\beta\nu)-\alpha(1-\beta)(1+d_1)+\alpha(1-\beta)d_2(\rho-\nu)]} < 0.$$

Note that $m+m_x=(\zeta/(1-\alpha))>0$, so $m>0$.

Proof of Proposition 2

The complete-market economy can be fully characterized by the following first-order conditions:

$$E_{ht}\omega_t - E_{ht}(A_t\theta N_t^{\theta-1}) = 0 \tag{A.27}$$

$$E_{ft}\omega_t^* - E_{ft}(P_t A_t^*\theta N_t^{*\theta-1}) = 0 \tag{A.28}$$

$$Y_t - A_t N_t^{\theta} = 0 \tag{A.29}$$

$$Y_t^* - A_t^* N_t^{*\theta} = 0 \tag{A.30}$$

$$C_t^{-\gamma} - C_t^{*-\gamma} = 0 \tag{A.31}$$

$$N_t^{\epsilon} - C_t^{-\gamma}\omega_t = 0 \tag{A.32}$$

$$N_t^{*\epsilon} - C_t^{*-\gamma}\omega_t^* = 0 \tag{A.33}$$

$$C_t + C_t^* - A_t N_t^{\theta} - A_t^* N_t^{*\theta} = 0. \tag{A.34}$$

Log-linearizing equations (A.27), (A.29) and (A.32) around the steady state, and jointly solving them, we have

$$y_t = c_0 + \zeta \log A_t + \alpha E_{ht} c_t. \tag{A.35}$$

Similarly, jointly solving equations (A.28), (A.30) and (A.33), we have

$$y_t^* = c_0^* + \zeta \log A_t^* + \alpha E_{ft} c_t^*, \tag{A.36}$$

where $c_0 = c_0^* = \theta/(1+\varepsilon-\theta)$, $\alpha \equiv -\theta\gamma/(1+\varepsilon-\theta)$ and $\zeta \equiv (1+\varepsilon)/(1+\varepsilon-\theta)$. Since $C_t = C_t^*$ and $C_t + C_t^* = Y_t + Y_t^*$, we have:

$$c_t = c_t^* = \frac{1}{2}[y_t + y_t^*]. \tag{A.37}$$

To solve the model, we use guess-verification approach and assume

$$\begin{aligned} y_t &= m_{-1}\log A_{t-1} + m_{-1}^*\log A_{t-1}^* + m\log A_t + m_x x_t \\ y_t^* &= m_{-1}\log A_{t-1}^* + m_{-1}^*\log A_{t-1} + m\log A_t^* + m_x x_t^*. \end{aligned}$$

Then we plug the two above equations into (A.37), and then (A.35), and we have

$$
\begin{aligned}
& m_0 + m_{-1}\log A_{t-1} + m^*_{-1}\log A^*_{t-1} + m\log A_t + m_x x_t \\
= \quad & c_0 + \zeta\log A_t + \alpha\{m_0 + \frac{m_{-1}+m^*_{-1}}{2}\log A_{t-1} + \frac{m_{-1}+m^*_{-1}}{2}\log A^*_{t-1} + \frac{m[k_a(\rho\log A^*_{t-1} + \nu\log A_{t-1}) + k_x x_t]}{2(k_a+k_x)} \\
& + \frac{m}{2}\log A_t + \frac{m_x}{2}x_t + \frac{m_x}{2}\log A_t\}.
\end{aligned}
$$

Comparing the coefficients in the above equation, we get

$$m_{-1} = \alpha\frac{m_{-1}+m^*_{-1}}{2} + \frac{m\alpha\nu k_a}{2(k_a+k_x)} \tag{A.38}$$

$$m^*_{-1} = \alpha\frac{m_{-1}+m^*_{-1}}{2} + \frac{m\alpha\rho k_a}{2(k_a+k_x)} \tag{A.39}$$

$$m = \zeta + \alpha\frac{m+m_x}{2} \tag{A.40}$$

$$m_x = \alpha[\frac{m_x}{2} + \frac{mk_x}{2(k_a+k_x)}] \tag{A.41}$$

Jointly solving equations (A.38) to (A.41), we have,

$$
\begin{cases}
m = \frac{(1-\frac{\alpha}{2})\zeta(k_a+k_x)}{(1-\frac{\alpha}{2})^2k_a+(1-\alpha)k_x}; \\
m_x = \frac{\frac{\alpha}{2}\zeta k_x}{(1-\frac{\alpha}{2})^2k_a+(1-\alpha)k_x}; \\
m_{-1} = \frac{[\frac{\alpha}{2}\rho+(1-\frac{\alpha}{2})\nu](1-\frac{\alpha}{2})\zeta\alpha}{2(1-\alpha)[(1-\frac{\alpha}{2})^2k_a+(1-\alpha)k_x]}k_a; \\
m^*_{-1} = \frac{[(1-\frac{\alpha}{2})\rho+\frac{\alpha}{2}\nu](1-\frac{\alpha}{2})\zeta\alpha}{2(1-\alpha)[(1-\frac{\alpha}{2})^2k_a+(1-\alpha)k_x]}k_a.
\end{cases}
;
$$

For part (ii), when $k_a/k_x \rightarrow 0$, it is straightforward to show that $(m_{-1}, m_{-1}^*, m, m_x)$ approaches (0,0,(1-α/2)/(1-α)ζ,(α/2)/(1-α)ζ.

Data

We choose the US versus Euro area as the two countries in our model. To construct the Euro aggregator we choose the following four countries: Austria, France, Germany, and Spain. The following files are used for average weekly hours worked per worker in manufacturing. All the raw data are seasonally adjusted by X-12-ARIMA.

Austria: we collect Monthly hours of work per month in manufacturing data from LABORSTA and then use the arithmetic mean of every three months to calculate the quarterly hours of work per month in manufacturing. The data cover wage earners from 1989M1 to 1995M12 and employees from 1996M1 to 2009M12[5].

France: we collect Quarterly hours of work per week in manufacturing data from LABORSTA. The data covers wage earners from 1989Q1 to 1992Q4 and employees from 1993Q1 to 2009Q4.

Germany: before 2005, LABORSTA has two different sequences for Germany: Western Germany and Eastern Germany. Western Germany covers both of two parts after 1990Q1. After 2005, there is only one Germany sequence. We choose the series of Western Germany to supplement the Germany series to have a complete series of Germany. The data covers wage earners.

Spain: we collect quarterly hours of work per week data from LABORSTA. The data cover wage earners. For the two missing observations, we replace them by means of their two closest observations.

We collect the data of average annual hours actually worked per worker in all sectors from OECD Main Economic Indicator. To construct the quarterly hours of work in all sectors data, we use the quarterly average hours of worker per week or per month in manufacturing as proportions to divide the annual hours worked per worker in all sectors.

Employment: we use quarterly seasonal adjusted average employment data from OECD Main Economic Indicator.

Output and consumption: the data for quarterly GDP and consumption are from OECD Quarterly National Accounts.

The series for US quarterly average hours of work per week and employment are from BLS. The series for US GDP and consumption are from OECD Quarterly National Accounts.

[5] Since we use the series of quarterly hours of work as proportions to divide the series of annual hours of work, we conjecture the change of coverage only has a minor effect.

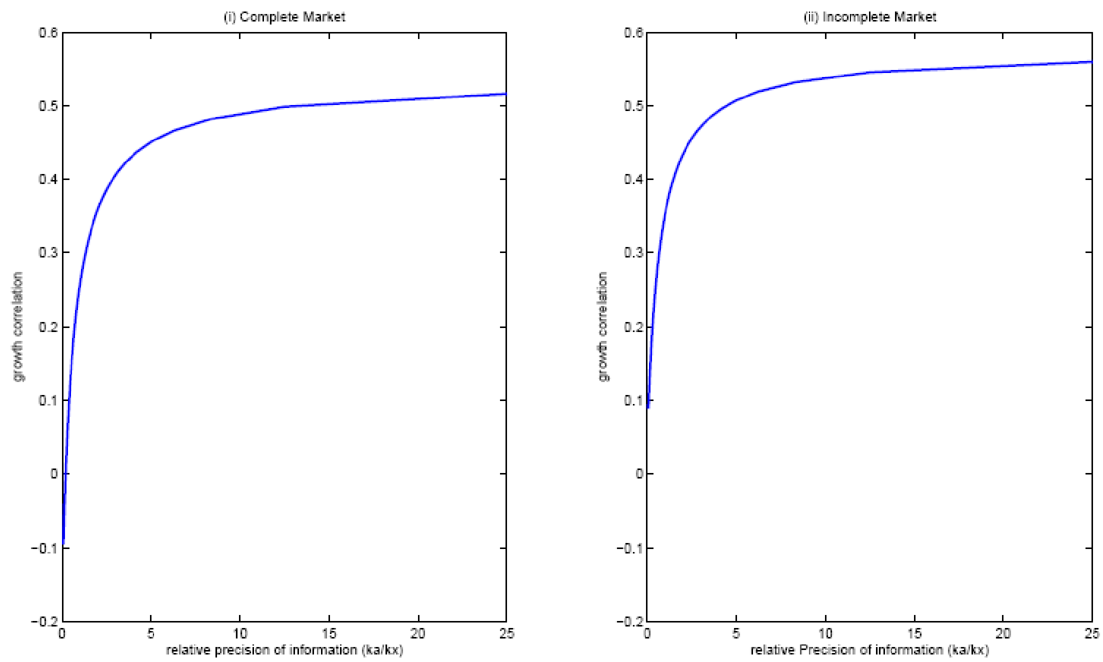

Figure 1: The correlations of outputs in different asset markets

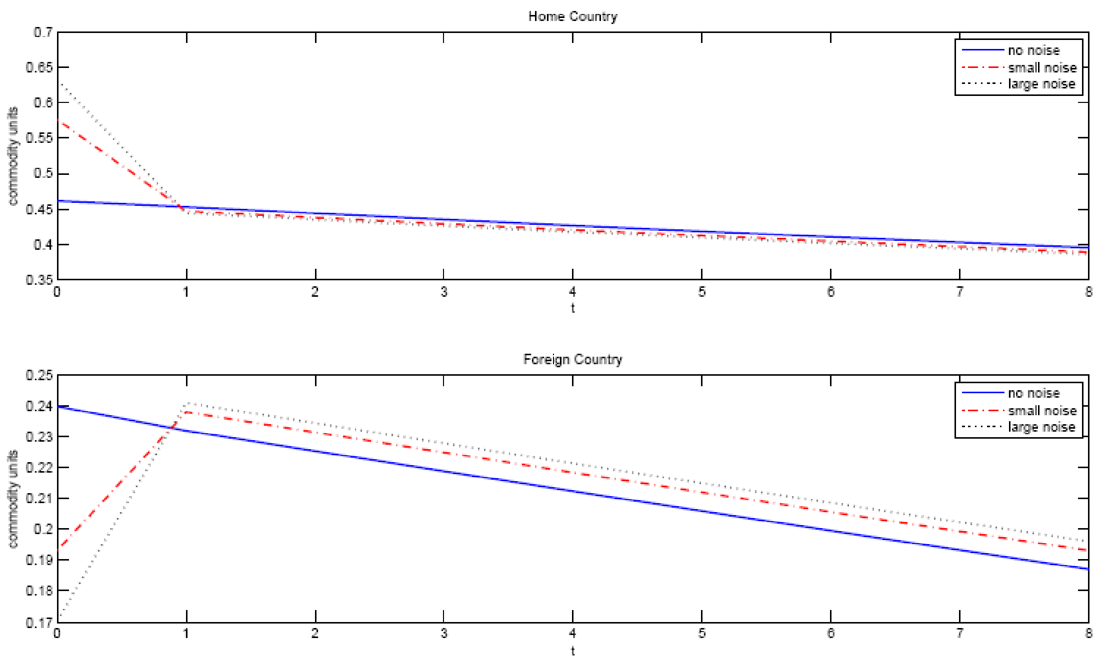

Figure 2: The impulse response of consumption

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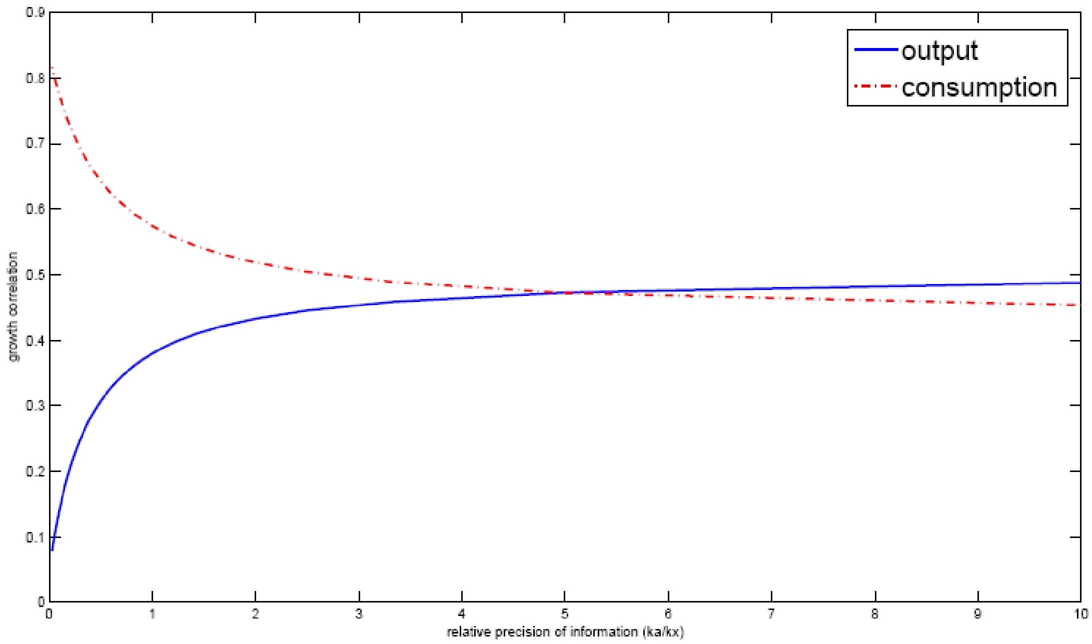

Figure 3: Outputs growth correlation and consumption growth correlation ($\kappa = \kappa^* = 0.6$)

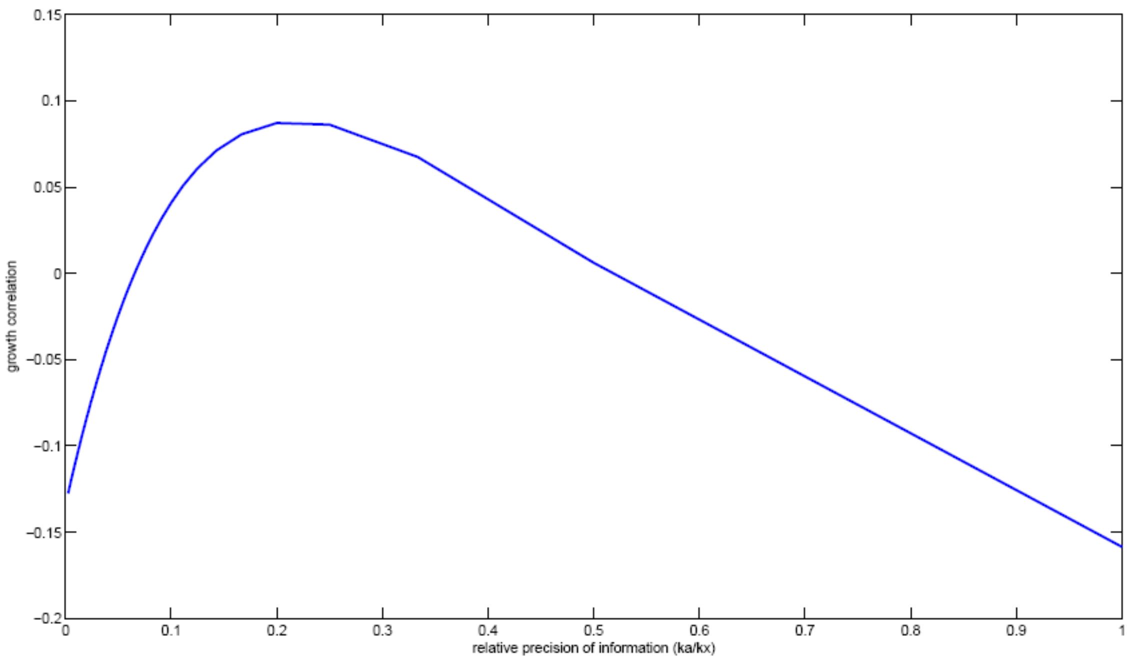

Figure 4: The correlations of hours worked growth

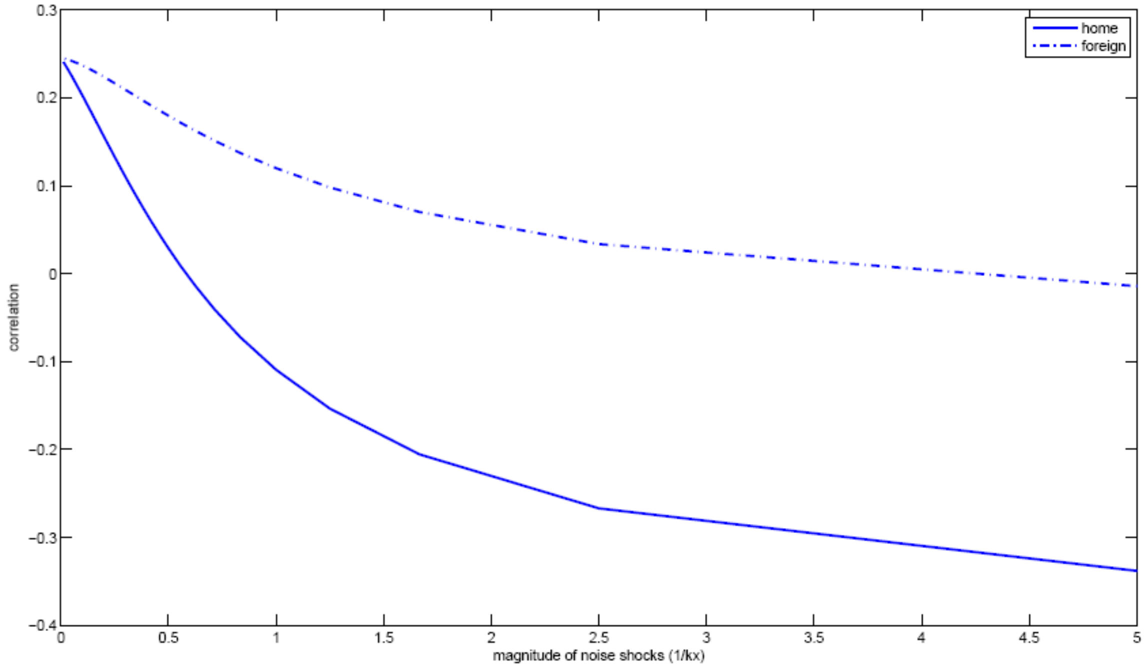

Figure 5: Productivity growth and hours growth correlation ($\kappa = 0.1$ and $\kappa^{*} = 0.7$)

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