

[DOI: 10.20472/EFC.2017.007.021](https://doi.org/10.20472/EFC.2017.007.021)

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## **EXPLORING FARMERS' SELECTION OF CROP PROTECTION LEVELS AS AN ADAPTATION STRATEGY TO CLIMATE RISKS**

### **Abstract:**

Among the challenges facing the European Union agricultural sector in the coming years, the impacts of climate change could lead to much greater variability in farmers' incomes. In this context, the insurance industry will have to develop new instruments to cover farmers' incomes against losses due to meteorological factors. Some protective technologies that farmers can use for climate risk management have associated costs that vary as a function of the losses involved. These sorts of instruments compete with other less flexible instruments such as crop insurance. We here analyse an issue of decision-making, where the farmer can decide how much to invest in protection, as in situations where the farmer chooses which portion of a loss to protect in the case of adverse weather conditions, and we propose optimal management to mitigate the increasing negative effects of climate uncertainty. By analysing the optimal policy in a continuous choice situation, we consider whether farmers, as part of their crop management duties, should opt to protect some portion of their harvest value with available technologies, or whether they should protect the entire crop. To analyse this decision-making problem, we employ the cost-loss ratio model and take risk aversion into account.

### **Keywords:**

Crop yield protection, climate risks, information value, cost-loss ratio, decision models

**JEL Classification:** Q00, C44, Q54

## 1 Introduction

Climate change is already happening and will continue to occur even if global greenhouse gas emissions are curtailed. There is now concern that global warming has the potential to affect the climatic regimes of entire regions (IPCC, 2014). Many studies document the implications of climate change for agriculture and show that effects vary among regions and at different scales (global, regional and local) (IPCC, 2014, 2007; GHF, 2010; González-Zeas, 2014; Quiroga *et al.*, 2009). The economic effects also vary between regions (IPCC 2014, Parry *et al.*, 2004), but in general they highlight the increasing vulnerability of farmers' incomes (Quiroga *et al.*, 2015). On the other hand, among the changes to be faced by the European Union agriculture sector in coming years, the so-called CAP Health Check could give rise to significantly higher variability in farmers' incomes (Quiroga *et al.*, 2016). The degree of attention being paid to the behaviour of agricultural producers operating under conditions of risk has been increased by the progressive liberalization of world agricultural markets (Hope and Lingard, 1992; Quiroga and Suarez, 2016; Morss *et al.*, 2010).

In this context, the insurance industry is faced with the development of new instruments in order to protect farmers' incomes from climate risks (Anwar *et al.*, 2013; Iglesias *et al.*, 2012). Moreover, some protective technologies that farmers can use for climate risk management have associated costs which vary as a function of the loss of value involved, and these kinds of instrument may be competitive with less flexible insurance mechanisms.

We here analyse a problem of decision-making where the farmer can decide how much to invest in protection, covering situations in which farmers choose the proportion of the loss that they may avoid in the event of adverse weather conditions. The aim of the paper is to propose optimal management decisions in this context, in order to mitigate the increasing negative effects of climate change. Information about climate and weather and risk aversion are key determinants for an optimal decision in this type of problem (Cerdá and Quiroga, 2010; Cerdá and Quiroga, 2011). By analysing the optimal policy in a continuous choice situation, we consider whether farmers, as an aspect of crop management, could better protect a portion of their harvest value with an available technology, or whether it might be optimal to protect the whole crop by purchasing fixed insurance, or else using that protection across the entire harvest value. To analyse this decision-making problem, we propose to employ the cost-loss ratio approach, already widely employed in the literature in assessing the economic value of weather forecasts (Katz, 1993; Katz and Murphy, 1997; Palmer, 2002; Katz and Ehrendorfer, 2006; Cerdá and Quiroga, 2015).

As far as the quality of meteorological information is concerned, recent improvements in atmospheric observational technology, methods of data assimilation, numerical model formulation, and the use of ensemble techniques have led to substantial increases in forecasting skill (Bauer *et al.*, 2015; Shapiro and Thorpe, 2004). However, despite these improvements, limitations persist in the ability to forecast high-impact weather events, and decision-making sometimes requires the use of such forecasts to minimize an expected expense. It should be noted that many users remain aware of the uncertainty

attributable to the forecast probability (Cerdá and Quiroga, 2015; Katz and Ehrendorfer, 2006), and the attitude towards risk is crucial to finding the optimal policy; thus, we analyse the results in terms of risk-aversion considerations.

The paper proceeds as follows: Section 2 provides the conceptual framework for our approach; Sections 3 and 4 describe the results of the optimal decision-making policy for risk-neutral and risk-averse behaviour, respectively; and discussion and conclusions are presented in Section 5. Finally, formal solutions to some of the optimization problems are provided in Annexes A and B.

## 2 The model

The cost-loss ratio is a decision-making approach widely analyzed in the literature for assessing the economic value of weather forecasts (Katz, 1993; Palmer, 2002; Katz and Ehrendorfer, 2006; Cerdá and Quiroga, 2015). The model involves two possible actions, to protect ( $\alpha = 1$ ) or not to protect ( $\alpha = 0$ ), and two possible events, adverse weather ( $\theta = 1$ ) and non-adverse weather ( $\theta = 0$ ).

The decision maker is assumed to incur a cost  $C > 0$  if protective action is taken, a loss  $L > 0$  if protective action is not taken and adverse weather occurs, and otherwise no cost or loss. We consider a variation of this prototype model to introduce the possibility of analyzing the farmer's decision when given the possibility of protecting some share  $\alpha$  of the total harvest value  $L$ . The type of protection chosen could avoid physical loss (i.e. applying protective technology to some plants) or simply provide an economic compensation (i.e. purchasing insurance for part of the crop). In both cases, we consider that the farmer can decide the proportion of loss protected from adverse weather, so:  $\alpha \in [0, 1]$ .

Protecting a part of the loss  $K = \alpha L$  has a positive associated cost, which we consider a proportion of the avoided loss  $\gamma K = \gamma \alpha L$ , where  $0 < \gamma < 1$ . We consider the common assumptions of the familiar prototype usually referred to as the cost-loss ratio situation (Murphy and Ehrendorfer, 1987, Katz, 1993), and a summary of the model structure is shown in Table 1.

**Table 1 Model structure summary**

Two states of nature	adverse weather $\theta = 1$ or non-adverse $\theta = 0$
Infinite possible actions	protect $\alpha \in [0, 1]$
Protection cost	$\gamma \alpha L$
Loss value in the event of adverse weather	$L - \alpha L$ (non avoided loss value)

The expense value expressions associated with each action and state of nature are presented in Table 2. The decision-maker is assumed to incur a cost  $(\gamma\alpha L + (1-\alpha)L)$  if protective action is taken over a portion  $\alpha$  of the loss and adverse weather occurs, and a cost  $(\gamma\alpha L)$  if the same portion is protected but no adverse weather occurs.

**Table 2 Expense matrix**

Action	State of nature	
	Adverse weather ( $\theta = 1$ )	Non- adverse weather ( $\theta = 0$ )
To protect $\alpha$	$\gamma\alpha L + L - \alpha L = \gamma\alpha L + (1-\alpha)L$	$\gamma\alpha L$

As mentioned above, in the following sections we consider two different risk approaches:

- A risk-neutral agent
- A risk-averse agent

### 3 Optimal level of protection for a risk-neutral farmer

#### 3.1 Climatological information

Climatological (or prior) information consists of probability of adverse weather,  $P_\theta = \Pr\{\theta = 1\}$  based on statistical and historical information which the decision-maker can receive each day. When this is the sole information that farmers have available, the decision to protect a portion  $\alpha \in [0, 1]$  of the loss value incurs for them the following expected expense:

$$E(\alpha) = P_\theta [\gamma\alpha L + (1-\alpha)L] + (1-P_\theta) [\gamma\alpha L] = P_\theta (1-\alpha)L + \gamma\alpha L.$$

Under risk-neutral behaviour, the farmer will minimize the expected expense, so he or she will choose  $\alpha \in [0, 1]$  such as:

$$\begin{aligned} & \underset{\alpha}{\text{Min}} \{ \alpha L [\gamma - P_\theta] \} \\ \text{s.t.} \quad & \alpha \in [0, 1] \end{aligned}$$

So the optimal policy would be:

$$\begin{aligned} \alpha^* &= 0 & \text{if } [\gamma - P_\theta] > 0 & \Leftrightarrow P_\theta < \gamma \\ \alpha^* &= 1 & \text{if } [\gamma - P_\theta] < 0 & \Leftrightarrow P_\theta > \gamma \end{aligned}$$

If  $P_\theta = \gamma$ ,  $E(\alpha) = 0, \forall \alpha \in [0,1]$  then the farmer has the same utility whatever the value of  $\alpha$ , including  $\alpha = 0$  and  $\alpha = 1$ . Accordingly, depending on the cost of protection and the adverse weather probability, the farmer should protect the overall harvest or none of it, so he or she should never purchase an insurance covering a portion of the loss value, the result being the same obtained in the literature when mid-way levels of protection are not an option. (Murphy *et al.*, 1985).

### 3.2 Imperfect information

Next, we analyze the decision where the farmer has access to an imperfect information system. An imperfect forecast is assumed to consist of the random variable  $Z$ , as a forecast of adverse weather ( $Z = 1$ ) or non-adverse weather ( $Z = 0$ ). As in Murphy *et al.* (1985), we consider the following conditional (or ex-post) probabilities of adverse weather:  $P_1 = \Pr\{\theta = 1|Z = 1\}$ ,  $P_0 = \Pr\{\theta = 1|Z = 0\}$ , so two parameters describing basic characteristics of the forecast must be specified in order to determine forecast quality alone (Murphy and Ehrendorfer, 1987). Without loss of generality, the following ordering is assumed:  $0 \leq P_0 \leq P_\theta \leq P_1$  (Katz and Murphy, 1997). If the farmer has access to this imperfect forecast system, the ex-post expected utility in the case of choosing the protected portion  $\alpha$  of the harvest value, will be:

- If  $Z=1$ :

$$E(\alpha^1) = P_1[\gamma\alpha^1 L + (1-\alpha^1)L] + (1-P_1)[\gamma\alpha^1 L]$$

- If  $Z=0$ :

$$E(\alpha^0) = P_0[\gamma\alpha^0 L + (1-\alpha^0)L] + (1-P_0)[\gamma\alpha^0 L],$$

Where:

$\alpha^1$  is the portion of loss value that is chosen to protect if  $Z$  takes the value 1,

$\alpha^0$  is the portion of loss value that is chosen to protect if  $Z$  takes the value 0.

Therefore, if the farmer takes into consideration the imperfect forecast, he or she would protect the fraction  $\alpha^1$  of the total loss if  $Z=1$ , and a different proportion  $\alpha^0$  if  $Z=0$ . In this case, the ex-ante expected utility would be:

$$E(\alpha^0, \alpha^1) = P_\theta \left\{ P_1 [\gamma\alpha^1 L + (1-\alpha^1)L] + (1-P_1) [\gamma\alpha^1 L] \right\} + (1-P_\theta) \left\{ P_0 [\gamma\alpha^0 L + (1-\alpha^0)L] + (1-P_0) [\gamma\alpha^0 L] \right\}$$

Therefore, the farmer that chooses the portions  $\alpha^0, \alpha^1 \in [0,1]$  that minimize the expected expense should solve the following problem:

$$\begin{aligned} \text{Min}_{\{\alpha^0, \alpha^1\}} E(\alpha^0, \alpha^1) &= P_\theta \left\{ P_1 [\gamma \alpha^1 L + (1 - \alpha^1) L] + (1 - P_1) [\gamma \alpha^1 L] \right\} + \\ &+ (1 - P_\theta) \left\{ P_0 [\gamma \alpha^0 L + (1 - \alpha^0) L] + (1 - P_0) [\gamma \alpha^0 L] \right\} \\ \text{s.t.} \quad & -\alpha^0 \leq 0 \\ & \alpha^0 - 1 \leq 0 \\ & -\alpha^1 \leq 0 \\ & \alpha^1 - 1 \leq 0. \end{aligned}$$

The  $\bar{\alpha}$  optimal solution of  $\text{Min}_{\bar{\alpha}} h(\bar{\alpha})$  is the same as the  $\bar{\alpha}$  solving  $\text{Max}_{\bar{\alpha}} \{-h(\bar{\alpha})\} = f(\bar{\alpha})$ , so this problem has the following general structure:  $\text{Max}_{\bar{\alpha}} f(\bar{\alpha})$  restricted to  $g_1(\bar{\alpha}) \leq 0$ ,  $g_2(\bar{\alpha}) \leq 0$ ,  $g_3(\bar{\alpha}) \leq 0$ ,  $g_4(\bar{\alpha}) \leq 0$  where  $f(\bar{\alpha}) = -E(\bar{\alpha})$ ,  $g_1(\bar{\alpha}) = -\alpha^0$ ,  $g_2(\bar{\alpha}) = \alpha^0 - 1$ ,  $g_3(\bar{\alpha}) = -\alpha^1$ ,  $g_4(\bar{\alpha}) = \alpha^1 - 1$ , being  $\bar{\alpha} = (\alpha^0, \alpha^1)$ .

Kuhn-Tucker conditions, which are necessary conditions for local optimality, are in this case:

$$\text{KT1) } \nabla f(\bar{\alpha}) + \lambda_1 \nabla g_1(\bar{\alpha}) + \lambda_2 \nabla g_2(\bar{\alpha}) + \lambda_3 \nabla g_3(\bar{\alpha}) + \lambda_4 \nabla g_4(\bar{\alpha}) = (0, 0), \text{ that is:}$$

$$-(1 - P_\theta) P_0 \gamma L + (1 - P_\theta) P_0 L - (1 - P_\theta)(1 - P_0) \gamma L - \lambda_1 + \lambda_2 = 0$$

$$-P_\theta P_1 \gamma L + P_\theta P_1 L - P_\theta(1 - P_1) \gamma L - \lambda_3 + \lambda_4 = 0$$

$$\text{KT2) } \lambda_1 \leq 0, \lambda_2 \leq 0, \lambda_3 \leq 0, \lambda_4 \leq 0$$

$$\text{KT3) } g_1(\bar{\alpha}) \leq 0, g_2(\bar{\alpha}) \leq 0, g_3(\bar{\alpha}) \leq 0, g_4(\bar{\alpha}) \leq 0$$

$$\text{KT4) } \lambda_1 g_1(\bar{\alpha}) = 0, \lambda_2 g_2(\bar{\alpha}) = 0, \lambda_3 g_3(\bar{\alpha}) = 0, \lambda_4 g_4(\bar{\alpha}) = 0,$$

where  $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ , are the multipliers associated to the respective restrictions  $g_1(\bar{\alpha}) \leq 0$ ,  $g_2(\bar{\alpha}) \leq 0$ ,  $g_3(\bar{\alpha}) \leq 0$  and  $g_4(\bar{\alpha}) \leq 0$ .

To apply the Kunt-Tucker conditions, we begin for the KT4) condition, which sources the following possibilities:

$$\text{P1) } \lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 = 0$$

$$\text{P2) } \lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0, \alpha^1 = 1$$

$$\text{P3) } \lambda_1 = 0, \lambda_2 = 0, \alpha^1 = 0, \lambda_4 = 0$$

$$\text{P4) } \lambda_1 = 0, \alpha^0 = 1, \lambda_3 = 0, \lambda_4 = 0$$

$$\text{P5) } \alpha^0 = 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 = 0$$

P6)  $\lambda_1 = 0, \lambda_2 = 0, \alpha^1 = 0, \alpha^1 = 1$  (which is not possible since  $\alpha^1 = 0$  and  $\alpha^1 = 1$  are incompatible).

P7)  $\lambda_1 = 0, \alpha^0 = 1, \lambda_3 = 0, \alpha^1 = 1$

P8)  $\alpha^0 = 0, \lambda_2 = 0, \lambda_3 = 0, \alpha^1 = 1$

P9)  $\lambda_1 = 0, \alpha^0 = 1, \alpha^1 = 0, \lambda_4 = 0$

P10)  $\alpha^0 = 0, \lambda_2 = 0, \alpha^1 = 0, \lambda_4 = 0$

P11)  $\alpha^0 = 0, \alpha^0 = 1, \lambda_3 = 0, \lambda_4 = 0$  (which is not possible since  $\alpha^0 = 0$  and  $\alpha^0 = 1$  are incompatible).

P12)  $\alpha^0 = 0, \alpha^0 = 1, \alpha^1 = 0, \alpha^1 = 1$  (also incompatible).

Assuming we are in P1) we have  $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 = 0$ .

In such a situation, the KT1) condition can be written as  $\nabla f(\bar{\alpha}) = (0, 0)$ , so the following conditions have to be satisfied:

$$-(1 - P_\theta)P_0\gamma L + (1 - P_\theta)P_0L - (1 - P_\theta)(1 - P_0)\gamma L = 0 \Rightarrow \gamma = P_0,$$

$$-P_\theta P_1\gamma L + P_\theta P_1L - P_\theta(1 - P_1)\gamma L = 0 \Rightarrow \gamma = P_1.$$

Then if  $\gamma = P_0 = P_1$ , whatever decision  $\alpha^0, \alpha^1 \in [0, 1]$  is the optimal policy.

Now, if we suppose we are in P2):  $\lambda_1 = \lambda_2 = \lambda_3 = 0, \alpha^1 = 1$ .

The KT1) condition can be written as:  $-(1 - P_\theta)P_0\gamma L + (1 - P_\theta)P_0L - (1 - P_\theta)(1 - P_0)\gamma L = 0 \Rightarrow \gamma = P_0$

Furthermore:

$$-P_\theta P_1\gamma L + P_\theta P_1L - P_\theta(1 - P_1)\gamma L + \lambda_4 = 0 \Rightarrow \lambda_4 = P_\theta P_1\gamma L - P_\theta P_1L + P_\theta(1 - P_1)\gamma L$$

To satisfy KT2) necessarily  $\lambda_4 \leq 0$ , so:  $P_\theta P_1\gamma L - P_\theta P_1L + P_\theta(1 - P_1)\gamma L \leq 0 \Rightarrow P_1 \geq \gamma$ . Therefore, when  $\gamma = P_0 \leq P_1$  the optimal solution is  $\alpha^1 = 1$  and the farmer is indifferent to what  $\alpha^0 \in [0, 1]$ .

If we are in P3):  $\lambda_1 = \lambda_2 = \lambda_4 = 0, \alpha^1 = 0$ , to satisfy the KT1) condition:

$$-(1 - P_\theta)P_0\gamma L + (1 - P_\theta)P_0L - (1 - P_\theta)(1 - P_0)\gamma L = 0 \Rightarrow \gamma = P_0$$

and:  $-P_\theta P_1\gamma L + P_\theta P_1L - P_\theta(1 - P_1)\gamma L - \lambda_3 = 0 \Rightarrow \lambda_3 = -P_\theta P_1\gamma L + P_\theta P_1L - P_\theta(1 - P_1)\gamma L$ .

To satisfy KT2) we necessarily have  $\lambda_3 \leq 0$ , which is possible if and only if:

$$-P_\theta P_1\gamma L + P_\theta P_1L - P_\theta(1 - P_1)\gamma L \leq 0 \Rightarrow \gamma \geq P_1. \text{ But } P_1 > P_0 \text{ is clearly impossible because } P_0 \leq P_1, \text{ so}$$

P3) offers no other feasible solution.

Now, if we consider P4):  $\lambda_1 = \lambda_3 = \lambda_4 = 0$ ,  $\alpha^0 = 1$ , the KT1) condition implies that:

$$-P_\theta P_1 \gamma L + P_\theta P_1 L - P_\theta (1 - P_1) \gamma L = 0 \Rightarrow \gamma = P_1, \text{ and also:}$$

$$-(1 - P_\theta) P_0 \gamma L + (1 - P_\theta) P_0 L - (1 - P_\theta)(1 - P_0) \gamma L + \lambda_2 = 0 \Rightarrow$$

$$\Rightarrow \lambda_2 = (1 - P_\theta) P_0 \gamma L - (1 - P_\theta) P_0 L + (1 - P_\theta)(1 - P_0) \gamma L$$

The KT2) condition implies that:

$$\lambda_2 \leq 0 \Leftrightarrow (1 - P_\theta) P_0 \gamma L - (1 - P_\theta) P_0 L + (1 - P_\theta)(1 - P_0) \gamma L \leq 0 \Rightarrow P_0 \geq \gamma$$

At the same time we have that  $P_0 \leq P_1$ , which is unfeasible when  $\gamma = P_1$ , so once more P4) does not offer any feasible solution.

Taking into account P5):  $\lambda_2 = \lambda_3 = \lambda_4 = 0$ ,  $\alpha^0 = 0$ , to satisfy KT1) it is necessarily:

$$-P_\theta P_1 \gamma L + P_\theta P_1 L - P_\theta (1 - P_1) \gamma L = 0 \Rightarrow \gamma = P_1, \text{ and also:}$$

$$-(1 - P_\theta) P_0 \gamma L + (1 - P_\theta) P_0 L - (1 - P_\theta)(1 - P_0) \gamma L = \lambda_1.$$

The KT2) condition implies that:

$$\lambda_1 \leq 0 \Leftrightarrow -(1 - P_\theta) P_0 \gamma L + (1 - P_\theta) P_0 L - (1 - P_\theta)(1 - P_0) \gamma L \leq 0 \Rightarrow \gamma \geq P_0.$$

Therefore, if  $P_0 < \gamma$ , then  $\alpha^0 = 0$  and the agent is indifferent among any proportion  $\alpha^1$ .

Dealing with P7):  $\lambda_1 = \lambda_3 = 0$ ,  $\alpha^0 = 1$ ,  $\alpha^1 = 1$ , the KT1) condition can be written as:

$$-P_\theta P_1 \gamma L + P_\theta P_1 L - P_\theta (1 - P_1) \gamma L + \lambda_4 = 0$$

From KT2) we have  $\lambda_4 \leq 0$ , which implies:

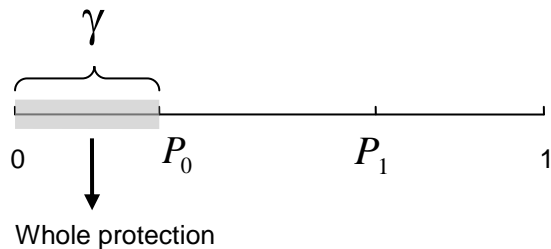
$$P_\theta P_1 \gamma L - P_\theta P_1 L + P_\theta (1 - P_1) \gamma L \leq 0 \Rightarrow \gamma \leq P_1, \text{ and also:}$$

$$-(1 - P_\theta) P_0 \gamma L + (1 - P_\theta) P_0 L - (1 - P_\theta)(1 - P_0) \gamma L + \lambda_2 = 0$$

The KT2) condition implies that:

$$\lambda_2 \leq 0 \Leftrightarrow (1 - P_\theta) P_0 \gamma L - (1 - P_\theta) P_0 L + (1 - P_\theta)(1 - P_0) \gamma L \leq 0 \Rightarrow \gamma \leq P_0.$$

So when  $\gamma \leq P_0 \Rightarrow \alpha^0 = 1$  and  $\alpha^1 = 1$ . The optimal policy is to protect the whole harvest regardless of the forecast received. (See Figure 1).



**Figure 1** Range of values where whole protection is optimal



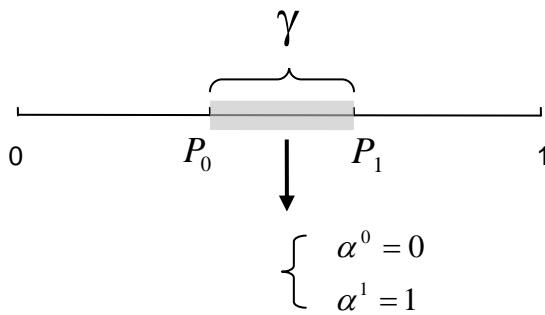
In P8):  $\lambda_2 = \lambda_3 = 0$ ,  $\alpha^0 = 0$ ,  $\alpha^1 = 1$ , the KT1) condition entails:

$$-P_\theta P_1 \gamma L + P_\theta P_1 L - P_\theta (1 - P_1) \gamma L + \lambda_4 = 0, \text{ and from KT2) we have } \lambda_2 \leq 0 \text{ and } \lambda_4 \leq 0, \text{ so:}$$

$$P_\theta P_1 \gamma L - P_\theta P_1 L + P_\theta (1 - P_1) \gamma L \leq 0 \Rightarrow \gamma \leq P_1, \text{ and also:}$$

$$-(1 - P_\theta) P_0 \gamma L + (1 - P_\theta) P_0 L - (1 - P_\theta) (1 - P_0) \gamma L \leq 0 \Rightarrow \gamma \geq P_0$$

Therefore, when  $P_0 \leq \gamma \leq P_1 \Rightarrow \alpha^0 = 0$  y  $\alpha^1 = 1$ ; that is, the agent should protect all the harvest value if an adverse weather forecast is received, and take no protective action in any other case. (See Figure 2).



**Figure 2 Range of values where partial protection is optimal**

If we are in P9):  $\lambda_1 = \lambda_4 = 0$ ,  $\alpha^0 = 1$ ,  $\alpha^1 = 0$ , the KT1) condition can be written as:

$$-(1 - P_\theta) P_0 \gamma L + (1 - P_\theta) P_0 L - (1 - P_\theta) (1 - P_0) \gamma L + \lambda_2 = 0, \text{ and also:}$$

$$-P_\theta P_1 \gamma L + P_\theta P_1 L - P_\theta (1 - P_1) \gamma L - \lambda_3 = 0$$

In addition, from KT2) we have  $\lambda_2 \leq 0$  and  $\lambda_3 \leq 0$ , which implies  $\gamma \leq P_0$  and  $\gamma \geq P_1$  at the same time. However, since  $P_0 \leq P_1$ , this is impossible and P9) offers no feasible solution.

Finally, if we consider P10):  $\lambda_2 = \lambda_4 = 0$ ,  $\alpha^0 = 0$ ,  $\alpha^1 = 0$ , the KT1) condition implies:

$$-(1 - P_\theta) P_0 \gamma L + (1 - P_\theta) P_0 L - (1 - P_\theta) (1 - P_0) \gamma L - \lambda_1 = 0, \text{ and also:}$$

$$-P_\theta P_1 \gamma L + P_\theta P_1 L - P_\theta (1 - P_1) \gamma L - \lambda_3 = 0$$

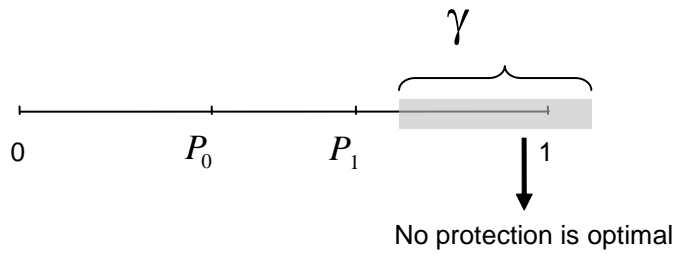
Moreover, from KT2) we have  $\lambda_1 \leq 0$ ,  $\lambda_3 \leq 0$ , which is satisfied if and only if:

$$-(1 - P_\theta) P_0 \gamma L + (1 - P_\theta) P_0 L - (1 - P_\theta) (1 - P_0) \gamma L \leq 0, \text{ and also:}$$

$$-P_\theta P_1 \gamma L + P_\theta P_1 L - P_\theta (1 - P_1) \gamma L \leq 0$$

Thus implying  $\gamma \geq P_0$  and  $\gamma \geq P_1$ .

So, given that  $P_0 \leq P_1$  and furthermore:  $\gamma > P_1$  the resultant solution is:  $\alpha^0 = 0$  and  $\alpha^1 = 0$ . Therefore in this case the optimal policy is to not protect the harvest (see Figure 3).

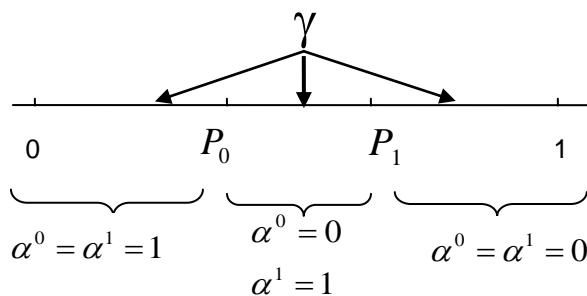


**Figure 3 Range of values where no protection is optimal**

So the optimal policy solving the problem (1) is:

1. If  $\gamma < P_0 \Rightarrow \alpha^0 = 1$  and  $\alpha^1 = 1$ . The optimal policy is to protect the overall harvest whatever the forecast received.
2. If  $\gamma = P_0 < P_1 \Rightarrow \alpha^1 = 1$  and any  $\alpha^0 \in [0,1]$  is an optimal solution.
3. If  $P_0 < \gamma < P_1 \Rightarrow \alpha^0 = 0$  and  $\alpha^1 = 1$ . The farmer should protect the overall harvest value if an adverse weather forecast is received, but take no protective action in any other case.
4. If  $P_0 < \gamma = P_1 \Rightarrow \alpha^0 = 0$  and any  $\alpha^1 \in [0,1]$  is an optimal solution.
5. If  $\gamma > P_1 \Rightarrow \alpha^0 = 0$   $\alpha^1 = 0$ . The farmer should take no protection whatever the forecast received.

Figure 4 summarizes the optimal policy.



**Figure 4 Optimal policy in the case of imperfect information and risk-neutral behaviour**

Therefore, a risk-neutral farmer should never prefer an intermediate level of protection, and the optimal policy is to protect the overall harvest value (or contrarily to take no protective action), but never to

choose an intermediate protective level, whatever the kind of weather information received (climatological or an imperfect forecast).

#### 4 Optimal level of protection for a risk-averse farmer

In this section we seek to prove that a farmer's optimal policy will be different when we consider risk behaviour; in particular, we expect that in this case, there is a chance of the farmer selecting an intermediate level of protection as the optimal solution. In the case of a risk-averse agent, he or she will maximize the expected utility. Here, we consider the CARA (Constant Absolute Risk Aversion) utility function as representative of agents' preferences,  $U(x) = -\exp\{-\rho x\}$ , where the Arrow-Pratt absolute risk aversion coefficient ( $\rho$ ) can be mathematically calculated as follows (Mas-Collel *et al.*, 1995):

$$\rho(X) = -\frac{U''(X)}{U'(X)}$$

This coefficient can be interpreted as the percentage change in marginal utility caused by each monetary unit of a gain or loss. If the coefficient does not change across the monetary level, the decision-maker exhibits constant absolute risk aversion (CARA), which implies that the level of the utility function argument does not affect his or her decisions under uncertainty. Since  $\rho$  cannot be considered as a non-dimensional measure of risk aversion, its value is dependent on the currency in which the monetary units are expressed (Gómez-Limón *et al.*, 2003), making the comparison among different economic agents difficult. However, it remains a good measure for decision-making problems involving a single economic agent.

##### 4.1 Climatological Information

In this case, expected utility when the portion of the loss value  $\alpha \in [0, 1]$  is preserved, will be:

$$\begin{aligned} UE(\alpha) &= P_\theta U[-\gamma\alpha L + (1-\alpha)L] + (1-P_\theta)U[-\gamma\alpha L] = \\ &= P_\theta U[-\gamma\alpha L - (1-\alpha)L] + (1-P_\theta)U[-\gamma\alpha L], \text{ assuming the CARA utility function can be written as} \\ &= P_\theta [-\exp\{-\rho[-\gamma\alpha L - (1-\alpha)L]\}] + (1-P_\theta)[- \exp\{-\rho[-\gamma\alpha L]\}] = \\ &= \exp\{\rho\gamma\alpha L\} [-P_\theta \exp\{\rho(1-\alpha)L\} - (1-P_\theta)]. \end{aligned}$$

So, the farmer should choose  $\alpha \in [0, 1]$  following:

$$\begin{aligned} \text{Max}_{\alpha} \text{UE}(\alpha) &= \exp\{\rho\gamma\alpha L\} \left[ -P_{\theta} \exp\{\rho(1-\alpha)L\} - (1-P_{\theta}) \right] \\ \text{s.a.} \quad &-\alpha \leq 0 \\ &\alpha - 1 \leq 0. \end{aligned}$$

We prove in Annex A that an internal solution to this problem exists, and is characterized by the expression (2):

$$\text{If } 1 < \frac{(1-P_{\theta})\gamma}{P_{\theta}(1-\gamma)} < \exp\{\rho L\} \Rightarrow \alpha^* = 1 - \left[ \frac{\ln \left\{ \frac{(1-P_{\theta})\gamma}{P_{\theta}(1-\gamma)} \right\}}{\rho L} \right] \in (0, 1) \quad (2)$$

Therefore, there is an interval for which a farmer with aversion to risky behaviour will obtain a greater utility if able to decide which portion of his or her harvest value to protect from climatic risk. We also obtain that, in the cases in which a risk-neutral agent decides to protect all of a harvest, (if  $P_{\theta} \geq \gamma$ ), a risk-averse agent should take the same decision. However, risk-averse behaviour is more preservative, and there are cases in which risk-neutral agents should protect none of the harvest while risk-averse agents should protect some part of it.

## 4.2 Imperfect information

Now we analyze the decision if a risk-averse farmer has access to some imperfect information system. In this case, the ex-post expected utility in the case of choosing the protected portion  $\alpha$  of the harvest value will be:

- If  $Z=1$ :

$$\text{UE}(\alpha^1) = P_1 U \left[ -[\gamma\alpha^1 L + (1-\alpha^1)L] \right] + (1-P_1) U \left[ -[\gamma\alpha^1 L] \right]$$

- If  $Z=0$ :

$$\text{UE}(\alpha^0) = P_0 U \left[ -[\gamma\alpha^0 L + (1-\alpha^0)L] \right] + (1-P_0) U \left[ -[\gamma\alpha^0 L] \right],$$

The ex-ante expected utility would be:

$$\begin{aligned} \text{UE}(\alpha^0, \alpha^1) &= P_{\theta} \left\{ P_1 U \left[ -[\gamma\alpha^1 L + (1-\alpha^1)L] \right] + (1-P_1) U \left[ -[\gamma\alpha^1 L] \right] \right\} + \\ &+ (1-P_{\theta}) \left\{ P_0 U \left[ -[\gamma\alpha^0 L + (1-\alpha^0)L] \right] + (1-P_0) U \left[ -[\gamma\alpha^0 L] \right] \right\} = \\ &= P_{\theta} \left\{ P_1 \left[ -\exp\{\rho[\gamma\alpha^1 L + (1-\alpha^1)L]\} \right] + (1-P_1) \left[ -\exp\{\rho[\gamma\alpha^1 L]\} \right] \right\} + \\ &+ (1-P_{\theta}) \left\{ P_0 \left[ -\exp\{\rho[\gamma\alpha^0 L + (1-\alpha^0)L]\} \right] + (1-P_0) \left[ -\exp\{\rho[\gamma\alpha^0 L]\} \right] \right\} = \end{aligned}$$

$$= -P_\theta P_1 \exp\{\rho\gamma\alpha^1 L + \rho(1-\alpha^1)L\} - P_\theta(1-P_1) \exp\{\rho\gamma\alpha^1 L\} - \\ -(1-P_\theta)P_0 \exp\{\rho\gamma\alpha^0 L + \rho(1-\alpha^0)L\} - (1-P_\theta)(1-P_0) \exp\{\rho\gamma\alpha^0 L\}.$$

Therefore, the farmer that chooses those portions  $\alpha^0, \alpha^1 \in [0,1]$  that maximise the expected utility should solve the following problem:

$$\begin{aligned} \text{Max}_{\{\alpha^0, \alpha^1\}} \text{UE}(\alpha^0, \alpha^1) = & -P_\theta P_1 \exp\{\rho\gamma\alpha^1 L + \rho(1-\alpha^1)L\} - P_\theta(1-P_1) \exp\{\rho\gamma\alpha^1 L\} - \\ & -(1-P_\theta)P_0 \exp\{\rho\gamma\alpha^0 L + \rho(1-\alpha^0)L\} - (1-P_\theta)(1-P_0) \exp\{\rho\gamma\alpha^0 L\} \\ \text{s.t.} \quad & -\alpha^0 \leq 0 \\ & \alpha^0 - 1 \leq 0 \\ & -\alpha^1 \leq 0 \\ & \alpha^1 - 1 \leq 0. \end{aligned}$$

In Annex B we solve this problem and prove that the solution verifying the Kuhn-Tucker conditions for this problem is:

$$\bullet \alpha^{0*} = 1 - \left[ \frac{\ln\left\{\frac{(1-P_0)\gamma}{P_0(1-\gamma)}\right\}}{\rho L} \right] \text{ and } \alpha^{1*} = 1 - \left[ \frac{\ln\left\{\frac{(1-P_1)\gamma}{P_1(1-\gamma)}\right\}}{\rho L} \right] \text{ if } \begin{cases} \gamma \geq P_1 \\ \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \leq \exp\{\rho L\} \end{cases}.$$

That is, to protect a positive portion of the harvest value, even if non-adverse weather is forecast, and to preserve a higher share of this value in the case of receipt of an adverse weather forecast.

$$\bullet \alpha^{0*} = 1 - \left[ \frac{\ln\left\{\frac{(1-P_0)\gamma}{P_0(1-\gamma)}\right\}}{\rho L} \right] \text{ and } \alpha^{1*} = 1 \text{ if } \begin{cases} P_0 \leq \gamma \leq P_1 \\ \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \leq \exp\{\rho L\} \end{cases}$$

In this situation, the farmer should also protect a positive share of harvest value when receiving a non-adverse weather forecast, but should protect the whole harvest value if adverse weather is predicted.

$$\bullet \alpha^{0*} = 0 \text{ and } \alpha^{1*} = 1 - \left[ \frac{\ln\left\{\frac{(1-P_1)\gamma}{P_1(1-\gamma)}\right\}}{\rho L} \right] \text{ if } \begin{cases} \gamma \geq P_1 \\ \frac{(1-P_1)\gamma}{P_1(1-\gamma)} \leq \exp\{\rho L\} \leq \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \end{cases}.$$

The optimal policy for this interval is to not protect unless the forecast shows adverse weather.

$$\bullet \alpha^{0*} = 1 \text{ and } \alpha^{1*} = 1 \text{ if: } \gamma \leq P_0.$$

In the situation of relatively cheap protection, the optimal policy is to protect the overall harvest value, whatever the forecast received.

$$\bullet \quad \alpha^{0*} = 0 \text{ and } \alpha^{1*} = 1 \quad \text{if: } \begin{cases} P_1 \geq \gamma \\ \exp\{\rho L\} \leq \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \end{cases}$$

Therefore, the optimal policy should be to take no protection if the forecast weather is non-adverse and to protect the overall harvest if receiving an adverse weather forecast.

$$\bullet \quad \alpha^{0*} = 0 \text{ y } \alpha^{1*} = 0 \quad \text{if: } \exp\{\rho L\} \leq \frac{(1-P_1)\gamma}{P_1(1-\gamma)}$$

Thus, the optimal action is to not protect, whatever the received forecast.

## 5 Application: A case study of freeze-risk insurance in southern Spain

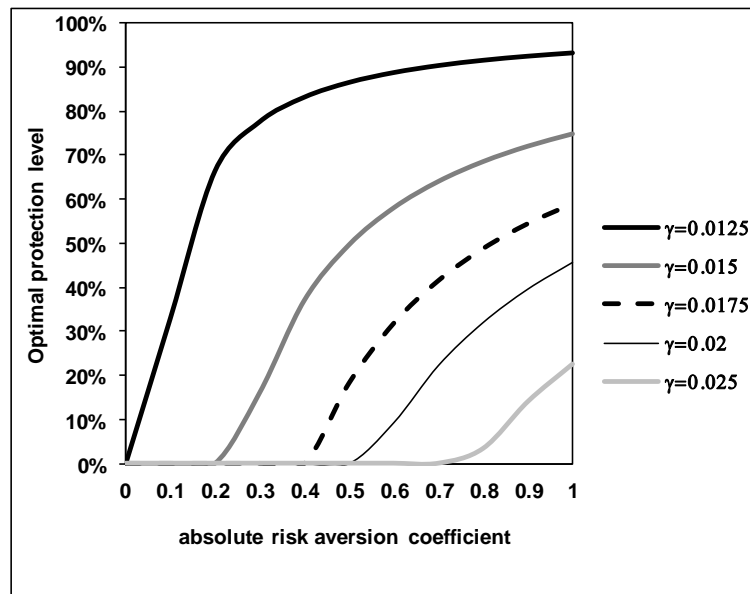
Freeze is one of the most significant risks affecting crop yields. Southern Spain is a Mediterranean climate region with continental influence. The probability of freeze is very small in this region, but when this extreme event occurs, crop yields suffer great losses. The probability of the temperature being below 0°C based on climatological information obtained at Córdoba (a location in southern Spain) for the period 1990-2000 is  $P_\theta = 0.0117$ . Currently, farmers can contract fixed insurance to protect their complete harvest, and the cost is around 2% of yield losses. (Quiroga *et al.*, 2011).

As we have demonstrated in this paper, if the probability of an extreme event is lower than the fixed cost of the risk premium, based on climatological information, farmers have no private incentives to contract the insurance that would seem compulsory in the current situation. We want to illustrate the effects of a potential, more flexible insurance that would allow farmers to protect a proportion of the harvest. In Table 3 this particular case is analysed for a range of costs for risk premium (normalized as a proportion of the loss protected). In our case study:  $\gamma = \{0.0125, 0.015, 0.0175, 0.02, 0.025\}$ ,  $L = 1$  and  $P_\theta = 0.0117$ .

This is a situation in which the minimum expected expense is achieved if the farmer does not protect the harvest. In Table 3, we observe how the proportion of harvest that should be optimal to protect is an increasing function of the absolute risk aversion coefficient of the farmers for a particular example ( $\gamma = 0.0125$ ). Figure 5 illustrates the proportion of the harvest that the farmer would be happy to protect with insurance for a range of risk premium costs as a function of the absolute risk aversion coefficient. The results show that there is a chance of establishing private incentives to purchase a more flexible form of insurance as an adaptation strategy to climate risks.

**Table 3 Proportion of harvest that should be optimal to protect ( $\alpha^*$ ) as a function of the absolute risk aversion coefficient  $\rho$ , when  $\gamma = 0.0125$ ;  $L = 1$  and  $P_\theta = 0.0117$**

$\rho$	$\alpha^*$
0.1	33.05%
0.2	66.53%
0.3	77.68%
0.4	83.26%
0.5	86.61%
0.6	88.84%
0.7	90.44%
0.8	91.63%
0.9	92.56%
1	93.31%



**Figure 5** Proportion of harvest that should be optimal to protect ( $\alpha^*$ ) as a function of the absolute risk aversion coefficient  $\rho$ , being  $\gamma = \{0.0125, 0.015, 0.0175, 0.02, 0.025\}$ ,  $L = 1$  and  $P_0 = 0.0117$

## 6 Discussion and conclusions

We have analyzed the optimal protection level in two different situations, a risk-neutral approach versus a risk-averse context. On one hand, the results show that in the case of a risk-neutral farmer, an intermediate level of protection should never be preferred, whatever the kind of weather information received (climatological or imperfect forecast).

On the other hand, in the case of risk-averse behaviour, comparing the optimal policy structure between situations in which climatological information is available and in which an imperfect forecast is available for the farmer, we observe that the information value is positive in almost all cases. However, there exist the following exceptions:

If  $\exp\{\rho L\} \leq \frac{(1-P_1)\gamma}{P_1(1-\gamma)}$ , the agent that receives a weather forecast should decide to take no protective

actions, whatever the forecast results. In addition, as  $P_\theta \leq P_1$  implies  $\frac{(1-P_1)\gamma}{P_1(1-\gamma)} \leq \frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)}$ , so that in

this interval  $\exp\{\rho L\} \leq \frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)}$  is maintained, the optimal decision in the case of climatological

information was therefore the same (not to protect). In this case, the imperfect forecast does not affect the farmers' decision, so the information has no economic value.

The same occurs if  $\gamma \leq P_0$ , (when protecting cost is low in relation to the assumed risk) the farmer should protect the overall harvest whatever the weather forecast. In addition, as  $P_0 \leq P_\theta$  always implies that  $\gamma \leq P_\theta$ , in this case the agent's decision would be the same protective behaviour if only climatological information was available; the information has no value in this case, because the information has no effect on the decision or the consequences.

Nevertheless, in the rest of situations, optimal policy depends on the meteorological information received; in fact,  $\alpha^{0*} \neq \alpha^{1*}$ , so the optimal decision is different if the forecast anticipates adverse or non-adverse weather. Therefore, the information is important since it reduces the farmer's expected expense. Consequently, the information value is positive in this situation.

In conclusion, this paper examines the situation in which a farmer can manage climate risk over a given harvest by choosing a protection level applied as a function of the associated cost. The results show that a risk-neutral agent would never choose an intermediate level of protection, but when risk-averse behaviour is taken into account, interesting management possibilities emerge. There do exist cases in which the lowest expected expense is achieved when no protection is taken, although the highest expected utility is reached when a part of the harvest is protected, and the larger the risk-aversion factor, the larger the optimal portion of loss value protected.



## 7 Annex A

$$\begin{aligned} \text{Max}_{\alpha} \text{UE}(\alpha) &= \exp\{\rho\gamma\alpha L\} \left[ -P_{\theta} \exp\{\rho(1-\alpha)L\} - (1-P_{\theta}) \right] \\ \text{s.a.} \quad &-\alpha \leq 0 \\ &\alpha - 1 \leq 0. \end{aligned}$$

The problem has the following general structure:  $\text{Max}_{\alpha} f(\alpha)$  restricted to  $g_1(\alpha) \leq 0, g_2(\alpha) \leq 0$  where

$$f(\alpha) = \text{UE}(\alpha), \quad g_1(\alpha) = -\alpha \quad \text{and} \quad g_2(\alpha) = \alpha - 1.$$

Kuhn-Tucker conditions, which are necessary conditions for local optimality, are in this case:

$$\text{KT1)} \quad f'(\alpha) + \lambda_1 g_1'(\alpha) + \lambda_2 g_2'(\alpha) = 0$$

$$\text{KT2)} \quad \lambda_1 \leq 0, \quad \lambda_2 \leq 0$$

$$\text{KT3)} \quad g_1(\alpha) \leq 0, \quad g_2(\alpha) \leq 0$$

$$\text{KT4)} \quad \lambda_1 g_1(\alpha) = 0, \quad \lambda_2 g_2(\alpha) = 0,$$

Where  $\lambda_1$  and  $\lambda_2$  are the multipliers associated to the respective restrictions  $g_1(\alpha) \leq 0$  and  $g_2(\alpha) \leq 0$ .

To apply the Kunt-Tucker conditions, we begin with the KT4) condition. Then we have the following four possibilities:

$$\text{P1)} \quad \lambda_1 = 0, \quad \lambda_2 = 0$$

$$\text{P2)} \quad \lambda_1 = 0, \quad \alpha = 1$$

$$\text{P3)} \quad \alpha = 0, \quad \lambda_2 = 0$$

$$\text{P4)} \quad \alpha = 0, \quad \alpha = 1 \quad (\text{which is obviously incompatible}).$$

If we assume we are in P1):  $\lambda_1 = 0$  y  $\lambda_2 = 0$ . In such case, the KT1) condition ends up as  $f'(\alpha) = 0$ .

That is:

$$\begin{aligned} 0 = f'(\alpha) &= \frac{d\text{UE}(\alpha)}{d\alpha} = \exp\{\rho\gamma\alpha L\} \rho\gamma L \left[ -P_{\theta} \exp\{\rho(1-\alpha)L\} - (1-P_{\theta}) \right] + \\ &+ \exp\{\rho\gamma\alpha L\} \left[ -P_{\theta} \exp\{\rho(1-\alpha)L\} (-\rho L) \right] = \exp\{\rho\gamma\alpha L\} \left[ P_{\theta} \exp\{\rho(1-\alpha)L\} [\rho L - \rho\gamma L] - (1-P_{\theta}) \rho\gamma L \right]. \end{aligned}$$

So it can be verified that:

$$f'(\alpha) = 0 \Leftrightarrow P_{\theta} \exp\{\rho(1-\alpha)L\} [\rho L - \rho\gamma L] - (1-P_{\theta}) \rho\gamma L = 0 \Leftrightarrow$$

$$\Leftrightarrow \exp\{\rho(1-\alpha)L\} = \frac{(1-P_{\theta})\rho\gamma L}{P_{\theta}\rho L(1-\gamma)} = \frac{(1-P_{\theta})\gamma}{P_{\theta}(1-\gamma)} \Leftrightarrow$$

$$\Leftrightarrow \rho(1-\alpha)L = \ln\left(\frac{(1-P_{\theta})\gamma}{P_{\theta}(1-\gamma)}\right) \Leftrightarrow 1-\alpha = \frac{\ln\left(\frac{(1-P_{\theta})\gamma}{P_{\theta}(1-\gamma)}\right)}{\rho L} \Leftrightarrow$$

$$\alpha = 1 - \frac{\ln\left(\frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)}\right)}{\rho L} = \alpha^*.$$

To satisfy the KT3) condition, it is necessarily the case that:  $0 \leq \alpha^* \leq 1$ .

Therefore, we have:

$$0 \leq \alpha^* \Leftrightarrow 0 \leq 1 - \frac{\ln\left(\frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)}\right)}{\rho L} \Leftrightarrow \ln\left(\frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)}\right) \leq \rho L \Leftrightarrow \frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)} \leq \exp\{\rho L\}.$$

$$\alpha^* \leq 1 \Leftrightarrow 1 - \frac{\ln\left(\frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)}\right)}{\rho L} \leq 1 \Leftrightarrow \ln\left(\frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)}\right) \geq 0 \Leftrightarrow \frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)} \geq 1 \Leftrightarrow \gamma - P_\theta\gamma \geq P_\theta - P_\theta\gamma \Leftrightarrow \gamma \geq P_\theta.$$

So, if  $1 \leq \frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)} \leq \exp\{\rho L\}$  is verified (note that only the first inequity can be satisfied in the case

$$\text{of } \gamma \geq P_\theta), \text{ then, the solution to the problem is } \alpha^* = 1 - \frac{\ln\left(\frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)}\right)}{\rho L}.$$

We can now consider the case in which P2) is satisfied:  $\lambda_1 = 0$  y  $\alpha = 1$ . In this case, KT1) condition ends up as  $f'(1) + \lambda_2 = 0$ , so:

$$\lambda_2 = -f'(1) = -\exp\{\rho\gamma L\} [P_\theta\rho L - P_\theta\rho\gamma L - \rho\gamma L + P_\theta\rho\gamma L] = -\exp\{\rho\gamma L\} \rho L (P_\theta - \gamma).$$

To satisfy KT2), then necessarily  $\lambda_2 \leq 0$ , which is only possible if  $P_\theta \geq \gamma$

So, if  $P_\theta \geq \gamma \Leftrightarrow 1 \geq \frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)}$ , we find that the single solution satisfying Kuhn-Tucker conditions is

$$\alpha = 1.$$

Finally, we can consider P3):  $\alpha = 0$  y  $\lambda_2 = 0$ . In this case, the KT1) condition ends up being  $f'(0) - \lambda_1 = 0$ , so:

$$\lambda_1 = f'(0) = P_\theta \exp\{\rho L\} [\rho L - \rho\gamma L] - (1 - P_\theta) \rho\gamma L.$$

To satisfy KT2) it is necessary that  $\lambda_1 \leq 0$ , which is verified if and only if

$$P_\theta \exp\{\rho L\} [\rho L - \rho\gamma L] - (1 - P_\theta) \rho\gamma L \leq 0 \Leftrightarrow \frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)} \geq \exp\{\rho L\}.$$

Consequently, the solution that verifies the Kuhn-Tucker conditions for this problem is:

$\alpha = 0$  (that is, zero protection), when  $\frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)} \geq \exp\{\rho L\}$ .

$\alpha = 1$  (that is, overall protection), when  $\frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)} \leq 1 \Leftrightarrow P_\theta \geq \gamma$ .

$\alpha = \alpha^* = 1 - \frac{\ln\left(\frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)}\right)}{\rho L}$  (protecting a positive portion of the harvest value), when

$$1 \leq \frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)} \leq \exp\{\rho L\}.$$

To tackle with sufficiency conditions for the maximization problem, we want to prove that the function  $f(\alpha) = UE(\alpha)$  is a concave function:

$$\begin{aligned} f''(\alpha) &= \exp\{\rho\gamma\alpha L\}(\rho\gamma L) \left[ P_\theta \exp\{\rho(1-\alpha)L\} [\rho L - \rho\gamma L] - (1-P_\theta)\rho\gamma L \right] + \\ &+ \exp\{\rho\gamma\alpha L\} \left[ P_\theta [\rho L - \rho\gamma L] (-\rho L) \exp\{\rho(1-\alpha)L\} \right] = \\ &= \underbrace{\exp\{\rho\gamma\alpha L\}}_{positivo} \left[ \underbrace{P_\theta \exp\{\rho(1-\alpha)L\} [\rho L - \rho\gamma L]}_{positivo} \underbrace{[\rho\gamma L - \rho L]}_{negativo} \right] - \underbrace{\exp\{\rho\gamma\alpha L\} (\rho\gamma L)^2 (1-P_\theta)}_{positivo} < 0. \end{aligned}$$

Therefore, the objective function of this maximization program is concave, and the available set is convex, so we have a convex program, so the Kuhn-Tucker conditions are at the same time necessary and sufficient conditions for global optimality and the obtained solution verifying Kuhn-Tucker conditions is a global optimum for the problem.

## 8 Annex B

$$\begin{aligned} \text{Max}_{\{\alpha^0, \alpha^1\}} \text{UE}(\alpha^0, \alpha^1) &= -P_\theta P_1 \exp\{\rho\gamma\alpha^1 L + \rho(1-\alpha^1)L\} - P_\theta(1-P_1) \exp\{\rho\gamma\alpha^1 L\} - \\ &\quad -(1-P_\theta)P_0 \exp\{\rho\gamma\alpha^0 L + \rho(1-\alpha^0)L\} - (1-P_\theta)(1-P_0) \exp\{\rho\gamma\alpha^0 L\} \\ \text{s.a.} \quad &-\alpha^0 \leq 0 \\ &\alpha^0 - 1 \leq 0 \\ &-\alpha^1 \leq 0 \\ &\alpha^1 - 1 \leq 0. \end{aligned}$$

The problem has the following general structure:  $\text{Max}_{\bar{\alpha}} f(\bar{\alpha})$  restricted to  $g_1(\bar{\alpha}) \leq 0$ ,  $g_2(\bar{\alpha}) \leq 0$ ,  $g_3(\bar{\alpha}) \leq 0$ ,  $g_4(\bar{\alpha}) \leq 0$  where  $f(\bar{\alpha}) = \text{UE}(\bar{\alpha})$ ,  $g_1(\bar{\alpha}) = -\alpha^0$ ,  $g_2(\bar{\alpha}) = \alpha^0 - 1$ ,  $g_3(\bar{\alpha}) = -\alpha^1$ ,  $g_4(\bar{\alpha}) = \alpha^1 - 1$ , being  $\bar{\alpha} = (\alpha^0, \alpha^1)$ .

Kuhn-Tucker conditions, which are necessary conditions for local optimality, are in this case:

$$\text{KT1) } \nabla f(\bar{\alpha}) + \lambda_1 \nabla g_1(\bar{\alpha}) + \lambda_2 \nabla g_2(\bar{\alpha}) + \lambda_3 \nabla g_3(\bar{\alpha}) + \lambda_4 \nabla g_4(\bar{\alpha}) = (0, 0), \text{ that is:}$$

$$-(1-P_\theta)P_0[\rho\gamma L - \rho L] \exp\{\rho\gamma\alpha^0 L + \rho(1-\alpha^0)L\} - (1-P_\theta)(1-P_0)(\rho\gamma L) \exp\{\rho\gamma\alpha^0 L\} - \lambda_1 + \lambda_2 = 0$$

$$-P_\theta P_1(\rho\gamma L - \rho L) \exp\{\rho\gamma\alpha^1 L + \rho(1-\alpha^1)L\} - P_\theta(1-P_1)(\rho\gamma L) \exp\{\rho\gamma\alpha^1 L\} - \lambda_3 + \lambda_4 = 0$$

$$\text{KT2) } \lambda_1 \leq 0, \lambda_2 \leq 0, \lambda_3 \leq 0, \lambda_4 \leq 0$$

$$\text{KT3) } g_1(\bar{\alpha}) \leq 0, g_2(\bar{\alpha}) \leq 0, g_3(\bar{\alpha}) \leq 0, g_4(\bar{\alpha}) \leq 0$$

$$\text{KT4) } \lambda_1 g_1(\bar{\alpha}) = 0, \lambda_2 g_2(\bar{\alpha}) = 0, \lambda_3 g_3(\bar{\alpha}) = 0, \lambda_4 g_4(\bar{\alpha}) = 0,$$

Where  $\lambda_1, \lambda_2, \lambda_3, \lambda_4$  are the multipliers associated to the respective restrictions  $g_1(\bar{\alpha}) \leq 0$ ,  $g_2(\bar{\alpha}) \leq 0$ ,  $g_3(\bar{\alpha}) \leq 0$  and  $g_4(\bar{\alpha}) \leq 0$ .

To apply the Kunt-Tucker conditions, we begin for the KT4) condition, that sources the following possibilities:

$$\text{P1) } \lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 = 0$$

$$\text{P2) } \lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0, \alpha^1 = 1$$

$$\text{P3) } \lambda_1 = 0, \lambda_2 = 0, \alpha^1 = 0, \lambda_4 = 0$$

$$\text{P4) } \lambda_1 = 0, \alpha^0 = 1, \lambda_3 = 0, \lambda_4 = 0$$

$$\text{P5) } \alpha^0 = 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 = 0$$

P6)  $\lambda_1 = 0, \lambda_2 = 0, \alpha^1 = 0, \alpha^1 = 1$  (which is not possible since  $\alpha^1 = 0$  and  $\alpha^1 = 1$  are incompatible).

P7)  $\lambda_1 = 0, \alpha^0 = 1, \lambda_3 = 0, \alpha^1 = 1$

P8)  $\alpha^0 = 0, \lambda_2 = 0, \lambda_3 = 0, \alpha^1 = 1$

P9)  $\lambda_1 = 0, \alpha^0 = 1, \alpha^1 = 0, \lambda_4 = 0$

P10)  $\alpha^0 = 0, \lambda_2 = 0, \alpha^1 = 0, \lambda_4 = 0$

P11)  $\alpha^0 = 0, \alpha^0 = 1, \lambda_3 = 0, \lambda_4 = 0$  (which is not possible since  $\alpha^0 = 0$  and  $\alpha^0 = 1$  are incompatible).

P12)  $\alpha^0 = 0, \alpha^0 = 1, \alpha^1 = 0, \alpha^1 = 1$  (also incompatible).

Assuming we are in P1):  $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 = 0$ .

In such a situation, KT1) condition can be written as:  $\nabla f(\bar{\alpha}) = (0, 0)$ , so the following has to be satisfied:

$$-(1-P_\theta)P_0 \exp\{\rho\gamma\alpha^0 L + \rho(1-\alpha^0)L\}(\rho\gamma L - \rho L) - (1-P_\theta)(1-P_0) \exp\{\rho\gamma\alpha^0 L\}(\rho\gamma L) = 0,$$

$$-P_\theta P_1 \exp\{\rho\gamma\alpha^1 L + \rho(1-\alpha^1)L\}(\rho\gamma L - \rho L) - P_\theta(1-P_1) \exp\{\rho\gamma\alpha^1 L\}(\rho\gamma L) = 0.$$

Therefore, in this situation, the optimal solution is:

$$\alpha^{0*} = 1 - \left[ \frac{\ln \left\{ \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \right\}}{\rho L} \right]$$

$$\alpha^{1*} = 1 - \left[ \frac{\ln \left\{ \frac{(1-P_1)\gamma}{P_1(1-\gamma)} \right\}}{\rho L} \right].$$

Besides, to satisfy KT3) it is necessary that:

$$0 \leq \alpha^{0*} \leq 1 \Leftrightarrow 1 \leq \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \leq \exp\{\rho L\} \Leftrightarrow \begin{cases} \gamma \geq P_0 \\ \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \leq \exp\{\rho L\} \end{cases},$$

Moreover, as well:

$$0 \leq \alpha^{1*} \leq 1 \Leftrightarrow 1 \leq \frac{(1-P_1)\gamma}{P_1(1-\gamma)} \leq \exp\{\rho L\} \Leftrightarrow \begin{cases} \gamma \geq P_1 \\ \frac{(1-P_1)\gamma}{P_1(1-\gamma)} \leq \exp\{\rho L\} \end{cases} .$$

So, as  $P_0 \leq P_1$  and therefore:  $\frac{(1-P_1)\gamma}{P_1(1-\gamma)} \leq \frac{(1-P_0)\gamma}{P_0(1-\gamma)}$ , we find that the optimal solution would be:

$$\bar{\alpha} = \left( 1 - \left[ \frac{\ln \left\{ \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \right\}}{\rho L} \right], 1 - \left[ \frac{\ln \left\{ \frac{(1-P_1)\gamma}{P_1(1-\gamma)} \right\}}{\rho L} \right] \right) \text{ if and only if: } \begin{cases} \gamma \geq P_1 \\ \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \leq \exp\{\rho L\} \end{cases} .$$

Now, if we suppose we are in P2):  $\lambda_1 = \lambda_2 = \lambda_3 = 0$ ,  $\alpha^1 = 1$ .

The KT1) condition can be written as:

$$\begin{aligned} & -(1-P_\theta)P_\theta \exp\{\rho\gamma\alpha^0 L + \rho(1-\alpha^0)L\}(\rho\gamma L - \rho L) - (1-P_\theta)(1-P_\theta) \exp\{\rho\gamma\alpha^0 L\}(\rho\gamma L) = 0 \Leftrightarrow \\ & \Leftrightarrow \alpha^{0*} = 1 - \left[ \frac{\ln \left\{ \frac{(1-P_\theta)\gamma}{P_\theta(1-\gamma)} \right\}}{\rho L} \right], \end{aligned}$$

Furthermore, as well:

$$-P_\theta P_1 \exp\{\rho\gamma L\}(\rho\gamma L - \rho L) - P_\theta(1-P_1) \exp\{\rho\gamma L\}(\rho\gamma L) + \lambda_4 = 0 .$$

To satisfy KT2) necessarily:

$$\lambda_4 \leq 0 \Leftrightarrow P_\theta P_1 \exp\{\rho\gamma L\}(\rho\gamma L - \rho L) + P_\theta(1-P_1) \exp\{\rho\gamma L\}(\rho\gamma L) \leq 0 \Leftrightarrow \gamma \leq P_1 .$$

Finally, for KT3) being satisfied it has to happen that:

$$0 \leq \alpha^{0*} \leq 1 \Leftrightarrow 1 \leq \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \leq \exp\{\rho L\} \Leftrightarrow \begin{cases} \gamma \geq P_0 \\ \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \leq \exp\{\rho L\} \end{cases} .$$

Consequently, the solution is:

$$\bar{\alpha} = \left( 1 - \left[ \frac{\ln \left\{ \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \right\}}{\rho L} \right], 1 \right) \text{ if and only if: } \begin{cases} P_0 \leq \gamma \leq P_1 \\ \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \leq \exp\{\rho L\} \end{cases} .$$

If we are in P3):  $\lambda_1 = \lambda_2 = \lambda_4 = 0$ ,  $\alpha^1 = 0$ , to satisfy KT1) condition:  $\alpha^{0*} = 1 - \left[ \frac{\ln \left\{ \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \right\}}{\rho L} \right]$ , and

due to KT3)  $0 \leq \alpha^{0*} \leq 1 \Leftrightarrow 1 \leq \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \leq \exp\{\rho L\} \Leftrightarrow \begin{cases} \gamma \geq P_0 \\ \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \leq \exp\{\rho L\} \end{cases}$  is satisfied.

Also due to KT1), as  $\alpha^1 = 0$ , the following is upheld:

$$-P_\theta P_1 \exp\{\rho L\}(\rho\gamma L - \rho L) - P_\theta(1-P_1)(\rho\gamma L) - \lambda_3 = 0.$$

Also, to satisfy KT2) we necessarily have  $\lambda_3 \leq 0$ , which is possible if and only if:

$$-P_\theta P_1 \exp\{\rho L\}(\rho\gamma L - \rho L) - P_\theta(1-P_1)(\rho\gamma L) \leq 0 \Leftrightarrow \exp\{\rho L\} \leq \frac{(1-P_1)\gamma}{P_1(1-\gamma)},$$

which is clearly incompatible with  $\frac{(1-P_0)\gamma}{P_0(1-\gamma)} \leq \exp\{\rho L\}$ , so that P3) offers no feasible solution.

Now if we consider P4):  $\lambda_1 = \lambda_3 = \lambda_4 = 0$ ,  $\alpha^0 = 1$ , the KT1) condition implies that:

$$\alpha^{1*} = 1 - \left[ \frac{\ln \left\{ \frac{(1-P_1)\gamma}{P_1(1-\gamma)} \right\}}{\rho L} \right],$$

and KT3) requires:  $0 \leq \alpha^{1*} \leq 1 \Leftrightarrow 1 \leq \frac{(1-P_1)\gamma}{P_1(1-\gamma)} \leq \exp\{\rho L\} \Leftrightarrow \begin{cases} \gamma \geq P_1 \\ \frac{(1-P_1)\gamma}{P_1(1-\gamma)} \leq \exp\{\rho L\} \end{cases}$ .

Also, to satisfy the KT1) condition, while  $\alpha^0 = 1$ , it is necessary that:

$$-(1-P_\theta)P_0 \exp\{\rho\gamma L\}(\rho\gamma L - \rho L) - (1-P_\theta)(1-P_0) \exp\{\rho\gamma L\}(\rho\gamma L) + \lambda_2 = 0.$$

KT2) condition implies  $\lambda_2 \leq 0$ , which is possible if and only if:

$$(1-P_\theta)P_0 \exp\{\rho\gamma L\}(\rho\gamma L - \rho L) + (1-P_\theta)(1-P_0) \exp\{\rho\gamma L\}(\rho\gamma L) \leq 0 \Leftrightarrow \gamma \leq P_0.$$

At the same time we have that  $P_0 < P_1$ , which is unfeasible when  $\gamma \geq P_1$ , so once more P4) does not offer any feasible solution.

Taking into account P5):  $\lambda_2 = \lambda_3 = \lambda_4 = 0$ ,  $\alpha^0 = 0$ , to satisfy KT1) it is necessarily the case that:

$$\alpha^{1*} = 1 - \left[ \frac{\ln \left\{ \frac{(1-P_1)\gamma}{P_1(1-\gamma)} \right\}}{\rho L} \right], \text{ and KT3) implies:}$$

$$0 \leq \alpha^{1*} \leq 1 \Leftrightarrow 1 \leq \frac{(1-P_1)\gamma}{P_1(1-\gamma)} \leq \exp\{\rho L\} \Leftrightarrow \begin{cases} \gamma \geq P_1 \\ \frac{(1-P_1)\gamma}{P_1(1-\gamma)} \leq \exp\{\rho L\} \end{cases} .$$

Moreover, from KT1), as  $\alpha^0 = 0$ , we have:

$$-(1-P_\theta)P_0 \exp\{\rho L\}(\rho\gamma L - \rho L) - (1-P_\theta)(1-P_0)(\rho\gamma L) - \lambda_1 = 0 .$$

From KT2)  $\lambda_1 \leq 0$ , which requires:

$$-(1-P_\theta)P_0 \exp\{\rho L\}(\rho\gamma L - \rho L) - (1-P_\theta)(1-P_0)(\rho\gamma L) \leq 0 \Leftrightarrow \exp\{\rho L\} \leq \frac{(1-P_0)\gamma}{P_0(1-\gamma)} .$$

Consequently the solution is:

$$\bar{\alpha} = \begin{cases} 0, & 1 - \left[ \frac{\ln \left\{ \frac{(1-P_1)\gamma}{P_1(1-\gamma)} \right\}}{\rho L} \right] \end{cases} \text{ if and only if: } \begin{cases} \gamma \geq P_1 \\ \frac{(1-P_1)\gamma}{P_1(1-\gamma)} \leq \exp\{\rho L\} \leq \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \end{cases} .$$

Dealing with P7):  $\lambda_1 = \lambda_3 = 0$ ,  $\alpha^0 = 1$ ,  $\alpha^1 = 1$ , the KT1) condition can be written as:

$$-(1-P_\theta)P_0 \exp\{\rho\gamma L\}(\rho\gamma L - \rho L) - (1-P_\theta)(1-P_0) \exp\{\rho\gamma L\}(\rho\gamma L) + \lambda_2 = 0 .$$

$$-P_\theta P_1 \exp\{\rho\gamma L\}(\rho\gamma L - \rho L) - P_\theta(1-P_1) \exp\{\rho\gamma L\}(\rho\gamma L) + \lambda_4 = 0 .$$

From KT2) we have  $\lambda_2 \leq 0$ ,  $\lambda_4 \leq 0$ , which implies:

$$-(1-P_\theta)P_0 \exp\{\rho\gamma L\}(\rho\gamma L - \rho L) - (1-P_\theta)(1-P_0) \exp\{\rho\gamma L\}(\rho\gamma L) \geq 0 \Leftrightarrow P_0 \geq \gamma ,$$

and,  $-P_\theta P_1 \exp\{\rho\gamma L\}(\rho\gamma L - \rho L) - P_\theta(1-P_1) \exp\{\rho\gamma L\}(\rho\gamma L) \geq 0 \Leftrightarrow P_1 \geq \gamma .$

Moreover, since  $P_0 \leq P_1$ , the solution is:

$$\bar{\alpha} = (1, 1) \text{ if and only if: } \gamma \leq P_0 .$$

In P8):  $\lambda_2 = \lambda_3 = 0$ ,  $\alpha^0 = 0$ ,  $\alpha^1 = 1$ , the KT1) condition entails:

$$-(1-P_\theta)P_0 \exp\{\rho L\}(\rho\gamma L - \rho L) - (1-P_\theta)(1-P_0)(\rho\gamma L) - \lambda_1 = 0 , \text{ and:}$$

$$-P_\theta P_1 \exp\{\rho\gamma L\}(\rho\gamma L - \rho L) - P_\theta(1-P_1) \exp\{\rho\gamma L\}(\rho\gamma L) + \lambda_4 = 0 .$$



From KT2) we have  $\lambda_1 \leq 0$ ,  $\lambda_4 \leq 0$ , so:

$$-(1-P_\theta)P_0 \exp\{\rho L\}(\rho\gamma L - \rho L) - (1-P_\theta)(1-P_0)(\rho\gamma L) \leq 0 \Leftrightarrow \exp\{\rho L\} \leq \frac{(1-P_0)\gamma}{P_0(1-\gamma)}, \text{ and:}$$

$$-P_\theta P_1 \exp\{\rho\gamma L\}(\rho\gamma L - \rho L) - P_\theta(1-P_1) \exp\{\rho\gamma L\}(\rho\gamma L) \geq 0 \Leftrightarrow P_1 \geq \gamma.$$

Therefore, the solution is:

$$\bar{\alpha} = (0, 1) \text{ if and only if: } \begin{cases} P_1 \geq \gamma \\ \exp\{\rho L\} \leq \frac{(1-P_0)\gamma}{P_0(1-\gamma)} \end{cases}.$$

If we are in P9):  $\lambda_1 = \lambda_4 = 0$ ,  $\alpha^0 = 1$ ,  $\alpha^1 = 0$ , the KT1) condition can be written as:

$$-(1-P_\theta)P_0 \exp\{\rho\gamma L\}(\rho\gamma L - \rho L) - (1-P_\theta)(1-P_0) \exp\{\rho\gamma L\}(\rho\gamma L) + \lambda_2 = 0, \text{ and:}$$

$$-P_\theta P_1 \exp\{\rho L\}(\rho\gamma L - \rho L) - P_\theta(1-P_1)(\rho\gamma L) - \lambda_3 = 0.$$

In addition, from KT2) we have  $\lambda_2 \leq 0$ ,  $\lambda_3 \leq 0$ , so:

$$-(1-P_\theta)P_0 \exp\{\rho\gamma L\}(\rho\gamma L - \rho L) - (1-P_\theta)(1-P_0) \exp\{\rho\gamma L\}(\rho\gamma L) \geq 0 \Leftrightarrow P_0 \geq \gamma, \text{ and:}$$

$$-P_\theta P_1 \exp\{\rho L\}(\rho\gamma L - \rho L) - P_\theta(1-P_1)(\rho\gamma L) \leq 0 \Leftrightarrow \exp\{\rho L\} \leq \frac{(1-P_1)\gamma}{P_1(1-\gamma)}.$$

However, since  $P_0 \leq P_1$ , if  $P_0 \geq \gamma \Rightarrow \gamma \leq P_1 \Rightarrow \frac{(1-P_1)\gamma}{P_1(1-\gamma)} \leq 1 \leq \exp\{\rho L\}$ , P9) does not offer a feasible solution.

Finally, if we consider P10):  $\lambda_2 = \lambda_4 = 0$ ,  $\alpha^0 = 0$ ,  $\alpha^1 = 0$ , the KT1) condition implies:

$$-(1-P_\theta)P_0 \exp\{\rho L\}(\rho\gamma L - \rho L) - (1-P_\theta)(1-P_0)(\rho\gamma L) - \lambda_1 = 0, \text{ and:}$$

$$-P_\theta P_1 \exp\{\rho L\}(\rho\gamma L - \rho L) - P_\theta(1-P_1)(\rho\gamma L) - \lambda_3 = 0.$$

Moreover, from KT2) we have  $\lambda_1 \leq 0$ ,  $\lambda_3 \leq 0$ , which is satisfied if and only if:

$$-(1-P_\theta)P_0 \exp\{\rho L\}(\rho\gamma L - \rho L) - (1-P_\theta)(1-P_0)(\rho\gamma L) \leq 0 \Leftrightarrow \exp\{\rho L\} \leq \frac{(1-P_0)\gamma}{P_0(1-\gamma)}, \text{ and:}$$

$$-P_\theta P_1 \exp\{\rho L\}(\rho\gamma L - \rho L) - P_\theta(1-P_1)(\rho\gamma L) \leq 0 \Leftrightarrow \exp\{\rho L\} \leq \frac{(1-P_1)\gamma}{P_1(1-\gamma)}.$$

So, given that  $P_0 \leq P_1$  and furthermore:  $\frac{(1-P_1)\gamma}{P_1(1-\gamma)} \leq \frac{(1-P_0)\gamma}{P_0(1-\gamma)}$ , the resultant solution is:

$$\bar{\alpha} = (0, 0) \text{ if and only if: } \exp\{\rho L\} \leq \frac{(1-P_1)\gamma}{P_1(1-\gamma)}.$$

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