DOI: 10.20472/TEC.2019.007.011

JUAN MIGUEL MORALES
University of Malaga, Spain

BENJAMÍN OLEA
Universidad de Málaga, Spain

MIGUEL ATENCIA
Universidad de Málaga, Spain

NICOLÁS MADRID
Universidad de Málaga, Spain

FOSTERING THE COOPERATIVE LEARNING OF MATHEMATICS IN ENGINEERING SCHOOLS

Abstract:

In this article, we report on the experience gained and the results obtained from an educational innovation project that has sought to introduce cooperative learning into the mathematics subjects of the first year of the Engineering degrees at the University of Malaga. In particular, we focus on the teamwork-based learning method that we have called the "Teacher-Apprentice" dynamics: Groups of 3-5 members are formed in which one of the members takes on the role of the Teacher, while the rest are the Apprentices. We introduce a system of incentives such that the group that progresses adequately in the subject has the right to group bonuses, which count towards the final grade awarded to each of the group members and which are different for the Teacher and for the Apprentices to allow for some level of competitiveness within the group. The selection of the Teacher is reviewed after each scheduled assessment test, where the student in the group who has obtained the highest score from among the group members will become the (possibly new) Teacher. The results collected to date reveal that group dynamics increase the motivation with which students face math learning, while helping them to correct conceptual errors through discussion with their peers.

Keywords:

Cooperative learning, Teaching methods, University Teaching, Mathematics, Engineering

JEL Classification: A22

1 Introduction

Mathematics is an essential and basic instrument for an engineer, which is why Mathematics subjects are fundamental and transversal pieces in the curricular design of the different Engineering degrees offered by higher education institutions worldwide. It is also recurrent that the subjects of mathematics are typically concentrated in the first or first few years of these degrees due to their instrumental and basic character for the study and comprehension of the different branches of Engineering. Thus, the first-year undergraduate student begins to walk the path to become an engineer through the study of Mathematics. However, the skills and competencies expected of an engineer may be quite different from those of a mathematician, as is their motivation.

On the other hand, it is assumed that the student of Engineering will develop competences that, although not necessarily linked to the assimilation of knowledge, and very particularly, not linked to the mastering of mathematical concepts and tools, will be, however, key for the later exercise of their profession. These are, for example, the ability to lead, to work in a team or the ability to present and communicate results, among others. In spite of this, the learning of Mathematics in many higher education institutions continues being mostly based on "classic" teaching methods, such as the master lecture and student’s individual work. However, it is well known that these methods do not encourage students to develop the competences and skills of an engineer. Students will have to wait for more advanced courses (or even for their integration into the labor market) to learn to be an engineer working with other engineers. In addition, teaching methods that are based almost exclusively on the master lecture and individualized work, often cause student’s frustration, demotivation or even withdrawal, especially among first-year students.

Within this context, this article reports on the experience gained and the results obtained from an educational innovation project that has sought to introduce cooperative learning into the Mathematics subjects of the first year of the Engineering degrees at the University of Malaga, in Spain. More specifically, in the course of this project, we have designed and implemented a teamwork-based learning method that we have called the "Teacher-Apprentice" dynamics. For the implementation of this method, groups of 3-5 students are formed at the beginning of the course, in which one of the students takes on the role of the Teacher, while the rest are the Apprentices. The learning within the groups is supported by a system of incentives whereby the group that progresses adequately in the course has the right to group bonuses in the summative assessment of the student’s performance. The rationale behind this teaching method is, therefore, to foster the unity and responsibility towards learning of the team members by assigning specific roles (teacher or apprentice) to them.

Cooperative learning is a topic that has drawn a great deal of attention from the educational community in the last decades. As a result, we can find today a wealth of information, practical guidelines and manuals for instructors to implement some form of cooperative learning in their courses. Here we mention the works of Rogers et al. (2001), Dubinsky, Matthews and Reynolds (1997), Leikin and Zaslavsky (1999) and Kaufman, Sutow and Dunn (1997), where the reader can discover a wide range of experiences with the *cooperative learning of Mathematics*, including a variety of implementation strategies and methods, tips, pros-and-cons analyses, etc. Furthermore, cooperative learning supports the *constructive alignment* principle whereby human beings and, in particular, students *learn by doing* (Biggs, 2011). In cooperative learning, naturally, students learn by doing through cooperation in groups.

The rest of this article is organized as follows. Section 2 describes, in detail, the teamwork-based learning method "Teacher-Apprentice" that we have investigated. Section 3 introduces the methodology we have followed to implement and evaluate its success. In Section 4, we present and discuss the most salient results we have obtained from the implementation of the "Teacher-Apprentice" dynamics. Finally, Section 5 concludes this article with some qualitative remarks.

2 The "Teacher-Apprentice" group dynamics

The overall objective of this teaching method is to develop and implement activities both outside and inside the classroom that encourage the students to address the study and resolution of math problems in groups.

From the very beginning of the course, the students are informed about the rules of the group dynamics. These rules are also made available to the students in a pdf document through the Moodle platform of the university.

Groups of four members are formed among the students of the course where the teaching method is to be implemented. One of those students will play the role of the “Teacher”, while the rest of the members of the group will take on the role of "Apprentices". Only in the exceptional case that the number of students does not allow that all the groups are of four members, it is allowed to form some group of three or five members.

At the beginning of the course, the Teachers of the groups are chosen from among the students who have obtained the highest marks in a preliminary test. Teachers are given the freedom to recruit their apprentices, although this process is supervised by the Professor to ensure the formation of balanced groups (according to the results of the preliminary test). Similarly, the Professor reserves the right

1. To intervene in the group dynamics in case of a perverse and malicious use of it, and implement corrective measures in such cases.

2. To modify the rules of the dynamics in order to encourage and facilitate the learning of the contents of the subject by the student.

Teachers have the responsibility to teach the Apprentices and oversee their training and learning, so that all members of the group can successfully pass the subject. They also have the responsibility to act as a spokesperson for the group regarding communications with the Professor. Failure to carry out these functions could imply the change of Teacher. On the other hand, being an Apprentice entails watching over the benefit of the group, allowing oneself to be guided by the Teacher and having the willingness and availability to participate in the group dynamics for the good of the whole group. Failure to carry out these functions could imply the expulsion of the student in question from the activity.

The selection of the Teacher is reassessed after each scheduled assessment test (for example, after each mid-term evaluation), where, for each group, the student who has obtained the highest score (without taking into account the group bonuses that are detailed below) from among the members of the group will become the (possibly new) Teacher.

For each thematic unit that covers the content of the subject, the Professor organizes a group session in the classroom in which the groups have to solve and discuss a set of exercises and problems under his or her supervision. It is expected that each group has worked on the resolution of these exercises and problems prior to the group session, so that the sessions can be devoted mainly to the clarification of doubts with the help of the rest of the group and the Professor.

Beyond the evidence-supported benefits of cooperative learning (see, for instance, the thorough and comprehensive analysis of this topic carried out in Smith et al. (2005), and references therein), the “Teacher-Apprentice” group dynamics pursues

1. To strengthen the student's feeling of responsibility in relation to his/her group.
2. To foster that students with lower performance can benefit from the possibility of having a partner with whom s/he can clarify doubts.
3. To facilitate that the students with greater performance can consolidate their knowledge about the subject by teaching the rest of their groupmates.
4. To generate a system of incentives that is not only aligned with the learning of all members of the group via pure cooperation, but also leaves a bit of room for competitiveness for those students with aspirations that are more ambitious.

We next elaborate on the incentive scheme that we have previously mentioned. This scheme distinguishes between the bonuses that the student playing the role of the Teacher may earn and the bonuses that can be obtained by the Apprentices.

Group bonuses for the Teacher:

1. They are applied after each assessment test (either a partial, a midterm or a final examination).
2. They are applicable as long as the Teacher has obtained a score in the partial/midterm/final examination higher than or equal to 4 out of 10 (or has passed the course by some form of continuous assessment).
3. The bonuses, if applicable, are added to the grade obtained by the Teacher in the partial/midterm/final examination.
4. The Teacher shall receive a bonus of
    a) 0.25 points if only one apprentice has passed the test.
    b) 0.75 points if two apprentices have passed.
    c) 1.5 points if three or more apprentices have passed.
5. The maximum score that can be obtained in a partial/midterm/final examination, including bonuses, is 10 out of 10.

Group bonuses for the Apprentices:

1. They are applied after each partial/midterm evaluation test.
2. They are applicable as long as the Apprentice has obtained a score in the partial/midterm examination greater than or equal to 4 out of 10.
3. The group average score is calculated in the assessment test, excluding group bonuses. The Apprentice's grade will be the maximum between the Apprentice's grade on the partial/midterm exam and the group's average grade on that exam.

Naturally, the “Teacher-Apprentice” group dynamics, and its associated incentive scheme, may be implemented in a variety of ways. In the following section, we describe how this cooperative learning method has been specifically conducted within the context of this research.

3 Methodology

The “Teacher-Apprentice” group activity has been implemented within the course *Calculus*, which is taught in the first year and first semester of most of the Engineering Bachelor degrees that are offered by the Industrial Engineering School of the University of Malaga, in South Spain. The experience that we report here corresponds to the academic courses 2017/2018 and 2018/2019. To be more precise, the team-based learning activity was first applied to one of the student groups enrolled in the Electrical Engineering Bachelor degree (course 2017/2018). Subsequently, in the course 2018/2019, the “Teacher-Apprentice” method was applied to one of the group of students enrolled in the Mechanical Engineering Bachelor degree. In what follows, we will refer to both group of students as the *Electrical* and *Mechanical Engineering Groups* for ease of exposition and comparison.

There were key differences in the actual application of the proposed team-based learning activity to each of the two group of students, namely:

1. *Size*: While the Electrical Engineering Group consisted of 33 students, the Mechanical Engineering Group was made up of 79 students (that is, this group was approximately two times and a half bigger than the former). These figures exclude those students that enrolled in the course, but never showed up in class or quitted a very short time after the course started.
2. *Intensity*: Whereas the teaching and learning activities *in the classroom* for the Electrical Engineering Group included *eight team sessions*, the Mechanical Engineering Group was only involved in *half of them*. The rest of the class activities other than the team sessions were fundamentally based on master lectures and problem-solving exercises by the Professor on the blackboard. Furthermore, the students were encouraged to study the subject out of the classroom together with their peers in their assigned group.
3. *Participation*: The participation of the Electrical Engineering Group in the "Teacher-Apprentice" activity was *compulsory*, in the sense that students not taking part in the team sessions were not given the option to pass the course via the continuous assessment system. In contrast, the participation in the team activity by the students in the Mechanical Engineering Group was *voluntary*, that is, students not involved in a team could still pass the course following the continuous assessment system, but could not benefit from group bonuses. Eventually, 72 out of the 79 students enrolled in the Mechanical Engineering Group decided to take part in the team activities, and the majority of those who did not were repeaters.

The summative assessment of the student's performance was based on three written tests. Two of these tests cover only a half of the course content each and are held in the middle and at the end of the semester (before lectures are over), respectively. Furthermore, these two tests constitute the basis for the continuous assessment system whereby the student can pass the course: if the average of the student's scores in these two tests is above five out of ten, with a minimum grade of four in both of them, then s/he passes the course. Otherwise, the student has to take a third exam covering the whole course content in the examination period that has been previously established by the board of the Industrial Engineering School. In this third exam (which we call "the final exam"), the student must get a minimum score of five (out of ten) to pass the course.

It is interesting to note that those students playing the role of Teachers in the groups may still profit from group bonuses even in the case that they have passed the subject through the continuous assessment system. For this purpose, they must keep on acting as Teachers for those Apprentices in their groups that need to take the final exam. Besides, these Teachers must provide the Professor with a plan of group study sessions where they must indicate when and where these sessions are taking place during the examination period and within the School.

In the following section, we discuss and compare the performance in the subject *Calculus* of those students that took part in the “Teacher-Apprentice” group dynamics, with respect to the group (Electrical or Mechanical Engineering) to which they belonged. However, given the relatively short period of time during which the “Teacher-Apprentice” group activity has been implemented, the small sample size of the students that took part in it, and the many factors that affect the learning process of a first-year undergraduate student, the figures that we provide in Section 4 must be taken with (statistical) caution.

Having the above in mind from the very beginning of the experiment, the primary aim of this research was to analyze the impact of our team-based learning activity, not on the students’ performance, but on their motivation to face the Mathematics subject, because it is well known that a high level of motivation facilitates deep learning and the student’s success. To this aim, a Google survey was carried out among the students participating in the “Teacher-Apprentice” group dynamics after the semester was over and before the final examination period. This survey included the following questions/inquiries:

1. *Indicate your subjective level of involvement in the teamwork (not involved at all; involved to a lower extent; involved to a considerable extent; very involved from beginning to end)*.
2. *The teamwork has helped you keep up with the subject (from 1 to 5, with 1 meaning “totally disagree” and 5 “totally agree”).*
3. *The teamwork has helped you better understand the contents of the subject (from 1 to 5, with 1 meaning “totally disagree” and 5 “totally agree”).*
4. *The teamwork has helped you identify and correct conceptual mistakes (from 1 to 5, with 1 meaning “totally disagree” and 5 “totally agree”).*
5. *The teamwork has increased your motivation to face the subject (from 1 to 5, with 1 meaning “totally disagree” and 5 “totally agree”).*
6. *The teamwork has made the subject more bearable for you (from 1 to 5, with 1 meaning “totally disagree” and 5 “totally agree”).*
7. *Indicate whether you have studied the subject with your group peers outside the classroom (never; occasionally; sometimes; regularly).*
8. *State your opinion about the number of group sessions that have taken place within the classroom (insufficient; adequate; too many).*
9. *Group bonuses have encouraged you to work with your group peers (from 1 to 5, with 1 meaning “totally disagree” and 5 “totally agree”).*
10. *State whether you have found useful to have a “Teacher” that can help you clarify doubts about the subject (from 1 to 5, with 1 meaning “totally disagree” and 5 “totally agree”).*
11. *State whether you have found helpful to discuss a problem/exercise within your group (from 1 to 5, with 1 meaning “totally disagree” and 5 “totally agree”).*

Additionally, the students were given the possibility in the survey to make any other comment on the "Teacher-Apprentice" group activity (for instance, what they liked the most/the least, suggestions for improvement …).

In the following section, we analyze and discuss the students' answers to these questions.

4 Results and Discussion

In this section, we first present and discuss the results of the survey that was conducted among the students that took part in the cooperative learning activities. In our analysis, we will try to put forward sensible explanations for the differences that can be observed between the responses given by the students of the Electrical Engineering Group and those provided by the students of the Mechanical Engineering Group. For this purpose, it is worth recalling that the size of the latter group was more than twice the size of the former. In both groups, however, the same single professor was responsible for supervising the group activities and mentoring the different teams. Consequently, the implementation of the "Teacher-Apprentice" team activity in the Mechanical Engineering Group was comparatively more challenging, with substantially less time for the Professor to follow up on the progress of each group independently. Furthermore, remember that the participation of the students from the Mechanical Engineering Group in the teamwork activities was voluntary and that they were only involved in half of the group sessions.

On a different front, it is important to remark that the participation of the students in the survey about their experience with the cooperative learning activities was voluntary. As a result, 23 (resp. 31) students out of the 33 (resp. 72) that partook in the teamwork activities from the Electrical (resp. Mechanical) Engineering Group completed the survey.

Table 1 provides the distribution in percentage of the students' reply to question 1 of the survey. Interestingly, the vast majority of the students perceive that they have been highly engaged in the group dynamics. Clearly, they are overstating their actual level of involvement, as inferred from their response to question 7 of the survey, where a bit less than 50% of the students claim that they sometimes or frequently met their group peers to work on the subject outside the classroom (see Table 2).

Table 1: Students' subjective level of involvement in the "Teacher-Apprentice" group activity (in percentage) based on their reply to question 1 in the survey.

Group			
Electrical Engineering		**Mechanical Engineering**	
Not involved or barely involved	Considerably or very involved	Not involved or barely involved	Considerably or very involved
0	100	3.2	96.8

In the students' answers to question 7 of the survey, we find the first relevant difference between the experiences of the Electrical and Mechanical Engineering Groups with the cooperative learning activities. Indeed, over a quarter of the students that completed the survey from the former group states that they met with their group peers to prepare the subject on a regular basis. This suggests that the students in the Electrical Engineering Group developed a higher degree of engagement in the teamwork activities than those in the Mechanical Engineering Group. This hypothesis is further confirmed by the students' replies to question 8 (see Table 3), to which 39% of the students that completed the survey from the Electrical Engineering Group answered that the number of group sessions within the classroom was insufficient. This percentage drops to 22.6% in the case of the Mechanical Engineering Group, even though this group only took part in half of the group sessions. Therefore, it seems clear that the Electrical Engineering Group perceived the "Teacher-Apprentice" group dynamics as a more productive activity to learn the subject.

Table 2: Distribution (in percentage) of students' response to question 7 about the frequency they have met their group peers to work on the subject outside the classroom.

		Never	Occasionally	Sometimes	Regularly
Group	**Electrical Engineering**	21.7	30.4	21.7	26.1
	Mechanical Engineering	16.1	35.5	41.9	6.5

 ISBN 978-80-87927-48-9, IISES

Table 3: Distribution (in percentage) of students' response to question 8 about the number of group sessions within the classroom.

		Insufficient	Adequate	Too Many
Group	Electrical Engineering	39.1	56.5	4.4
	Mechanical Engineering	22.6	74.2	3.2

Table 4 below provides the mean and standard deviation of the scores given by the students to questions 2-6 and 9-11 of the survey. Again, it is apparent that, in general, the students in the Electrical Engineering Group report a higher level of satisfaction about the conducted experiment. As mentioned before, this outcome is not surprising as the considerable size of the Mechanical Engineering Group made the implementation of the experiment more intricate, leaving less room and time for a closer interaction between the Professor and the groups during the teamwork sessions in the classroom. Furthermore, four group sessions might not be enough for the students to fully perceive the benefits of cooperative learning. Notwithstanding this, it is interesting to notice that the majority of the students in both the Electrical and Mechanical Engineering Groups did realize that discussing with their peers about a math exercise problem was useful (question 11) and increased their motivation and readiness to learn the subject (questions 5 and 6). Likewise, the possibility to improve the subject mark by way of cooperation was also perceived as a motivating factor in general (question 9), albeit to a lesser extent.

Finally, attendance and success rates for the three tests that comprise the summative assessment system of *Calculus* are collated in Table 5. Attendance rates are given as a percentage of those students that enrolled in the "Teacher-Apprentice" teamwork activity at the beginning of the course, while success rates are calculated as a percentage of those students that actually took the corresponding test. The table also includes the percentage of students (enrolled in the cooperative learning activities) that eventually passed the subject. Figures in this table reveals that the students in the Electrical Engineering Group were significantly more successful than those of the Mechanical Engineering Group. Based on the experience gained by the authors of this work during the experiment, such a difference in success rates between the two groups can be attributed, at least in part, to the higher level of engagement of the Electrical Engineering Group in the "Teacher-Apprentice" teamwork activities. The cooperative learning in this group was, without any doubt, much more effective and intense. Nevertheless, we also acknowledge that attributing the observed difference to that only would not be scientifically rigorous, as we cannot exclude the influence of other possible explanatory factors such as students' academic background, level of difficulty of the different tests, etc.

Table 4: Mean and standard deviation of the scores given by the students to questions 2-6 and 9-11 in the survey.

	Group			
	Electrical Engineering		Mechanical Engineering	
Question	Mean	Std	Mean	Std
2	3.70	0.82	3.35	1.02
3	3.61	1.03	3.58	0.99
4	3.70	1.02	3.61	0.95
5	3.96	0.93	3.35	1.05
6	4.13	0.97	3.74	1.18
9	3.96	0.98	3.77	0.99
10	3.57	1.16	3.58	1.21
11	4.26	0.86	4.13	0.76

Table 5: Attendance and success rates (in percentage) of the students that enrolled in the cooperative learning activities. Attendance rates are computed with respect to the number of students that joined a group at the beginning of the course, whereas success rates are given with respect to the students that actually took the corresponding test.

	Test 1		Test 2		Final Exam		Pass rate
	Attendance	Success	Attendance	Success	Attendance	Success	
Electrical Engineering	100	30.3	78.8	34.6	66.7	30.7	45.6
Mechanical Engineering	96.0	23.2	75.0	40.7	51.4	10.8	26.4

5 Conclusions

For a long time in Higher Education Centers, the learning of Mathematics, perhaps because of its large theoretical and abstract component, has been based almost exclusively on the master lecture on the part of the teacher and on the individual study on the part of the student. Such a system, when applied in the first courses of the different Engineering degrees, may cause demotivation and frustration in the students, at the same time that it differs notably from the way in which they will have to learn during the subsequent exercise of their profession as engineers. In fact, society expects the engineer to work within a team and, therefore, teachers should encourage engineering students to learn with the help of their peers.

This article reports on an educational innovation project which aimed at introducing cooperative learning into some mathematics subjects of the first year of the Engineering degrees offered by the University of Malaga, in the south of Spain. More specifically, within the context of this project, we have developed a teamwork-based learning method called the "Teacher-Apprentice" group dynamics, whereby teams of 3-5 members are formed in which one of the members plays the role of the Teacher, while the remaining ones are the Apprentices. The experience here reported is linked to the implementation of this teamwork-based learning method for the subject *Calculus* during the academic courses 2017/2018 and 2018/2019 and involved groups of students enrolled in the Electrical and Mechanical Engineering degrees, respectively.

We have drawn the following main qualitative conclusions from our experiment:

1. The cooperative learning of mathematics is not only possible, but also highly recommendable and effective, especially when involving students that do not aspire to become mathematicians (like, for instance, those enrolled in Bachelors of Engineering programs).
2. Giving the students the possibility to discuss a math problem or question with their peers increases their motivation and readiness to participate and learn mathematics.
3. The cooperative learning of mathematics can replace classical teaching methods such as master lectures fully or largely. Teachers should not be afraid of increasing the share or intensity of the team activities in the classroom.
4. The effectiveness of the proposed “Teacher-Apprentice” teamwork activity diminishes with the number of groups that the Professor needs to supervise and monitor. Cooperative learning may be more resource demanding.
5. Designing a system of incentives that encourages students to cooperate may increase the effectiveness of cooperative learning methods.

Lastly, we highlight that, in the first year of the Engineering Bachelor’s degrees offered by the University of Malaga, it is quite frequent to have students who drop out of the subject or even of the degree sometime after the course has begun. This may be due to a variety of reasons (e.g., they realize that they made a bad choice from the very beginning or they find out that the degree does not live up to their expectations). These students may have a negative impact on the cooperative learning activities, as they may suddenly abandon their groupmates. As future work, it would be interesting to design measures to early detect those students and to dynamically adjust the groups accordingly, so that any group can fully recover from the demotivating experience of losing a member in the middle of a semester.

6 Acknowledgements

This research was carried out within the framework of the Educational Innovation Project PIE17-025, funded by the University of Malaga.

7 References

ROGERS, E. C.; DAVIDSON, N.; REYNOLDS, B. E., and THOMAS, A. D. (editors) (2001). *Cooperative Learning in Undergraduate Mathematics: Issues that Matter and Strategies that Work*. Mathematical Association of America, 2001. ISBN: 978-0-88385-166-1.

DUBINSKY, E; MATHEWS, D. and REYNOLDS, B. E. (editors) (1997), *Readings in Cooperative Learning for Undergraduate Mathematics.* Mathematical Association of America, 1997. ISBN: 978-0-88385-153-1.

LEIKIN, R. and ZASLAVSKY, O. (1999). Cooperative learning in Mathematics. *The Mathematics Teacher*, vol. 92, no. 3, pp. 240-246, 1999.

BIGGS, J. B. (2011). *Teaching for Quality Learning at University: What the Student Does*. McGraw-Hill/Society for Research into Higher Education/Open University Press, 2011. ISBN: ISBN 978-0-33-524275-7.

KAUFMAN, D.; SUTOW, E. and DUNN, K. E. N. (1997). Three Approaches to Cooperative Learning in Higher Education. *The Canadian Journal of Higher Education*, vol. XXVII, no. 2-3, pp. 37-66, 1997.

SMITH, K. A.; SHEPPARD, S. D.; JOHNSON, D. W. and JOHNSON, R. T. (2005). Pedagogies of Engagement: Classroom-Based Practices. *Journal of Engineering Education*, 94(1):87-101, Jan. 2005.