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AN EFFICIENT WAY TO OPTIMIZING MULTIPLE QUALITY CHARACTERISTICS IN QUALITY MANAGEMENT: POSTERIOR PREFERENCE ARTICULATION METHOD

Abstract:

Response Surface Methodology (RSM) is a major research field of quality management, which studies the relationship between a response (quality characteristic) and a number of input variables. In real-world RSM application problems, it is quite common that several responses are of interest. In this case, determination of optimum conditions on the input variables would require simultaneous consideration of all the responses. This is called a Multiple Response Surface (MRS) problem. One of the most important issues in MRS Optimization is how to obtain a satisfactory “compromise” solution considering a Decision Maker (DM)’s preference information on the tradeoffs among multiple responses. A promising alternative to incorporate the DM’s preference information well into the problem is the posterior preference articulation approach, which first generates all (or most) of the nondominated solutions and then makes the DM select the best one from the set of nondominated solutions a posteriori. This paper proposes a new posterior method to MRSO, which does not generate all the nondominated solutions before the selection stage, unlike other posterior methods which go on straightforward from generation to selection only once. Instead, it generates only the required nondominated solutions while making the response space of interest narrower gradually. The proposed method can improve the efficiency by minimizing the number of nondominated solutions generated.

Keywords:

Quality Management, Response Surface Methodology, Multiple Response Surface Optimization, Posterior Preference Articulation Method

JEL Classification: M11, C90, C61

Introduction

Response Surface Methodology (RSM) consists of a group of techniques used in empirical study of the relationship between a response and a number of input variables. Consequently, the experimenter attempts to find the optimal setting for the input variables that maximizes (or minimizes) the response (Box and Draper, 1987; Khuri and Cornell, 1996; Myers and Montgomery, 2002). In product or process development, it is quite common that several response variables are of interest. In this case, determination of optimum conditions on the input variables would require simultaneous consideration of all the responses. This is called a Multiple Response Surface (MRS) problem (Khuri, 1996).

An MRS problem is solved through three major steps: data collection, modeling, and optimization. Among those steps, this paper is focused on the last optimization step. A problem in the MRS Optimization (MRSO) is formulated below in general.

$$\begin{aligned} &\text{Optimize}\left[\hat{y}_1(\mathbf{x}), \hat{y}_2(\mathbf{x}), \cdots, \hat{y}_l(\mathbf{x})\right] \\ &\text{s.t.}\ \mathbf{x} \in \boldsymbol{\Omega}. \end{aligned} \tag{1}$$

where $\hat{y}_1(\mathbf{x})$ denotes the estimated *i*th response (i = 1, …, l), $\mathbf{x}$ is an input variable vector, and $\boldsymbol{\Omega}$ is the experimental region. To date, various methods have been proposed for MRSO, including desirability function approach (Harrington, 1965; Derringer and Suich, 1980; Derringer, 1994) and loss function approach (Pignatiello, 1993; Vining, 1998; Ko et al., 2005). For various literature review studies on MRSO, see Myers et al. (1989), Myers (1999) Myers et al. (2004), and Ardakani and Wulff (2013).

One of the most important issues in MRSO is how to resolve the conflict among multiple responses. In many cases of MRS problems, improving one response is likely to make one or some of the other responses get worse. In order to obtain a satisfactory “compromise” solution in such a case, a Decision Maker (DM)’s preference information on the tradeoffs among multiple responses should be considered in optimizing them.

MRSO can be considered as one of various application areas of Multiple Objective Optimization (MOO). Therefore, MRSO methods also can be classified into the categories generally accepted in the MOO literature (Park and Kim, 2005). The MOO literature classifies MOO methods into three major approaches in terms of the timing of the DM’s preference information articulation: the prior, progressive, and posterior preference articulation approach. The prior approach requires that the DM provides his/her preference information a priori and then solves the MOO problem straightforward without any interaction with the DM. The progressive approach – often referred to as the interactive approach – solves the problem by allowing the DM to articulate his/her preference information progressively. The posterior approach first generates all (or most) of the nondominated solutions, which can be obtained without the DM's preference information. Then, the DM selects the best one from the set of nondominated solutions a posteriori.

Most of the MRSO methods proposed so far are classified into prior approach. The prior approach assumes that the DM can specify all the required preference information before solving a problem. But, it is quite difficult for the DM to provide such information in advance, because he/she cannot experience the relationships or conflicts among the responses. A good alternative is to show the various nondominated solutions to improve the DM's understanding on the tradeoffs among the responses and make the DM select the most preferred solution. Such an alternative is the posterior approach.

In this paper, we propose a new posterior method to MRSO. The distinctive advantage of the proposed method is that it does not generate all the nondominated solutions before the selection stage, unlike other posterior methods which go on straightforward from generation to selection only once. Instead, it generates only the required nondominated solutions while making the response space of interest narrower gradually. Therefore, it can improve the efficiency by minimizing the number of nondominated solutions generated.

Literature Review on MRSO

The existing studies in MRSO can be categorized into five approaches: priority-based, desirability function, loss function, process capability, and probability-based approaches. These five approaches take a common strategy that reduces a multidimensional problem in (1) into a one-dimensional problem and then solves it. Each approach is reviewed below.

The priority-based approach selects the most important response among a number of ones and then uses it as the objective function. The other responses are employed as constraints. Assuming there are only two responses of interest, Myers and Carter (1973) proposed an optimization formulation that maximizes (or minimizes) the primary response with an equality constraint on the other response. Biles (1975) extended this idea by allowing not only more than two responses, but also inequality constraints on the secondary responses.

The desirability function approach transforms an estimated response into a scale-free value, called a desirability. It is a value between 0 and 1, and increases as the corresponding response value becomes more desirable. The overall desirability, another value between 0 and 1, is defined by combining the individual desirability values. Then, the optimal setting is determined by optimizing the overall desirability. Harrington (1965) first proposed a simple form of a desirability function. Derringer and Suich (1980) extended Harrington's approach by suggesting a more systematic transformation scheme from an estimated response to the corresponding individual desirability. The overall desirability can be obtained by aggregating the individual desirability functions using the geometric mean. Later, different forms of aggregation have been proposed. For example, Derringer (1994) proposed the use of a weighted geometric mean. Kim and Lin (2000) suggested maximizing the lowest (individual) desirability.

The loss function approach originates from Taguchi’s robust design concept. It aims to find the optimal parameter setting by minimizing the expected loss function. Pignatiello (1993) first proposed the use of a squared error loss function. Then, the expected loss is derived and minimized. Vining (1998) proposed a modification to Pignatiello’s model, which employs the estimated responses in the loss function. Ko et al. (2005) proposed an improvement over Pignatiello’s and Vining’s models. They employ the predicted responses in the loss function. Ko et al.’s model is a more generalized model thus and includes both Pignatiello’s and Vining’s models as special cases.

The process capability approach derives a process capability index using the estimated mean and standard deviation of a response. The overall capability index is obtained by combining the individual process capability indices. Then, the optimal setting is determined by maximizing the overall capability index. Barton and Tsui (1991) proposed a performance centering as a process capability index. Then, they suggested maximizing the minimum of process capability indices. Plante (1999) extended the Barton and Tsui’s approach by developing several multicriteria models based on the performance centering. Plante (2001) proposed the use of two typical process capability indices, C_{pk} and C_{pm}. Then, he suggested maximizing the (weighted) geometric mean of individual C_{pk}’s (or individual C_{pm}’s). Ch’ng et al. (2005) proposed to maximize the weighted sum of individual C_{pm}’s. Köksalan and Plante (2003) proposed an interactive optimization method to incorporate the DM’s preference information in their proposed method.

The probability-based approach assumes a multivariate probability distribution of a multivariate response. It first models the distributional parameters in terms of input variables and then finds the optimal setting which maximizes the probability that all responses simultaneously meet their specifications. Chiao and Hamada (2001) assumed the multivariate normal distribution with mean and variance-covariance matrix. The distributional parameters are modeled. Then, they suggested maximizing the proportion of conformance. Peterson (2004) and Miró-Quesada et al. (2004) estimated the distributional parameters in the multivariate t distribution using a Bayesian approach.

Proposed Method

The proposed method consists of 3 steps: generation of candidate spaces (Step 1), generation of representative solutions (Step 2), and selection of the most preferred space (Step 3).

Step 1: Generation of Candidate Spaces

Step 1 partition a response space into smaller subspaces. For a single response (say, *i*th response), the response space is divided into r_i equal-sized intervals. For the whole (I – 1) responses the response space is partitioned into ($r_1 \times r_2 \times \cdots \times r_{I-1}$) equal-sized subspaces. The DM determines r_i values of (I – 1) responses in this step. Note that one response is excluded in the response space partitioning process. In Step 2, nondominated solutions are generated within each subspace by the ε-constraint method (Haimes et al., 1971; Chankong and Haimes, 1983, p. 274). In this method, one response is used as the objective function and the other (I – 1) responses are used as constraints. We need to determine the (I – 1) numbers of ε values of in the method and they are determined within the (I – 1) dimensional subspace. For this reason, one response is excluded in the response space partitioning process.

Step 2: Generation of Representative Solutions

When the number of responses is two, the partitioned subspaces can be plotted in the two-dimensional space which is easy to recognize. However, in case of more than two responses, a graphical representation of the partitioned spaces may not provide enough insights for selecting the preferred subspace. To overcome this difficulty, we choose to use a numeric approach instead of a visual one to understand the subspaces. That is, the subspaces are compared based on their representative nondominated solution. In this paper, we determine a single nondominated solution which is located in the center of the subspace as the representative nondominated solution. The representative nondominated solution is generated by the ε-constraint method as well. To obtain the representative nondominated solution, the values of ε_i in the constraints are set as the middle point for each response.

Step 3: Selection of the Most Preferred Space

The DM selects the most preferred subspace by comparing the representative nondominated solutions. The total number of the representative nondominated solutions is $n(r_1 \times r_2 \times \cdots \times r_{I-1})$, where n is the number of the representative nondominated solutions for each subspace. For example, n = 1, when a single center point is adopted as the representative nondominated solution. However, it gets large if the number of responses (i.e., I) increases. In such a case, it is not easy to select a single solution. To support the DM's selection process, the interactive selection method (Köksalan and Sagala, 1995) is adopted. The DM evaluates whether the (representative) nondominated solution within the selected subspace is satisfactory or not. If the solution is satisfactory, the procedure ends. Otherwise, it goes back to Step 1.

Illustrative Example

In this section, we illustrate the proposed IP-MRSO via a well-known MRS problem, ‘‘tire tread compound problem,” originally presented in Derringer and Suich (1980).

Problem Description

The problem aims to improve the tire tread performance characterized in four different responses by controlling three chemical ingredients. The four responses are PICO abrasion index (y_1), 200% modulus (y_2), elongation at break (y_3), and hardness (y_4). The three chemical ingredients are silica (x_1), silane (x_2), and sulfur (x_3). The experiment was conducted in a central composite response surface design. The experimental region $\mathbf{\Omega}$ was given as $-1.63 \le x_i \le 1.63$, $i = 1, 2, 3$. The response surfaces for the four responses were fitted as follows (Derringer and Suich, 1980):

$$\hat{y}_1(\mathbf{x}) = 139.12 + 16.49x_1 + 17.88x_2 + 10.91x_3 - 4.01x_{12} - 3.45x_{22} - 1.57x_{32} + 5.13x_1x_2 + 7.13x_1x_3 + 7.88x_2x_3, \tag{2}$$

$$\hat{y}_2(\mathbf{x}) = 1261.11 + 268.15x_1 + 246.5x_2 + 139.48x_3 - 83.55x_{12} - 124.79x_{22} + 199.17x_{32} + 69.38x_1x_2 + 94.13x_1x_3 + 104.38x_2x_3, \tag{3}$$

$$\hat{y}_3(\mathbf{x}) = 400.38 - 99.67x_1 - 31.4x_2 - 73.92x_3 + 7.93x_{12} + 17.31x_{22} + 0.43x_{32} + 8.75x_1x_2 + 6.25x_1x_3 + 1.25x_2x_3, \tag{4}$$

$$\hat{y}_4(\mathbf{x}) = 68.91 - 1.41x_1 + 4.32x_2 + 1.63x_3 + 1.56x_{12} + 0.06x_{22} - 0.32x_{32} - 1.63x_1x_2 + 0.13x_1x_3 - 0.25x_2x_3. \tag{5}$$

For an illustration purpose, we assume that the DM’s underlying true utility functions are known and he/she is satisfied with a solution whose utility function value is larger than a specific value (0.780). It should be noted that the underlying true utility function is unknown in reality.

Solving Results

Table 1 shows the nondominated solutions generated finally in the second round. The DM selected the 1st solution (the gray area) in the table after conducting 7 times of pairwise comparisons. Since the utility function value of the 1st solution is larger than the predetermined value (i.e., 0.785 > 0.780), the procedure stopped with the selected solution.

Table 1. The Final (2nd round) Nondominated Solutions in the illustrative Example.

No.	(ε_2, ε_3, ε_4)	(x_1, x_2, x_3)	($\hat{y}_1$, $\hat{y}_2$, $\hat{y}_3$, $\hat{y}_4$)	Utility Value
1	**(625, 12.5, 0.9375)**	**(-0.10, 0.35, -1.15)**	**(126.01, 1375, 487.5, 68.44)**	**0.785**
2	(625, 12.5, 2.8125)	(-0.13, 0.69, -1.19)	(126.60, 1375, 487.5, 70.10)	0.760
3	(625, 37.5, 0.9375)	(0.05, 0.36, -1.04)	(129.75, 1375, 462.5, 68.44)	0.784
4	(625, 37.5, 2.8125)	(0.03, 0.77, -1.07)	(131.21, 1375, 462.5, 70.31)	0.759
5	(875, 12.5, 0.9375)	(-0.26, 0.13, -0.86)	(126.90, 1250.86, 487.5, 68.44)	0.760
6	(875, 12.5, 2.8125)	(-0.37, 0.42, -0.80)	(128.32, 1230.67, 487.5, 70.31)	0.730
7	(875, 37.5, 0.9375)	(-0.10, 0.16, -0.76)	(130.55. 1271.08. 462.5. 68.44)	0.763
8	(875, 37.5, 2.8125)	(-0.21, 0.48, -0.70)	(132.66. 1262.58. 462.5. 70.31)	0.740

Concluding Remarks

There have been few attempts to apply a posterior approach to MRSO problems to obtain the best compromise solution through a true understanding on the tradeoffs among the multiple responses. In this paper, we proposed a new posterior method to MRSO to overcome the problem. The proposed method first divides a response space into smaller subspaces. Then, the DM selects the preferred space among the subspaces. Once the preferred space is selected, the other subspaces are not considered in the subsequent process. Therefore, it can generate the smaller number of nondominated solutions required only for selection. Through a case problem, we have found that the proposed method is very efficient while producing a quite satisfactory solution.

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