DOI: 10.20472/IAC.2019.046.017

JORGE OMAR RAZO-DE ANDA
INSTITUTO POLITÉCNICO NACIONAL, Mexico

ANA CECILIA PARADA-ROJAS
INSTITUTO POLITÉCNICO NACIONAL, Mexico

SALVADOR CRUZ-AKÉ
INSTITUTO POLITÉCNICO NACIONAL, Mexico

THE CREDIT CYCLE AND THE FINANCIAL FRAGILITY HYPOTHESIS: AN EVOLUTIONARY POPULATION APPROACH

Abstract:

Minsky's idea of triggering a financial crisis is the adoption of risky financial positions by companies and their relationship with the financial system through banks and the credit they provide. The present work seeks to provide an explanation from a microeconomic point of view through the behavior of agents and their decision making under a Theory of evolutionary games, especially population games. The great advantage of this type of games is that it allows us to obtain proportions of the different decisions that a population or subpopulation is taking and how their interaction promotes equilibrium and the dynamics towards (or around) them.
This allows us to determine the dynamics and equilibria of the credit cycle, following Minsky's idea of financial fragility. Additionally, the dynamics of the replicator allows transforming the differential equations in a Lotka-Volterra system, from which it can be concluded that both companies and banks adopt a predatory prey relationship in order to survive.

Keywords:

Capital Structure, Evolutionary Games, Financial crises

JEL Classification: C73, G02, G01

Introduction

It has been some years since the Hypothesis of Financial Fragility in Minsky (1956) emerged as an alternative theory to the mainstream that tried to explain the emergence of financial crises. Minsky's idea of triggering a financial crisis is the adoption of risky financial positions by companies and their relationship with the financial system through banks and the credit they provide. Rationality makes its appearance again in the sense that all companies seek to maximize their benefits and if the decision regarding the capital structure allows it, then the logical response would be to increase external financing through leverage. According to the work of Modigliani and Miller (1958), in an environment with distortions such as taxes, one of the factors that increases the value of a company is the deduction of interest. In this sense, it could be thought that the greater the indebtedness, the greater the interests and therefore the greater the fiscal shield, which by deduction would imply 100% debt financing. However, aggressive leverage entails a greater difficulty for debt services to be paid in a timely manner, leading to penalizing the value of the company with the probability of default on its obligations to creditors. On the other hand, as Hayek and Wicksell mention, in a monetary economy, there is a natural and nominal interest rate. The nominal rate according to the studies of Studart, is manipulated by the injection of monetary mass through the credits granted. This is a function of the demand for loans by companies, which is why banks act as agents that compete on Bertrand prices in order to obtain the largest amount of loans granted. The relationship between banks and companies determines the so-called credit cycle. Although there is a lot of literature regarding the previous point, the majority adopts a macroeconomic point of view, and does not allow to identify the interrelation of the agents. Additionally, traditional models suppose a unique balance, when they can exist more. If we assume that the decisions of both types of agents are taken simultaneously and that their benefits depend on what the opposite does, they set the perfect scenario for the application of Game Theory from a microeconomic point of view. The Game Theory approach has three advantages over traditional growth models; in first place allows the synchronization of the microeconomic point with the macroeconomic point through the study of population games, in second place it allows to know the dynamics and transition of the populations towards the points of equilibrium considering the evolutionary games. Finally, considering equilibria as proposed by John Nash, there may be more than one equilibrium

This paper seeks to provide an explanation from a microeconomic point of view through the behavior of agents and their decision making under the theory of evolutionary games, especially population games. The great advantage of this type of games is that it allows us to obtain proportions of the different decisions that a population or subpopulation is taking and how their interaction promotes equilibrium and the dynamics towards (or around) them.

The article is integrated as follows. First, a review of the literature is carried out in order to identify the most recent models that adopt the Fragility and instability approach from

Minsky's point of view. Subsequently, a section related to Game Theory is incorporated, especially the application of evolutionary population games, for which the elements of a game in general are reviewed in a very summary way and a revision of the concept of mixed strategy and equilibrium is made. of Nash. Additionally, the formalization of an evolutionary game n is presented, which includes the fundamental concepts of Evolutively Stable Strategy (ESS, for its acronym in English) for monomorphic games, as well as the dynamics of the replicator for polymorphic games. The third section focuses mainly on the analysis of joint decision making between companies and banks. In principle, the business valuation model is developed considering the sum of the expected value of the generated cash flows minus the expected value of the default costs. Dividends are calculated from a modification of the original Merton model to determine Equity, while default costs are calculated using the Merton model and a leverage adjustment factor. It is further assumed that the leverage increases the expected value of the operating flows and that the production has decreasing returns. Companies must choose the magnitude of financial leverage according to the strategy that maximizes expected flows, while banks must choose the price of financing granted based on their benefit. The decisions are made under the assumption of static games, where the process of choice is given simultaneously. The mixed strategies allow to consider proportions and the dynamics of the replicator, incorporates the dynamics of the previous ones towards the Nash equilibria based on the stability characteristic

This allows us to determine the dynamics and equilibria of the credit cycle, following Minsky's idea of financial fragility. Additionally, the dynamics of the replicator allows transforming the differential equations in a Lotka-Volterra system, from which it can be concluded that both companies and banks adopt a predatory prey relationship in order to survive.

Literature Review

The maximization of the value of a company is and will always be one of the objectives of the board of directors of the same. One of the tools that allows this to happen is the decision about the capital structure, that is, in what proportion the assets of a company are financed with their own resources, whether they were contributed by the partners at the beginning or by resources coming from of the operation of the company, and on the other hand resources of external investors such as loans. Modigliani and Miller (1958) showed that in a frictionless environment the capital structure had no effect on the final value of the company, however, when distortions such as taxes appear, then decisions regarding the type of financing took effect. on the valuation of assets.

In this way, in an environment with frictions arise studies that try to optimize the value of the company in terms of the debt incurred. Some points in which most of the above coincide, is that an operative leverage is beneficial because the interest is tax deductible, which allows obtaining an additional flow for the company. Another common point is that the debt settlement reduces the amount available for the investment or the payment to the partners depending on the dividend policy, therefore, a greater leverage implies a greater return on the equity due to a increase in risk. Another point is that the higher the leverage, the greater the difficulties in complying with the obligations on the part of the company, which translates into a probability of default, increasing the counterparty risk of the lenders.

Articles such as this or that, analyze the optimal capital structure based on the net present value adjusted by valuation of discounted flows. The use of perpetuities is a fairly useful simplification of reality. In this sense, Equity can be seen as a perpetuity, of the dividends paid that are the free cash flows after the payment of interest. However, if at any point the leverage is such that the pre-tax result is less than zero, then with the use of perpetuity and the Gordon model, Equity should be zero. On the other hand, if the investment policy were a 100% rate on profits obtained, we would have the same case. However, there are many examples in which a company does not pay dividends and the value of its Equity in the market is positive. An explanation to the above is that the value of Equity not only considers assets at this time, but the expectation of creation of assets with such profitability in the future. An alternative way of calculating Equity is through Merton's asset valuation model, which incorporates expectations that the expected value of assets is greater than indebtedness. Derived from the previous model, it is also possible to identify the probability of default, measured as the average distance to default.

With respect to the costs of non-compliance, some authors, such as TAL and TAL, suppose a constant cost on the assets, however, some authors consider that the cost must be a variable depending on the leverage. Charles (2008) proposes a measure of alternative leverage as the reason for such and such

When such leverage is such that it no longer allows a company to comply with the debt service it is known as a Ponzi financing position, on the contrary when it can meet its obligations it is known as Hedge hedged position. However, the fact that you can or can not meet your commitments to creditors depends on factors beyond your control such as interest rates. An increase in the interest rates increases the debt service in general, deteriorating its position allowing a Hedge company to become a Ponzi overnight depending on the magnitude of the change.

Commonly the dynamics of Ponzi financing is determined by the common leverage ratios (debt as a proportion of income and debt as a proportion of assets) and where their behavior is increasing monotony. However, Tymoigne (2010) indicates that Ponzi financing does not have to do with the above reasons, since the classification of the type of financing has to do strongly with the cash flows and liquid assets; and these reasons are not related to the latter, that is, the previous classification has to do with the quality of the leverage. According to Tymoigne (2010) it may happen that the interest rate is higher than the rate of income, even though the condition that the net cash flow is greater than the obligations which would indicate that there is no need for refinancing. Contrary may happen that it is necessary for the company to take position defense operations, even when the income growth rate is higher than the interest rate. With the above it is demonstrated that even when the reasons for leverage may be indicators of a change in position, they are not determinant for the classification

In Foley (2003) Minsky's financing conditions are formally established. This is done through the modeling of cash flows and a differential equation on the new debt. Assuming growth rates on the flows, determines the dynamics based on the previous ones, for which the system tends to be greater than one (Ponzi) or failing less than one (Speculative) and less than one without a level (Hedge) . It also identifies the conditions for the above and its relationship between the rate of growth or return on profits, on the interest rate and the growth rate of the company. Once the system is determined, it applies to the economy assuming a Kaleckian model of producing a single well with two factors of production such as labor and capital. Meirelles and Lima (2006) and Lima and Meirelles (2007) make an extension of the Foley model (2003) in which closed forms are now considered in terms of growth rates, assuming proportions over the benefits of capital, which they correspond to the physical capitalists (shareholders) and financing capitalists (creditors).

Determine additionally in terms of combinations of capitalist accumulation rate, interest rates and the growth rate, the proportions of companies in each position as the areas under the curve when the debt capital ratio is less than one.

El modelo

Population games: mixed strategies seen as proportions

As has been well mentioned, mixed strategies on a space of finite pure strategies is nothing more than a density function that indicates with what probability or frequency the players choose their each of their strategies. It should be remembered that the probability or frequency is simply a proportion.

In the branch of biology, one of the most common objects of study is the density of a population of an ecosystem. It is in this way that this social science adapts game theory for the study of population density, transforming the perception of mixed strategies into a grouped vision of population proportions. Even more that it is a single population density can be referred to a type of behavior or characteristic. For example, one of the pioneering studies concerning population games was the symmetrical game proposed by Maynard () in which players from the same population faced each other. The strategies in this case denoted an aggressive behavior (hawk) or a peaceful behavior (dove) in members of the same population. Its reward was the number of descendants for future generations.

For this game, the individuals that were born to each period had "preloaded" a type of behavior according to the distribution or the natural rule of behavior. However, randomly, a different behavior could arise, which would favor a change in the original distribution, promoting what Charles Darwin denoted as natural selection.

So far only the members of a single population have been talked about, however, this is not limited. When speaking of two populations in advance to the evolutionary games they are known as population games. These have the characteristic that the strategy space of the players are different (asymmetric games) and where each population can have different objectives. Hausbauer and Sigmund () mention distinguish three basic situations. On the one hand, there is competition in which the two populations struggle to take ownership or control of a particular resource. This type of games are known as match-matching games in which a random individual from a population with a pre-loaded population with a defined characteristic confronts another individual at random with another predefined characteristic of the population 2.

On the other hand, we have mutualism, where it represents the opposite of competition games. In this case individuals benefit from each other. Finally, the host-parasite relationship can be seen as predatory prey games developed in their origins by Lotka-Volterra.

In these games, the analysis is made on two fundamental components. On the one hand, it is evaluated if the strategies are stable over time, and on the other hand, the existing dynamics towards the equilibrium points is analyzed. From the above, both for evolutionary games and population games, the study of strategy stability and dynamics are of vital importance, for which the following sections of this chapter are dedicated.

Evolutionary Stable Strategy and Nash Equilibrium

As mentioned in the previous section, there is a relationship between stability and Nash equilibrium. When there is a stable mixed strategy, it is by itself seems to correspond to a Nash equilibrium, the former being a refinement of the latter. It is worth defining in this case an Evolutionary Stable Strategy in a formal way.

As defined in Maschler (), a mixed strategy σ^* it is an ESS[1], if for each different mixed strategy σ, exist an $\epsilon > 0$ such that:

$$(1-\epsilon)\pi_1(\sigma,\sigma^*) + \epsilon\pi_1(\sigma,\sigma) < (1-\epsilon)\pi_1(\sigma^*,\sigma^*) + \epsilon\pi_1(\sigma^*,\sigma)$$

The interpretation consists of the following; suppose that there is an initial configuration of the population which is stable and could be said that up to a certain point it is the normal distribution. Spontaneously, a mutation appears with a very small proportion ε, which modifies the initial distribution. When this mutation interacts with another agent of the population, an individual with the normal configuration can be found with Probability $(1-\epsilon)$

[1] A mixed strategy is used as a reference, since when finding the optimal distribution σ^*, the expected value to use any pure strategy that is in the support of said mixed strategy, will give the same profit, so it would be indifferent among all your alternatives.

and he would get a payment equal to $\pi_1(\sigma, \sigma^*)$, or he might encounter a mutation with probability ϵ and get a payment of $\pi_1(\sigma, \sigma)$. On the other hand, looking at the left part of the inequality, it would indicate what the payment would be if a normal individual meets another normal individual with Probability $(1-\epsilon)$ and obtaining a payment of $\pi_1(\sigma^*, \sigma^*)$. For the original distribution to remain unchanged, the expected value of an individual with the normal configuration would have to have an expected value greater than that of the mutation, regardless of the type of agent he or she is facing. If this happens, the mixed strategy σ would not be optimal returning to adopt the original strategy σ^* and condemning the mutation to extinction. This offers as a corollary, the fact that if σ^* is ESS, then it is also a Nash Equilibrium. There may be a case that there is no an ESS, however, it is guaranteed that there is always at least one Nash equilibrium in mixed strategies, with which it can be said that the relationship is unidirectional; that is, an ESS is always a Nash Equilibrium, but a Nash Equilibrium is not always ESS.

From the above, an optimal mixed strategy would result in all the support strategies giving the same expected value with which the previous expression could pass from an inequality to an equality. In order to refine the above, the Stability condition of a mixed strategy is defined as:

$$\pi_1(\sigma, \sigma^*) < \pi_1(\sigma^*, \sigma^*)$$

If happens that:

$$\pi_1(\sigma, \sigma^*) = \pi_1(\sigma^*, \sigma^*)$$

Then it must be satisfied that an individual using a normal mixed strategy against a mutation gives a higher expected value than using a mutation against a mutation.

$$\pi_1(\sigma, \sigma) < \pi_1(\sigma^*, \sigma)$$

It is worth mentioning that the ESS analysis is only functional when there are symmetrical or monomorphic games that indicate that they have the same strategy space. However, in nature and in an economic system hardly the two conditions above are met, either because the game has different payments for each player or because their decisions are different, which indirectly indicates that you have more than one population. The dynamics of the replicator tries to give an answer about the stability of the system when we speak of polymorphic games or in its absence of asymmetric games.

Replicator Dynamics

The dynamics of the replicator tries to explain how the size of a population is susceptible to changes depending on the interactions between the agents and their payment functions. Suppose for the moment that each strategy is a behavior gene which dictates in advance to each individual how to act. In this way a mixed strategy on this behavioral space would indicate what is the proportion of the population that acts with this strategy as a base and how the system evolves towards equilibrium or moves away from it.

Consider an individual who uses a preset strategy which inherits his configuration from his offspring. These individuals only use pure strategies when they have the assumption of a replicating agent. Suppose there is a space of strategies at the moment generic for all players

$$S = \{s_1, s_2, \dots, s_z\}$$

Suppose further that the number of individuals using a pure strategy is n_i so that the total population is represented by:

$$N = \sum_{i=1}^{z} n_i$$

From the above it can be deduced that the proportion of the population using a pure strategy is determined by:

$$x_i = \frac{n_i}{N}$$

In such a way that there is a distribution or proportion for each strategy such that:

$$X = \{x_1, x_2, \dots, x_z\}$$

With the condition of:

$$\sum_{i=1}^{z} x_i = 1$$

Consider that individuals have the possibility to change their strategy and with that the proportion or distribution of the initial configuration.

The changes in the proportions or distribution have a dynamic such that:

$$\dot{n}_i = (M + \pi(s_i, x))n_i$$

Where M represents the average growth rate of the population that uses pure strategy s_i and $\pi(s_i, x)$ represents the surcharge of the population provided that $\pi(s_i, x)$ would be positive.

The change in the total population in a given time, turns out to be simply the sum of all the changes in the proportions or subpopulations:

$$\dot{N} = \sum_{i=1}^{z} \dot{n}_i$$

Substituting the previous equation and considering that $n_i = Nx_i$ y, it is obtained:

$$\dot{N} = MN\sum_{i=1}^{z} x_i + \sum_{i=1}^{z} x_i\,\pi(s_i, x) = [M + \bar{\pi}(x)]N$$

Where $\bar{\pi}(x)$ represents the average payment of the population. For this case, the question that really matters is how the proportions of individuals with preferences vary over a particular mixed strategy through the tempo, for which, considering the expression $n_i = Nx_i$ suppose that:

$$\dot{n}_i = N\dot{x}_i + \dot{N}x_i \Rightarrow N\dot{x}_i = \dot{n}_i - \dot{N}x_i$$

Substituting the above equation and reducing terms, we obtain the equation known as the replicator dynamics:

$$\dot{x}_i = \big(M + \pi(s_i, x)\big)x_i - [M + \bar{\pi}(x)]x_i \Rightarrow \dot{x}_i = [\pi(s_i, x) - \bar{\pi}(x)]x_i$$

Actually the dynamics of the replicator indicates that changes in the proportion x_i are the result of the difference between the profit or payment of using a strategy s_i in specific and the average gain of the game for the player i. If this difference is positive, the proportion tends to grow until the payments are equal. When this happens, the growth rate is equal to zero, which coincides with the condition of long-term equilibrium. In this way the proportions or optimal distribution is defined by $\dot{x}_i$=0.

A game of evolutionary crisis: an approach of financial instability.

Once the methodology of the evolutionary games and the dynamics of the replicator have been developed, the basic elements of the game (corresponding agents, strategies and payment functions) are established. Consider a game with two agents $G = \{E, B\}$ where the first one is a firm (E) and the second one a commercial bank (B). Under the context of an evolutionary and population game, players can adopt a behavior for each available strategy. The space of pure strategies of the players are the following:

$S_1 = \{s_1^1, s_1^2,\}$ Y $S_2 = \{s_2^1, s_2^2\}$

Where it is clearly observed that the company has two pure strategies available as the bank.

Capital structure as financing decision

Suppose that the strategies of the company are related to the financing of its operations and can only request loans to the banks; On the other hand, banks obtain benefits by charging a certain percentage of the loan depending on the strategy on the active interest rate. The rule for a company to remain in the market is that the EBIT operating income is equal to or greater than the flows for payments to creditors (debt service, SD) and payment to investors (I) such that:

$$EBIT = SD + I$$

A firm that has sufficient resources to meet its obligations (payment to its investors and creditors) does not need to request any credit from the banking sector; It is worth mentioning that for this to happen, the expected flow of Operating Revenues (EBIT) is greater than the costs of indebtedness (SD) plus capital costs (I), such that:

$$EBIT > SD + I$$

When a company does not generate enough resources to make the payments, it turns to the banking sector to complete the missing part. As an additional assumption, it is clear that the resources generated by the operation of the company must be destined in the first instance to the payment of debt service (creditors), and the remainder to the investors. If the resources in a period are sufficient to pay the interest plus the principal of the debts, but not enough to pay the cost of the investment; Then the alternative is to request a bank loan in order to boost the investment. From the above, there are quite a few implications. Operating income is greater than the service of the debt but not greater than the sum of the above plus the investment, so the amount of debt should be a constant fraction α_1 of the investment I so that the additional flows for the concept of new indebtedness cover the shortfall.

$$(EBIT > SD); (EBIT < SD + I) \Rightarrow EBIT + \alpha_1 I > SD + I$$

In this way, it follows that the company needs to resort to debt so that once the debt service is covered, it can maintain its operation. The foregoing has some degree of similarity, with the description of "Speculative" financing made by Minsky (). In fact, the financing "Hedge", also resembles our company that does not require additional debt described above in this section.

Finally, "Ponzi" financing is described as the situation in which an agent can not meet its obligations (debt service and investment). In this way, the agent uses compulsory bank credit for increasingly large amounts, which is known as the degree of financial fragility (Minsky,). In the context of the payment function of the "Ponzi" strategy, the implication is that the sum of the debt service plus the investment is greater than the operating income, and in this way the amount of additional debt would have to be a fraction of the previous sum. It is in this way that companies aim to maximize the difference between their operating income and their costs.

$$EBIT < SD + I \Rightarrow EBIT + \alpha_2 (SD + I) > SD + I$$

The above means that the problem of maximization of companies is reduced to maximize the market value of the same. From this point of view, a modified version of the Modigliani and Miller (1958) thesis of the capital structure can be adapted. The traditional version of their work indicates that the market value of the company is not affected by financing decisions between debt or capital in a frictionless environment (transaction costs, taxes, etc.). The above means that the combination of debt and capital does not matter, the market value will always be the same, which does not contribute to the maximization problem raised above, since this is dependent on the amount of debt absorbed by the company. The solution consists in incorporating frictions to the original model.

One of the reasons why companies decide to finance themselves with debt is that interest is tax deductible. The incorporation of the taxes to Modigliani and Miller's model (1958) allows to change the size of the pizza and in this way maximize the value of the company subject to the amount of indebtedness. Although in this model, the value of the company if it turns out to be a function of the amount of indebtedness, the optimal response is to borrow as much as possible. However, in reality this does not happen since greater indebtedness entails a higher degree of default risk. In this way, the third version of Modigliani and Miller is determined as follows:

$$V_L = V_u + V_A - V_p$$

Where V_L represents the market value of the leveraged firm, V_u the market value of the unleveraged firm, V_A is the present value of tax savings and V_p indicates the present value of the bankruptcy cost as simile of credit risk or default.

The market value of the unleveraged firm V_u can be defined as:

$$V_u = \frac{EBIT(1-T) - I}{r_A}$$

Where $EBIT(1-T)$ are the average operating net income or also called Earnings after taxes and r_A it is the return on the assets of the company in a period, which in turn are only financed with capital; from the above the required return on capital must be equal to the return on assets, only when the company is financed 100% with capital $r_A = r_E$

The tax shield, assuming perpetuities, can be denoted as:

$$V_A = \frac{Dr_D T}{r_D} = TD$$

Where T is the tax rate and D is debt. Finally the value of the cost of bankruptcy (or financial difficulties) can be denoted by:

$$V_p = \frac{P(Q)C_Q}{r_{wacc}}$$

$P(Q)$ is the probability of bankruptcy, C_Q is the fixed cost of bankruptcy associated with indebtedness and r_{wacc} is the weighted average cost of capital. Substituting the previous equations and assuming that for the moment only the flows are considered, it is necessary to:

$$EBIT(1-T) + Dr_D T - P(Q)C_Q$$

From a flow point of view, the Net Operating Revenues are destined for the payment of dividends; On the other hand, the value of the company can be seen as the sum of the generated flows divided by the average cost of capital. From the above it is possible that the flows can be defined as follows:

$$FE = DIV + INT - P(Q)C_Q$$

In such a way that the value of the company would be determined by:

$$FE = (EBIT - INT)(1 - T) - I + INT - P(Q)C_Q$$

Determination of payment functions

From the previous section we can see that $EAT = EBIT - SD - IMP - I$ are the net operating income or profit after taxes and Interest minus the retained earnings for investment, which are determined by the difference between operating income or Earnings before Interest and Taxes ($EBIT$) minus debt service taxes and retained earnings for investment (I), such that:

$$FE = (EBIT - SD)(1 - T) - I + D_0 r_D T - P(Q)C_Q$$

Debt services are the debt interests contracted previously. Assume that the initial debt D_0 is the present value of a perpetuity, so it can be determined as:

$$D_0 = \frac{SD}{r_D}$$

r_D represents the cost of the debt. Suppose additionally that D_T the amount of additional debt that came into the savings account due to insufficient resources generated in the period plus the previous debt. In the case of a Hedge company, $D_T = D_0$ clearing SD and including the above variables in the above equation, it is obtained:

$$FE = (EBIT - D_0 r_D)(1 - T) + D_T - I + D_T r_D - P(Q)C_Q$$

Considering now the level of indebtedness as the aforementioned strategies of the company and after some algebraic manipulations, it is possible to define the payment functions of the Hedge, Speculative and Ponzi positions as:$U_H = EBIT(1 - T) + D_0(1 + r_D T) - I - P(Q)C_Q$

$$U_S = EBIT(1 - T) + (D_0 + D_S)(1 + r_D T) - I - P(Q)C_Q$$

$$U_P = EBIT(1 - T) + (D_0 + D_P)(1 + r_D T) - I - P(Q)C_Q$$

Where D_S y D_P represent the amount of new indebtedness of the strategy, which had previously been defined as:$D_S = \alpha_1 I$

$$D_P = \alpha_2(D_0 r_D + I)$$

Now consider player 2, that is, bank (B). His strategy space was only composed of two elements, which were related to the price of the credits offered. It is important to emphasize that the price offered is simply the active interest rate. For which your strategy is determined by maintaining a high active interest rate or a low active interest rate denoted by r_{DA} y r_{DB} respectively.

The decision to maintain high or low interest rates depends largely on the benefit obtained and the probability of default of the recipient of the credit, which is again a function of the active rate. In this way, the payment function of a bank that charges a high interest rate is determined by:

$$U_A = r_{DA}L - C_1 - LP(Q)$$

Where L represents the amount of debt that the firms mantain, C_1 is the cost associated with the passive interest rate, for which it is considered a constant for the time being. In other words, the bank's payment function is determined by the service of the debt contracted by the company minus its costs, for part of the deposits and, on the other hand, for the breaches.

On the other hand, the payment function of a bank with a low active interest rate is as follows:

$$U_B = r_{DB}L - C_1 - LP(Q)$$

An additional assumption is that companies always meet their obligations as long as they have sufficient resources, so the probability of default would only depend on the probability that their expenses are greater than their income. Thus the probability of default would be equal to the probability of bankruptcy. For practicality purposes with respect to the calculation of mixed strategies, consider now a game with only two positions, that is, a company can be Hedge or can be Ponzi.

Suppose additionally that the expected value of operating income is a random variable and grows depending on the amount of debt requested, since financing productive activities with debt instead of own resources lowers costs and increases profitability.

Nash equilibrium in mixed strategies

From the previous game, we look for the equilibria in mixed strategies for which suppose that pure strategies an associated distribution function $\sigma_B = (x_1, x_2)$ for banks and $\sigma_E = (y_1, y_2)$ for firms, where each $x_1$represents the probability that the bank uses a high rate; in the same way y_1 indicates the probability of the company becoming indebted with Hedge-type risk. Due to the characteristics of the distribution functions, the sum of all the probabilities must be equal to one, so the mixed strategies can be represented as follows:

$$\sigma_B = (x, 1 - x) \; and \; \sigma_E = (y, 1 - y)$$

Depending on the bank's mixed strategy, companies can determine the expected value by using each of their strategies,

 ISBN 978-80-87927-52-6, IISES

Companies are indifferent between their strategies when the previous ones have the same expected value regardless of what the other player does. The expected values depending on the distribution of banks can be written as follows:

$$\boldsymbol{VE}(\boldsymbol{HEDGE}) = (1-T)EBIT - I + xL_{HA}[\varphi - (\varphi - r_A)T + 1] + (1-x)L_{HB}[\varphi - (\varphi - r_B)T + 1] - xP(r_A, L_{HA})C_{HA} - (1-x)P(r_B, L_{HB})C_{HB}$$

$$\boldsymbol{VE}(\boldsymbol{PONZI}) = (1-T)EBIT - I + xL_{PA}[\varphi - (\varphi - r_A)T + 1] + (1-x)L_{PB}[\varphi - (\varphi - r_B)T + 1] - xP(r_A, L_{PA})C_{PA} - (1-x)P(r_B, L_{PB})C_{PB}$$

By matching the expected values of the strategies of the companies, it is possible to find the optimal mixed strategy of the bank, such that the company does not have incentives to change its strategy, according to the definition of equilibrium. As can be seen, the expected value of the "Hedge" strategy is contained in the "Speculative" and "Ponzi" strategy, thus replacing the first and equaling the expected values of the two remaining strategies, we have an equation with an unknown

$$0 = x[\varphi - (\varphi - r_A)T + 1](L_{PA} - L_{HA}) + (L_{PB} - L_{HB})[\varphi - (\varphi - r_B)T + 1] - x(L_{PB} - L_{HB})[\varphi - (\varphi - r_B)T + 1] - x[P(r_A, L_{PA})C_{PA} - P(r_A, L_{HA})C_{HA}] - [P(r_B, L_{PB})C_{PB} - P(r_B, L_{HB})C_{HB}] + x[P(r_B, L_{PB})C_{PB} - P(r_B, L_{HB})C_{HB}]$$

After algebraic reductions, the optimal mixed strategy of the banks with respect to the interest rate is obtained:

$$x^*_1 = \frac{\delta_B \Delta L_B - \Delta PC(L_{PB}, L_{HB})}{[\pi_B \Delta L_B - \Delta PC(L_{PB}, L_{HB})] - [\pi_A \Delta L_A - \Delta PC(L_{PA}, L_{HA})]}$$

$$x^*_2 = 1 - x^*_1$$

Where

$$\Delta PC(L_{PA}, L_{HA}) = P(r_A, L_{PA})C_{PA} - P(r_A, L_{HA})C_{HA}$$

$$\Delta PC(L_{PB}, L_{HB}) = P(r_B, L_{PB})C_{PB} - P(r_B, L_{HB})C_{HB}$$

$$\Delta L_B = (L_{PB} - L_{HB})$$

$$\Delta L_A = (L_{PA} - L_{HA})$$

$$\delta_B = [\varphi - (\varphi - r_B)T + 1]$$

$$\delta_A = [\varphi - (\varphi - r_A)T + 1]$$

On the other hand, the optimal mixed strategy for companies is in the same way, for which we proceed to calculate the expected values of the bank's strategies:

$$\boldsymbol{VE}(\boldsymbol{r_A}) = yr_AL_{HA} + (1-y)r_AL_{PA} - C_1 - yL_{HA}P(r_A, L_{HA}) - (1-y)L_{PA}P(r_A, L_{PA})$$

$$\boldsymbol{VE(r_B)} = yr_BL_{HB} + (1-y)r_BL_{PB} - C_1 - yL_{HB}P(r_B, L_{HB}) - (1-y)L_{PB}P(r_B, L_{PB})$$

It proceeds to equal the expected values $\boldsymbol{VE(r_A)} = \boldsymbol{VE(r_B)}$

$$-y(r_BL_{HB} - r_AL_{HA}) + y(r_BL_{PB} - r_AL_{PA}) + y\Delta PC(L_{HB}, L_{HA}) - y\Delta PC(L_{PB}, L_{PA}) = (r_BL_{PB} - r_AL_{PA}) - \Delta PC(L_{PB}, L_{PA})$$

Where

$$\Delta PC(L_{PB}, L_{PA}) = L_{PB}P(r_B, L_{PB}) - L_{PA}P(r_A, L_{PA})$$

$$\Delta PC(L_{HB}, L_{HA}) = L_{HB}P(r_B, L_{HB}) - L_{HA}P(r_A, L_{HA})$$

So the mixed strategy of the companies is expressed by:

$$y_1{}^* = \frac{(r_BL_{PB} - r_AL_{PA}) - \Delta PC(L_{PB}, L_{PA})}{[(r_BL_{PB} - r_AL_{PA}) - \Delta PC(L_{PB}, L_{PA})] - [(r_BL_{HB} - r_AL_{HA}) - \Delta PC(L_{HB}, L_{HA})]}$$

$$y^*{}_2 = 1 - y^*{}_1$$

Dynamics of the population configuration

Consider the dynamics of the replicator and the payment matrices for the players Company and Bank such that:

A=

$U_B(r_A, H)$	$U_B(r_A, P)$
$U_B(r_B, H)$	$U_B(r_B, P)$

B=

$U_E(r_A, H)$	$U_E(r_B, H)$
$U_E(r_A, P)$	$U_E(r_B, P)$

Donde A y B are the payment matrices for players 1 and 2 respectively, and where $U_E(r_A, H)$ is the company's payment when it uses pure strategy “Hedge” and the bank the pure strategy "High Rate"; the subscript E refers to the firm y B to the bank. On the other hand,$(Ay)_i$ refers to the expected payment of player 1 (in this case banks) when they use strategy 1 (high rates) considering the proportions of companies with Hedge and Ponzi financing. Thus, $(Bx)_1$ the expected value of the player "Firm" considering the distribution of the banks. It is also important to define the term xAy, which indicates the average expected profit for the banks considering both populations (the respective distributions of the companies and the banks), while yBx is the simile for firms.

In the case of the game of crisis with respect to the decision on indebtedness, the dynamics of the replicator is modified because, although only two players participate, the game is polymorphic. From the above, the dynamics of the replicator becomes a system of 2 dependent differential equations. For the case of the bank that has two strategies, the condition that the sum of the proportions must be equal to 1, allows that for every two strategies, there is a unique differential equation, that is, the game has a dimension of

$(2X2)$ so the system should be made up of 4 equations, however, given that $x_2 = 1 - x$ a single equation would suffice to determine both x_1 and x_2, while for the case of firms, the same condition must be maintained. The system is determined as follows:

$$\dot{x} = [(Ay)_1 - xAy]x$$

$$\dot{y}_1 = [(Bx)_1 - yBx]y$$

In order to be as objective as possible, the development of the replicator dynamics equations is not presented, however, they remain as follows:

$$\begin{aligned}\dot{\boldsymbol{x}} = \boldsymbol{x}(\mathbf{1} - \boldsymbol{x})\{(\boldsymbol{r_A L_{PA}} - \boldsymbol{r_B L_{PB}}) + \Delta \boldsymbol{PC}(\boldsymbol{L_{PB}}, \boldsymbol{L_{PA}}) \\ + \boldsymbol{y}\{[(\boldsymbol{r_A L_{HA}} - \boldsymbol{r_B L_{HB}}) + \Delta \boldsymbol{PC}(\boldsymbol{L_{HB}}, \boldsymbol{L_{HA}})] \\ - [(\boldsymbol{r_A L_{PA}} - \boldsymbol{r_B L_{PB}}) + \Delta \boldsymbol{PC}(\boldsymbol{L_{PB}}, \boldsymbol{L_{PA}})]\}\}\end{aligned}$$

$$\begin{aligned}\dot{\boldsymbol{y}} = \boldsymbol{y}(\mathbf{1} - \boldsymbol{y})\{-\boldsymbol{\delta_B}(\Delta \boldsymbol{L_B}) + \Delta \boldsymbol{PC}(\boldsymbol{L_{PB}}, \boldsymbol{L_{HB}}) \\ + \boldsymbol{x}\{[-\boldsymbol{\delta_A}(\Delta \boldsymbol{L_A}) + \Delta \boldsymbol{PC}(\boldsymbol{L_{PA}}, \boldsymbol{L_{HA}})] - [-\boldsymbol{x\delta_B}(\Delta \boldsymbol{L_B}) + \boldsymbol{x}\Delta \boldsymbol{PC}(\boldsymbol{L_{PB}}, \boldsymbol{L_{HB}})]\}\}\end{aligned}$$

As both equations can be observed, they are a function of cross-lagged values, which leads us to think of the Lotka-Volterra equations of predatory prey systems, which does not sound strange thinking that it is the banks that act as predators and the companies as dams, however, if there were no individuals or agents to lend to, the banks would tend to disappear.

Analysis, simulation and results

For this case, to make the distinction between positions, as mentioned in Charles (2008), the interest rate on the income flow can be used as a measure of instability, which results in an equivalent ratio in cash flow to the capital debt ratio of Post Keynesian models. This ratio allows to identify the positions.

The ratio is as follows:

$$f = \frac{r * D}{\pi}$$

When the numerator $r * D > \pi$ it would be talking about a Ponzi position and the reason would have a value greater than 1, if both are equal, it is a Speculative position, since it can pay the interest but not the principal one. On the contrary, if the previous relationship is lower then it is a Hedge position.

Conclusions

References

Charles, S. (2008). A Post-Keynesian model of accumulation with a Minskyan financial structure. *Review of Political Economy, 20*(3), 319-331.

Detzer, D., & Herr, H. (2014). Theories of Financial Crises-An Overview. *Institute for International Political Economy Berlin, Working Paper*.

Keen, S. (1995). Finance and Economic Breakdown: Modeling Minsky's "Financial Instability Hypothesis". *Journal of Post Keynesian Economics, 17*(4), 607-635.

Lavoie, M. (1986). Systemic financial fragility: a simplified view. *Journal of Post Keynesian Economics, 9*(2), 258-266.

Lima, G. T., & Meirelles, A. J. (2007). Macrodynamics of debt regimes, financial instability and growth. *Cambridge Journal of Economics, 31*(4), 563-580.

Meirelles, A. J., & Lima, G. T. (2006). Debt, financial fragility, and economic growth: a Post Keynesian macromodel. *Journal of Post Keynesian Economics, 29*(1), 93-115.

Minsky, H. (1974). The modeling of financial Instability: An Introduction. *Working Papers, Levy Economics Institute* , 4-24.

Minsky, H. (2008). *Stabilizing an unstable economy.* McGraw Hill.

Minsky, H. P. (1964). Longer waves in financial relations: financial factors in the more severe depresions. *The American Economic Review*, 324-335.

Modigliani, F., & Miller, M. (1958). The cost of capital, corporation finance and the theory of investment. *The American economic review, 48*(3), 261-297.

Narayanan, M. P. (1988). Debt Versus Equity under Asymmetric Information. *Journal of Financial and Quantitative Analysis, 23*(1), 39-51.

Nikolaidi, M., & Stockhammer, E. (2017). Minsky models: A structured survey. *Journal of Economic Surveys, 31*(5), 1304-1331.

Taylor, L., & O'Connell, S. A. (1985). A Minsky Crisis. *The Quarterly Journal of Economics, 100*(Suplement), 871-885.

Tymoigne, É. (2010). Detecting Ponzi finance: An evolutionary approach. *Working Paper, Levy Economics Institute*(605).