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DIGITAL TRANSFORMATION OF THE MASTER'S COURSE "STRUCTURAL ANALYSIS OF SYSTEMS"

Abstract:

The paper discusses the digitalization of the teaching process for the Master's course "Structural Analysis of Systems." Over the course of one semester, students explore research issues, quantitative assessment, and analysis of the efficiency (including flexibility, reliability, and safety) of economic, technical, organizational, human-machine, and other types of complex systems. During the teaching process, the practical tasks of quantitative evaluation criteria of efficiency were solved manually, which required a lot of lecture time and less time was left for analysis. In order to increase the effectiveness of teaching, as a result of the implementation of the internal university grant project, a special web application and a new manual with practical exercises were created. Now the students perform the tasks of quantitative evaluation of the system efficiency criteria using the web application based on logical-probabilistic methods, models and algorithms.
The practical task in the web application is carried out in the following order: 1) Building a graphical scheme of system reliability and security models; 2) Determination of logical functions of the shortest paths for the successful functioning of the system and their representation in disjunctive normal form (DNF); 3) Input of logical functions received in the form of disjunctive normal form to the application as initial data; 4) Transformation of DNF into orthogonal disjunction normal form (ODNF) using the orthogonalization algorithm; 5) Replacing logical variables with probabilistic data in ODNF and probabilistic assessment of reliability/safety; 6) Ranking and constructing a diagram of probabilistic estimates of the shortest paths of the successful functioning of the system; 7) Calculation of the "weight" of each element in system reliability and structural analysis of system.
As a result, using the web application, the learning process became more efficient, productive and interesting. Students were given the opportunity to solve a large number of varied and complex tasks during the learning process. In addition, students successfully use the new teaching technology in the process of preparing semester abstracts, conference presentations, articles, master's and doctoral theses.

Keywords:

Complex systems, logical-probabilistic methods, orthogonalization algorithm, reliability, safety, Weight of element

JEL Classification: C60

Introduction

The effectiveness of economic, organizational, technical, human-machine and other types of complex systems functioning significantly depends on the structure of the system. Under the structure we mean the composition of elements in the system, their interrelationship, their location in the structure and the scheme of distribution of functions between the elements. To achieve high efficiency, it is important to design, create and manage complex systems based on the evaluation of efficiency indicators and structural analysis. One of the important efficiency criteria is the reliability of the system, which depends on the components of the structure. Evaluating the reliability of intricate systems involves a sophisticated computational procedure, making it prudent to conduct this assessment with the aid of a computer. Hence, the topic of digitizing the Master's course "Structural Analysis of Systems" within the "Informatics" and "IT Management" program at the University of Georgia's School of Science and Technology was a key item on the agenda. During the teaching process, the practical tasks of quantitative evaluation criteria of efficiency were solved manually, which required a lot of lecture time and less time was left for analysis. In order to increase the effectiveness of teaching, as a result of the implementation of the internal university grant project, a special web application and a new manual with practical exercises were created. Now the students perform the tasks of quantitative evaluation of the system efficiency criteria using the web application based on logical-probabilistic methods, models and algorithms.

In *Igor Ryabinin's* book "*Reliability and Safety of Structural Complex Systems"* (Rus., 2000) discusses in detail logical-probabilistic methods, models and algorithms. After describing the structure of the system by logical functions, they are converted into probabilistic representations and a polynomial is obtained to evaluate the criteria for the effectiveness of the system.

In the book "*System Reliability: Organizational Design of Highly Reliable Human-Machine Systems Based on Multifunctional Operators*" (Rus., 2017) discussed into structural analyses and evaluating of the system efficiency criteria for adaptable human-machine systems built on multifunctional elements. This topic is also incorporated into the lecture series on structural analysis of systems.

In the scientific paper Todinov M.T. (PDF) "Reliability analysis of complex systems based on the losses from failures" discusses issues into efficient models and algorithms for reliability cost analysis of complex repairable systems, which relate reliability and damage due to failure.

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Problem Formulation

Solving problems of system reliability assessment by logical-probabilistic methods is based on the description of structural schemes of systems with logical functions and their transformation into probabilistic functions using minimum intersection, orthogonalization and recurrent algorithms. Carrying out this task manually, without computer modeling, is quite time-consuming. Based on the experience of solving practical problems for small-dimensional systems in the University of Georgia when the number of ways of their operation was $N \leq 10$, obtaining the reliability calculation polynomial required a lot of lecture and exam time. For high-dimensional systems, when the number of functional paths was $N > 10$, it was impossible to perform the task manually.

This circumstance significantly hindered the use of structural analysis and research methods of large-scale complex systems in master's and doctoral theses. Accordingly, made it difficult to achieve the results of the structural analysis of systems course.

In order to eliminate the problems, it became relevant and important to create a computer software for structural analysis of systems, research and evaluation of efficiency indicators. Accordingly, in 2023, the internal grants competition of the University of Georgia funded the digitization project of the "Structural Analysis of Systems" course, within the framework of which, in addition to software development, the teaching methodology was changed, the number and complexity of practical examples were increased, and the course manual and syllabus were changed taking into account the modern requirements of teaching.

Orthogonalization Algorithm

The web application for structural analysis of systems created by us was based on one of the logical-probabilistic methods - the orthogonalization algorithm (Ryabinin I.A., 2000; Yavuz Can and Georg Fischer, 2015; Weiguo Gao, Yingzhou Li, Bichen Lu, 2020, Andreas Stathopoulos, Kesheng Wu, 2000).

Using this algorithm, it was possible to transform the logical function of the system operability condition described in disjunction normal form (DNF) into orthogonal disjunction normal form (ODNF). As is known, in a logical function written by ODNF, each of its members is pairwise orthogonal (their logical product is equal to 0), which allows replacing the logical function with a probabilistic representation. Our goal was to use an algorithm in the application that would allow us to obtain an ODNF with a minimum number of members. For this, we used two theorems, which we offer without proof (the proof of both theorems is given in the book of Igor Ryabinin, 2000).

Theorem 1. The negation C_i' of an elementary conjunction of $C_i = x_1 x_2 \ldots x_r$ r rank, which does not contain the negations of logical elements, is equivalent to the following disjunction

$$C_i' = x_1' \vee x_1 x_2' \vee x_1 x_2 \ldots x_{r-1} x_r' \quad (1)$$

Its members are pairwise orthogonal.

Theorem 2. Boolean logical function $y(x_1, x_2, \ldots, x_m)$ represented as DNF

$$y(x_1, \ldots, x_m) = C_1 \vee C_2 \vee \ldots \vee C_n,$$

is equivalent to the function

$$y(x_1, \ldots, x_m) = C_1 \vee C_1' C_2 \vee C_1' C_2' C_3 \vee \ldots \quad (2)$$

The correctness of given transformations is easily proven by De Morgan's formulas and Shannon's decomposition theorem. If instead of each representation $C_i'(i<n)$ we insert its value obtained in the form of (1), we get the ODNF of the Boolean function $y(x_1,...,x_m)$.

The following steps should be performed to convert the Boolean function $y(x_1,...,x_m)$ to ODNF:

1. Boolean function $y(x_1,...,x_m)$, which describes the structural scheme of system reliability, must first be converted into DNF.
2. Renumber the members of DNF from 1 to n for $(n<2m)$ so as to assign lower numbers to members of lower rank.
3. Using the formula (2), get the ODNF of the function $y(x_1,...,x_m)$.

To reduce the number of operations, let's set the orthogonal members of DNF to 0 in the given conjunctions.

As a result of the transformation of the logical representation of the system operability condition into ODNF, it is possible to replace the logical elements with element-free probabilities and to replace logical operations with arithmetic operations. As a result, we get the formula for the probability of the system working flawlessly (system reliability):

$$P\{y(x_1,...,x_m)=1\}=\sum_{i=1}^{s}P(R_i) \quad (3)$$

where the orthogonal terms of the function are written as ODNF.

In order to demonstrate the orthogonalization algorithm, consider an example in which the system is composed of multi-functional elements (MFE). Building a structural reliability assessment model of a system composed of MFEs by hand is quite a difficult task, since the structure of such a system cannot be reduced to a sequence, parallel or sequence-parallel scheme. Accordingly, such systems belong to the class of complex structure systems.

MFE is an element with functional redundancy, which has the ability to perform any one of the functions assigned to the system at any time from the set of its functional resources $F_a=\{f_e/e\in[1,k]\}$, $k>1$.

Partial failure of MFE is a case when an element loses the ability to perform the function assigned to it, but retains the ability to perform other functions assigned to the system based on its functional capabilities.

The properties of multifunctional elements allow us to create reconfigurable systems with a reconfigurable structure based on them, which have the ability to rearrange the structure and continue successful operation in case of partial failure of any of the constituent elements.

Thus, in the case of partial failure of the MFE, when the failure occurs only with respect to the function $f_i\in F_a$ in the process of execution, the MFE is switched to another $f_j\in F_a$ of the set $F_a=\{f_e / e\in[1,k]\}$ in the allowed time interval to perform the function, while the missing function f_i starts to be performed by another MFE of the given system, which previously performed the function f_j (ie, there is a mutual exchange of elements).

Consider a specific example of using the orthogonalization algorithm. Let's get $n=m=k_i=3$, of the $i\in[1,3]$ system reliability evaluation polynomial, where n is the number of elements, m is the number of functions assigned to the system, and k_i is the i^{th} number of functional resources of the multifunctional element.

In the matrix of functional resources $B(3\times3)$ of the $n=m=k_i=3$ system, let's introduce the designations of the structural elements:

$a_1(f_1)=x_1,\ a_1(f_2)=x_2,\ a_1(f_3)=x_3,\ a_2(f_1)=x_4,\ a_2(f_2)=x_5,\ a_2(f_3)=x_6,\ a_3(f_1)=x_7,\ a_3(f_2)=x_8,$
$a_3(f_3)=x_9.$

Since we actually have three three-functional elements in the system – $\{a_1, a_2, a_3\}$, logical variables $\{x_1, x_2, \dots, x_9\}$ represent the conditional elements for evaluating the probability of the system's reliable operation (system reliability), which correspond to the functional capabilities (functional resources) of the elements. With the introduced notations, the system functional resources matrix $B(3\times3)$ will be written as follows

$$B(3\times3)=\begin{matrix} x_1 & x_4 & x_7 \\ x_2 & x_5 & x_8 \\ x_3 & x_6 & x_9 \end{matrix}$$

Consider the case when, for the successful operation of the system, it is necessary that all three functions assigned to the system $\{f_1, f_2, f_3\}$ are performed simultaneously, and any element performs only one function at any moment in time. Taking into account the mentioned condition, the shortest paths for the successful functioning of the system will be recorded by the following DNF:

$$\begin{aligned} C_1 &= x_1x_5x_9, \\ C_2 &= x_1x_6x_8, \\ C_3 &= x_2x_4x_9, \\ C_4 &= x_2x_6x_7, \\ C_5 &= x_3x_4x_8, \\ C_6 &= x_3x_5x_7. \end{aligned} \tag{4}$$

The system operability condition is written by a logical function, which is a disjunction of the shortest paths:

$$F_A\{x_1,\dots,x_9\} = C_1 \vee C_2 \vee \dots \vee C_6. \tag{5}$$

Using the formula of the orthogonalization algorithm (2), we transform the function $F_A(x_1, \dots, x_9)$ into orthogonal disjunctive normal form (ODNF), which we write in matrix form:

$$F_A = C_1 \vee \dots \vee C_6 = \begin{matrix} C_1 \\ C_1'C_2 \\ C_1'C_2'C_3 \\ C_1'C_2'C_3'C_4 \\ C_1'C_2'C_3'C_4'C_5 \\ C_1'C_2'C_3'C_4'C_5'C_6 \end{matrix} \tag{6}$$

In (6), let's express the negations of C_i conjunctions of Theorem 1 in the form of such disjunctions, the members of which are pairwise orthogonal:

$$C_1' = \begin{matrix} x_1' \\ x_1x_2' \\ x_1x_5x_9' \end{matrix}\ \dots,\ C_6' = \begin{matrix} x_3' \\ x_3x_5' \\ x_3x_5x_7' \end{matrix} \tag{7}$$

If we insert the logical expressions obtained in (7) into (6) and perform logical operations, we get ODNF. In the obtained logical representation, we can replace the logical elements with the probability of non-failure of the elements, and the logical operations with mathematical operations. (3) using the formula and assuming equal probabilities of the functional resources of the elements, we get

$$P\{F_A = 1) = 6p^3 - 9p^5 - 6p^6 + 18p^7 - 9p^8 + p^9.$$

Obviously, when the structural components of the system composed of MFE are *n, m,* $k_i > 3$, the number of ways of successful functioning of the system $i \in [1, n]$ increases factorially, and the reliability assessment of such systems can be carried out only by use of a computer.

The logical-probabilistic model of system performance and the corresponding reliability function allow us to evaluate the contribution and role of each element in ensuring system reliability. Determining the value and weight of an individual element is necessary to solve such tasks as detecting failure, developing an optimal reservation strategy, determining the demand for reliability characteristics, etc. In the web application created by us, it is possible to estimate the weight of elements or their functional resources, which is also based on the orthogonalization algorithm.

The "weight" of x_i element in the reliability of system, is defined as

$$\gamma_{x_i} = \sum_{l=1}^{s} 2^{-(r_l - 1)} - \sum_{j=1}^{q} 2^{-(r_j - 1)},$$

where s, r_l is the number of orthogonal conjunctions containing x_i elements and their ranks; The number of orthogonal conjunctions containing the element q, r_j - x_i' (the negation of the logical element x_i) and the corresponding ranks are. It should be noted that the "weight" of an individual element of the system depends on the location of the element in the system structure and does not depend on the probabilities of the element's work without failure (Ryabinin I.A., Parfenov Yu.M. 1978)

In the web application developed by us, the algorithms described above were implemented using the following systems:

Back-end is written on LARAVEL PHP Framework, and Front-end side is developed using HTML, CSS (Bootstrap), JavaScript (Vue.JS). As for the logical scheme of the system structure, it is organized using the GOJS JavaScript library. Graphs are built using Chart.js.

The web application is distinguished by its user-friendly interface and ease of use. At the initial stage, the user draws a graphic logical scheme of the system structure on the computer screen, according to which the program forms the conditions for the system to work. After the user enters the probabilities of faultlessness of the elements, the calculation of the probabilities of the shortest paths of the successful functioning of the system is performed, the results are ranked and displayed on the screen in the form of numerical data and graphics. Using the orthogonalization algorithm, the Boolean function of the system operability condition is transformed into ODNF. By inserting the failure probabilities of the elements into the ODNF representation, the system reliability, structural perfection factor and element weights are calculated. The analysis of the obtained results allows the user to identify the weak points of the system structure and to make the right decision on the selection of the best option of the structure.

4 Conclusion

1. For the purpose of digital transformation of the master's course "Structural Analysis of Systems", a web application was created based on logical-probabilistic methods, models

and algorithms. Logical-probabilistic methods are universal and with their help it is possible to model the functioning processes and quantify the efficiency indicators of various classes of complex systems.

2. Using the web application it is possible to quantify the efficiency indicators - flexibility, reliability, security of complex systems. Using the results of quantitative assessment, it is possible to design systems of high efficiency and optimal structure.
3. Digital transformation of the “Structural Analysis of Systems” course allowed us to reduce the time required to complete tasks on quantitative assessment of the performance indicators of complex systems and increase the time for research and structural analysis of systems. Students successfully can use new learning technology when preparing semester works, articles, master's and doctoral dissertations.

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